Supplementary Appendix to

Double Filter Instrumental Variable Estimation of Panel Data Models with Weakly Exogenous Variables

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Mathematical appendix

S.1 Derivation of $\mathbf{F}_T$

We derive the form of $\mathbf{F}_T^{\iota}$ and $\mathbf{F}_T^{\tau}$. Although a brief derivation of $\mathbf{F}_T^{\iota}$ is given in Arellano (2003), a complete derivation is not provided. Hence, we fill that gap. Let us define the following $T_1 \times T$ matrix that takes the first difference:

$$\mathbf{D}_T = \begin{bmatrix} -1 & 1 & 0 & & & 0 \\ 0 & -1 & 1 & & & \\ & & & \ddots & \ddots & \\ 0 & & & & -1 & 1 \end{bmatrix}.$$

Multiplying $\mathbf{D}_T$ by (1) and noting that $\mathbf{D}_T \boldsymbol{\iota}_T = \mathbf{0}$, we have

$$\mathbf{D}_T \mathbf{y}_i = \mathbf{D}_T \mathbf{W}_i + \mathbf{D}_T \mathbf{v}_i$$

where it is simply assumed that $Var(\mathbf{v}_i) = \sigma_v^2 \mathbf{I}_T$. Since $Var(\mathbf{D}_T \mathbf{v}_i) = \sigma_v^2 \mathbf{D}_T \mathbf{D}_T'$, the transformed error is serially correlated. To correct for the serial correlation, we use the following transformation matrix, which is a GLS transformation:

$$\mathbf{F}_T^{\iota} = \left(\mathbf{D}_T \mathbf{D}_T'\right)^{-1/2} \mathbf{D}_T,$$

where $\left(\mathbf{D}_T \mathbf{D}_T'\right)^{-1/2}$ is the *upper* triangular Cholesky factorization of $\left(\mathbf{D}_T \mathbf{D}_T'\right)^{-1}$ with[S.1]

$$\underset{(T_1 \times T_1)}{\mathbf{D}_T \mathbf{D}_T'} = \begin{bmatrix} 2 & -1 & 0 & & & 0 \\ -1 & 2 & -1 & & & \\ 0 & -1 & 2 & -1 & & \\ & & \ddots & \ddots & \ddots & 0 \\ & & & -1 & 2 & -1 \\ 0 & & & 0 & -1 & 2 \end{bmatrix}.$$

To compute $\left(\mathbf{D}_T \mathbf{D}_T'\right)^{-1/2}$, we need to derive the inverse matrix $\mathbf{H}^{\iota} = \left(\mathbf{D}_T \mathbf{D}_T'\right)^{-1} = \{h_{st}^{\iota}\}$.[S.2] Using the results by El-Mikkawy and Karawia (2006), we have

$$h_{st}^{\iota} = \begin{cases} \frac{T_1}{T_1+1} & \text{if } s = t = 1 \text{ or } s = t = T_1 \\ \frac{s(T_1-s+1)}{T_1+1} & \text{if } s = t < T_1 \\ \frac{s(T_1-t+1)}{T_1+1} & \text{if } s < t \\ \frac{t(T_1-s+1)}{T_1+1} & \text{if } s > t \end{cases}$$

Next, we need to compute the Cholesky factorization to $\mathbf{H}^{\iota}$. For a $K \times K$ matrix $\mathbf{A} = \{a_{ij}\}$, its Cholesky factorization is given by

$$\mathbf{A} = \mathbf{L}\mathbf{L}'$$

where $\mathbf{L} = (\ell_{ij})$ is the lower triangular matrix. Then using ℓ_{ij}, we can write the elements of $\mathbf{A}$ as follows:

$$a_{11} = \ell_{11}^2,$$

[S.1] A matrix with this structure is called *tridiagonal* matrix.

[S.2] Arellano (2003) does not provide the details how the upper triangular Cholesky factorization can be computed.

$$\begin{aligned}
a_{21} &= \ell_{21}\ell_{11}, \qquad a_{22} = \ell_{21}^2 + \ell_{22}^2, \\
a_{31} &= \ell_{31}\ell_{11}, \qquad a_{32} = \ell_{31}\ell_{21} + \ell_{32}\ell_{22}, \qquad \ell_{33} = \ell_{31}^2 + \ell_{32}^2 + \ell_{33}^2, \\
&\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad\qquad\qquad \ddots \\
a_{K1} &= \ell_{K1}\ell_{11}, \qquad a_{K2} = \ell_{K1}\ell_{21} + \ell_{K2}\ell_{22}, \qquad \cdots \qquad , a_{KK} = \ell_{K1}^2 + \cdots + \ell_{KK}^2.
\end{aligned}$$

ℓ_{ij} can be solved sequentially as follows:

$$\begin{aligned}
\ell_{11} &= \sqrt{a_{11}}, \\
\ell_{21} &= a_{21}/\ell_{11}, \qquad \ell_{22} = \sqrt{a_{22} - \ell_{21}^2}, \\
\ell_{31} &= a_{31}/\ell_{11}, \qquad \ell_{32} = (a_{32} - \ell_{31}\ell_{21})/\ell_{22}, \qquad \ell_{33} = \sqrt{a_{33} - \ell_{31}^2 - \ell_{32}^2}, \\
&\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad\qquad\qquad \ddots \\
\ell_{K1} &= a_{K1}/\ell_{11}, \qquad \ell_{K2} = (a_{K2} - \ell_{K1}\ell_{21})/a_{K2}, \cdots \; \ell_{KK} = \sqrt{a_{KK} - \ell_{K1}^2 - \cdots - \ell_{K,K-1}^2}.
\end{aligned}$$

The explicit form of $\mathbf{F}_T^\iota$ is obtained by letting $\mathbf{A} = \mathbf{H}^\iota$.

Next, we consider a model with individual effects and heterogeneous time trends given by (12). To remove both η_i and λ_i from the model, we take second differences. In terms of a model in matrix, this corresponds to multiplying by $\mathbf{D}_{T_1}\mathbf{D}_T$, $(T_2 \times T)$, we have

$$\mathbf{D}_{T_1}\mathbf{D}_T\mathbf{y}_i = \mathbf{D}_{T_1}\mathbf{D}_T\mathbf{W}_i\boldsymbol{\delta} + \mathbf{D}_{T_1}\mathbf{D}_T\mathbf{v}_i.$$

Since the transformed error is serially correlated, we consider the following GLS-type transformation matrix:

$$\mathbf{F}_T^\tau = \left(\mathbf{D}_{T_1}\mathbf{D}_T\mathbf{D}_T'\mathbf{D}_{T_1}'\right)^{-1/2}\mathbf{D}_{T_1}\mathbf{D}_T,$$

where $\left(\mathbf{D}_{T_1}\mathbf{D}_T\mathbf{D}_T'\mathbf{D}_{T_1}'\right)^{-1/2}$ is the *upper* triangular Cholesky factorization of $\left(\mathbf{D}_{T_1}\mathbf{D}_T\mathbf{D}_T'\mathbf{D}_{T_1}'\right)^{-1}$. To compute $\mathbf{F}_T^\tau$, we need to derive the inverse matrix $\mathbf{H}^\tau = \left(\mathbf{D}_{T_1}\mathbf{D}_T\mathbf{D}_T'\mathbf{D}_{T_1}'\right)^{-1} = \{h_{st}^\tau\}$ with[S.3]

$$\underset{(T_2\times T_2)}{\mathbf{D}_{T_1}\mathbf{D}_T\mathbf{D}_T'\mathbf{D}_{T_1}'} = \begin{bmatrix} 6 & -4 & 1 & & & 0 \\ -4 & 6 & -4 & & & \\ 1 & -4 & 6 & -4 & & \\ & & \ddots & \ddots & \ddots & 1 \\ & & & -4 & 6 & -4 \\ 0 & & & 1 & -4 & 6 \end{bmatrix}.$$

Using the results by Dow (2003), we have[S.4]

$$h_{st}^\tau = \begin{cases} a_{t0}s^3 + a_{t1}s^2 + a_{t2}s, & s \le t+1 \\ b_{t0}s^3 + b_{t1}s^2 + b_{t2}s + b_{t3}, & s \ge t+1 \end{cases}$$

where

$$\begin{aligned}
a_{t0} &= -(3 + 2t + T_2)d_t/c, \qquad a_{t1} = 3t(1 + T_2)d_t/c, \qquad a_{t2} = (3 + 5t + T_2 + 3tT_2)d_t/c, \\
b_{t0} &= (5 - 2t + 3T_2)e_t/c, \qquad b_{t1} = -3(1 + T_2)(4 - t + 2T_2)e_t/c, \\
b_{t2} &= (1 + 5t + 12T_2 + 3tT_2 + 12T_2^2 + 3T_2^3)e_t/c, \qquad b_{t3} = (1 - t)e_t/6, \\
d_t &= (T_2 - t + 1)(T_2 - t + 2), \qquad e_t = t(t + 1), \qquad c = 6(T_2 + 1)(T_2 + 2)(T_2 + 3).
\end{aligned}$$

Using these results and applying the algorithm of Cholesky factorization introduced above where $\mathbf{A} = \mathbf{H}^\tau$, after a lengthy calculation, we obtain the explicit expression of $\mathbf{F}_T^\tau$ as in (13).

[S.3] A matrix with this structure is called *pentadiagonal* matrix.

[S.4] See also Chen (2013) for an alternative expression.

S.2 Derivation of asymptotic variances (36) and (37)

Let us define $\mathbf{y}_{i,-1} = (y_{i0}, ..., y_{i,T-1})'$ and $Var(\mathbf{y}_{i,-1}) = \mathbf{V}_T = \sigma_\mu^2 \boldsymbol{\iota}_T \boldsymbol{\iota}_T' + \sigma_v^2 \boldsymbol{\Phi}_T$ where $\sigma_\mu^2 = \sigma_\eta^2/(1-\alpha)^2$ and $\boldsymbol{\Phi}_T = \{\phi_{st}\} = \alpha^{|s-t|}/(1-\alpha^2)$. Also, let $\mathbf{f}_t'$ be the tth row of $\mathbf{F}_T^\iota$, $\mathbf{L}_{(s:t)}$ be the sth to tth rows of $\mathbf{I}_T$ and $\mathbf{B}_{(s:t)}$ be the sth to tth rows of $\mathbf{B}_T^\iota$. Then, we obtain $y_{i,t-1}^* = \mathbf{f}_t' \mathbf{y}_{i,-1}$ and

$$\mathbf{z}_{it}^L = (y_{i,t-\ell}, ..., y_{i,t-1})' = \mathbf{L}_{(t-\ell+1:t)} \mathbf{y}_{i,-1}, \qquad \mathbf{z}_{it}^B = (y_{i,t-\ell}^{**}, ..., y_{i,t-1}^{**})' = \mathbf{B}_{(t-\ell+1:t)} \mathbf{y}_{i,-1}.$$

Using this, we have

$$\begin{aligned} E\left(\mathbf{z}_{it}^{L(\ell)} y_{i,t-1}^*\right) &= \mathbf{L}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{f}_t, \qquad E\left(\mathbf{z}_{it}^B y_{i,t-1}^*\right) = \mathbf{B}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{f}_t, \\ E\left(\mathbf{z}_{it}^{L(\ell)} \mathbf{z}_{it}^{L\prime}\right) &= \mathbf{L}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{L}_{(t-\ell+1:t)}', \qquad E\left(\mathbf{z}_{it}^{B(\ell)} \mathbf{z}_{it}^{B\prime}\right) = \mathbf{B}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{B}_{(t-\ell+1:t)}'. \end{aligned}$$

Hence, (36) and (37) can be written as

$$Avar\left(\widehat{\alpha}_{GMM}^L\right) = \sigma_v^2 \left[\sum_{t=1}^{T-1} \mathbf{f}_t' \mathbf{V}_T \mathbf{L}_{(t-\ell+1:t)}' \left[\mathbf{L}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{L}_{(t-\ell+1:t)}'\right]^{-1} \mathbf{L}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{f}_t\right]^{-1} \quad \text{(S.1)}$$

$$Avar\left(\widehat{\alpha}_{GMM}^B\right) = \sigma_v^2 \left[\sum_{t=2}^{T-1} \mathbf{f}_t' \mathbf{V}_T \mathbf{B}_{(t-\ell+1:t)}' \left[\mathbf{B}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{B}_{(t-\ell+1:t)}'\right]^{-1} \mathbf{B}_{(t-\ell+1:t)} \mathbf{V}_T \mathbf{f}_t\right]^{-1} \quad \text{(S.2)}$$

Figure 1 is described based on (S.1) and (S.2) numerically without deriving the explicit form of expectations such as $E\left(\mathbf{z}_{it}^{L(\ell)} y_{i,t-1}^*\right)$.

S.3 Proof of Theorem 1

We derive the explicit formula for asymptotic variances of IV and GMM estimators. First, consider $Avar\left(\widehat{\alpha}_{GMM}^{L(t)}\right)$. Note that under Assumption 1, $y_{i,t-1}$ can be written as

$$y_{i,t-1} = \frac{\eta_i}{1-\alpha} + \xi_{i,t-1} \quad \text{(S.3)}$$

where $\xi_{i,t-1} = \sum_{j=0}^\infty \alpha^j v_{i,t-1-j}$. Also, from (A43) of Alvarez and Arellano (2003), we have

$$y_{i,t-1}^* = \psi_t \xi_{i,t-1} - c_t \left(\frac{\phi_{T-t} v_{it} + \cdots + \phi_1 v_{i,T-1}}{T-t}\right). \quad \text{(S.4)}$$

where ϕ is defined in (45). Using (S.3) and (S.4), and under Assumption 1, we have

$$\begin{aligned} E(\mathbf{z}_{it}^L y_{i,t-1}^*) &= \psi_t \left(\frac{\sigma_v^2}{1-\alpha^2}\right) (\alpha^{\ell-1}, \cdots, 1)', & \text{(S.5)} \\ \left[E\left(\mathbf{z}_{it}^L \mathbf{z}_{it}^{L\prime}\right)\right]^{-1} &= \mathbf{V}_\ell^{-1} = \frac{1}{\sigma_v^2} \left[(\sqrt{\lambda} \boldsymbol{\iota}_\ell)(\sqrt{\lambda} \boldsymbol{\iota}_\ell)' + \boldsymbol{\Phi}_\ell\right]^{-1} & \text{(S.6)} \end{aligned}$$

where $\lambda = \sigma_\mu^2/\sigma_v^2 = r/(1-\alpha)^2$, $\boldsymbol{\iota}_\ell$ is an ℓ dimensional column vector of ones, and $\mathbf{V}_\ell$ is the upper-left $\ell \times \ell$ matrix of $\mathbf{V}_T$. The explicit expression of (S.6) is obtained as follows. By using the Sherman-Morrison-Woodbury inversion formula

$$[\mathbf{A} + \mathbf{b}\mathbf{b}']^{-1} = \mathbf{A}^{-1} - \left[\frac{1}{1 + \mathbf{b}'\mathbf{A}^{-1}\mathbf{b}}\right] \mathbf{A}^{-1} \mathbf{b}\mathbf{b}' \mathbf{A}^{-1}$$

and the decomposition of $\mathbf{V}_\ell^{-1}$ [S.5]

$$\mathbf{V}_\ell^{-1} = \mathbf{C}'\mathbf{C}$$

where

$$\mathbf{C} = \begin{bmatrix} \sqrt{1-\alpha^2} & 0 & 0 & \cdots & 0 & 0 \\ -\alpha & 1 & 0 & \cdots & 0 & 0 \\ 0 & -\alpha & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\alpha & 1 \end{bmatrix}$$

we obtain

$$\left[E(\mathbf{z}_{it}^L \mathbf{z}_{it}^{L\prime})\right]^{-1} = \sigma_v^{-2}\left[\mathbf{C}'\mathbf{C} - \frac{\lambda}{1+\lambda\boldsymbol{\iota}_\ell'\mathbf{C}'\mathbf{C}\boldsymbol{\iota}_\ell}\mathbf{C}'\mathbf{C}\boldsymbol{\iota}_\ell\boldsymbol{\iota}_\ell'\mathbf{C}'\mathbf{C}\right]. \tag{S.7}$$

By substituting (S.5) and (S.7) into (36), we obtain (38). The results (39) and (40) are obtained from (38). Next, we consider $Var(\widehat{\alpha}_{GMM}^{B(1)})$. First, note that $y_{i,t-1}^{**}$ can be written as

$$y_{i,t-1}^{**} = c_{T-t+1}^{\iota}\left[\xi_{i,t-1} - \frac{\xi_{i,t-2}+\cdots+\xi_{i0}}{t-1}\right]. \tag{S.8}$$

Then, using (S.4) and (S.8), we obtain

$$E\left(y_{i,t-1}^{**}y_{i,t-1}^{*}\right) = \left(\frac{\sigma_v^2}{1-\alpha^2}\right)\psi_t c_{T-t+1}^{\iota}\left(1-\frac{\phi_{t-1}}{t-1}\right). \tag{S.9}$$

Also, from (S.3), we obtain

$$\begin{aligned} E[(y_{i,t-1}^{**})^2] &= c_{T-t+1}^{\iota 2}E\left[\xi_{i,t-1} - \frac{1}{t-1}(\xi_{i,0}+\cdots+w_{i,t-2})\right]^2 \\ &= c_{T-t+1}^{\iota 2}\left[\frac{\sigma_v^2}{1-\alpha^2}\left(1-\frac{2\alpha\phi_{t-1}}{t-1}\right) + \frac{1}{(t-1)^2}E(\xi_{i0}+\cdots+\xi_{i,t-1})^2\right]. \end{aligned} \tag{S.10}$$

Using the result of (A8) in Alvarez and Arellano (2003), we have

$$E(\xi_{i0}+\cdots+\xi_{i,t-1})^2 = \frac{\sigma_v^2}{1-\alpha^2}\left[\frac{(t-1)(1+\alpha)}{1-\alpha} - \frac{2\alpha(1-\alpha^{t-1})}{(1-\alpha)^2}\right]. \tag{S.11}$$

By substituting this into (S.10), we get

$$E[(y_{i,t-1}^{**})^2] = \left(\frac{\sigma_v^2}{1-\alpha^2}\right)c_{T-t+1}^{\iota 2}A_t \tag{S.12}$$

where A_t is defined in (46). The result (42) is obtained by substituting (S.9) and (S.12) into (36).

Next, we derive the asymptotic variances of $\widehat{\alpha}_{IV}^{L}$ and $\widehat{\alpha}_{IV}^{B}$. Using (S.3) and (S.4) and the fact that v_{it}^* is serially uncorrelated, we obtain

$$\begin{aligned} \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T-1}E\left(y_{i,t-1}y_{i,t-1}^*\right) &= \left(\frac{\sigma_v^2}{1-\alpha^2}\right)\sum_{t=1}^{T-1}\psi_t, \\ Var\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T-1}y_{i,t-1}v_{it}^*\right) &= Var\left(\sum_{t=1}^{T-1}y_{i,t-1}v_{it}^*\right) = \sigma_v^2\sum_{t=1}^{T-1}E\left(y_{i,t-1}^2\right) = \sigma_v^2\sigma_\mu^2 + \frac{\sigma_v^4}{1-\alpha^2}. \end{aligned}$$

[S.5] See Amemiya (1985, p.164) and Hamilton (1994, p.120).

Using these, we obtain the asymptotic variance of $\widehat{\alpha}_{IV}^{L}$ as in (41). The asymptotic variance of $\widehat{\alpha}_{IV}^{B}$ can be derived similarly. Using (S.9) and (S.12), and under Assumption 1, we obtain

$$\begin{aligned}
\frac{1}{N}\sum_{i=1}^{N}\sum_{t=2}^{T-1} E\left(y_{i,t-1}^{**}y_{i,t-1}^{*}\right) &= \left(\frac{\sigma_v^2}{1-\alpha^2}\right)\sum_{t=2}^{T-1}\psi_t c_{T-t+1}^{\iota}\left(1-\frac{\phi_{t-1}}{t-1}\right), \\
Var\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sum_{t=2}^{T-1} y_{i,t-1}^{**}v_{it}^{*}\right) &= Var\left(\sum_{t=2}^{T-1} y_{i,t-1}^{**}v_{it}^{*}\right) = \sigma_v^2\sum_{t=2}^{T-1}E\left[(y_{i,t-1}^{**})^2\right] = \left(\frac{\sigma_v^4}{1-\alpha^2}\right)\sum_{t=2}^{T-1}c_{T-t+1}^{\iota 2}A_t.
\end{aligned}$$

From these, the asymptotic variance of $\widehat{\alpha}_{IV}^{L}$ is obtained as (43).

S.4 Proof of Lemma 1 for models with time trend

We provide the proof of Lemma 1 for models with time trend.[S.6] First, we decompose $T_d \times T$ matrix $\mathbf{F}_T$ as

$$\mathbf{F}_T = \begin{bmatrix} \mathbf{F}_{11} & \mathbf{F}_{12} & \mathbf{F}_{13} \\ \mathbf{0}_{T_{2d}\times d} & \mathbf{F}_{22} & \mathbf{F}_{23} \end{bmatrix} = \{f_{st}\}, (s=1,...,T_d; t=1,...,T)$$

where $\mathbf{F}_{11}$ is $d\times d$, $\mathbf{F}_{12}$ is $d\times T_{2d}$, $\mathbf{F}_{13}$ is $d\times d$, $\mathbf{F}_{22}$ is $T_{2d}\times T_{2d}$, and $\mathbf{F}_{23}$ is $T_{2d}\times d$. Note that $\mathbf{B}_T$ and $\mathbf{F}_T$ have the following relationship

$$\mathbf{B}_T = \mathcal{I}_{T_d}\mathbf{F}_T\mathcal{I}_T \tag{S.13}$$

where

$$\mathcal{I}_T = \begin{bmatrix} 0 & & 1 \\ & \iddots & \\ 1 & & 0 \end{bmatrix}$$

and $\mathcal{I}_T^2 = \mathcal{I}_T'\mathcal{I}_T = \mathbf{I}_T$. Furthermore, using (47), $\mathbf{W}_i$ can be written as

$$\mathbf{W}_i = \boldsymbol{\iota}_T\boldsymbol{\mu}_i' + \boldsymbol{\tau}_T\boldsymbol{\kappa}_i' + \boldsymbol{\Xi}_i = \mathbf{C}_T\boldsymbol{\Psi}_i + \boldsymbol{\Xi}_i$$

where $\boldsymbol{\Xi}_i = (\boldsymbol{\xi}_{i1}',...,\boldsymbol{\xi}_{iT}')'$ and $\boldsymbol{\Psi}_i = (\boldsymbol{\mu}_i, \boldsymbol{\kappa}_i)'$.

Proof of (a): Note the following decomposition:

$$\frac{1}{NT}\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{K}_T'\mathbf{B}_{T_d}'\mathbf{F}_{T_d}\mathbf{L}_T\mathbf{W}_i = \frac{1}{NT}\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{Q}_T\mathbf{W}_i + \frac{1}{NT}\sum_{i=1}^{N}\mathbf{W}_i'\left(\mathbf{K}_T'\mathbf{B}_{T_d}'\mathbf{F}_{T_d}\mathbf{L}_T - \mathbf{Q}_T\right)\mathbf{W}_i. \tag{S.14}$$

Using $\mathbf{F}_{T_d}\mathbf{L}_T\mathbf{C}_T = \mathbf{B}_{T_d}\mathbf{K}_T\mathbf{C}_T = \mathbf{Q}_T\mathbf{C}_T = \mathbf{0}$ and (20), the second term of (S.14) can be further decomposed as

$$\frac{1}{NT}\sum_{i=1}^{N}\mathbf{W}_i'\left(\mathbf{K}_T'\mathbf{B}_{T_d}'\mathbf{F}_{T_d}\mathbf{L}_T - \mathbf{Q}_T\right)\mathbf{W}_i = \frac{1}{NT}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\left(\mathbf{K}_T'\mathbf{B}_{T_d}'\mathbf{F}_{T_d}\mathbf{L}_T - \mathbf{I}_T\right)\boldsymbol{\Xi}_i + \frac{1}{NT}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{R}_T\boldsymbol{\Xi}_i. \tag{S.15}$$

[S.6] Although some equations are already introduced in the main body, we provide them for completeness.

To consider the first term of right-hand side of (S.15), we derive the explicit form of $\mathbf{A}_T = \mathbf{K}'_T\mathbf{B}'_{T_d}\mathbf{F}_{T_d}\mathbf{L}_T - \mathbf{I}_T$. Using (S.13), $\mathbf{F}_{T_d} = \mathbf{L}_{T_d}\mathbf{F}_T\mathbf{L}'_T$ and

$$
\begin{aligned}
\mathbf{K}'_T\mathcal{I}_{T_d}\mathbf{L}_T &= \begin{bmatrix} \mathbf{0}_{d\times d} & \mathbf{0}_{d\times T_{2d}} & \mathcal{I}_d \\ \mathbf{0}_{T_{2d}\times d} & \mathcal{I}_{T_{2d}} & \mathbf{0}_{T_{2d}\times d} \\ \mathbf{0}_{d\times d} & \mathbf{0}_{d\times T_{2d}} & \mathbf{0}_{d\times d} \end{bmatrix}, \quad \mathbf{L}'_{T_d}\mathcal{I}_{T_{2d}}\mathbf{L}_{T_d} = \begin{bmatrix} \mathbf{0}_{d\times d} & \mathbf{0}_{d\times T_{2d}} \\ \mathbf{0}_{T_{2d}\times d} & \mathcal{I}_{T_{2d}} \end{bmatrix}, \\
\mathbf{L}'_T\mathbf{L}_T &= \begin{bmatrix} \mathbf{0}_{d\times d} & \mathbf{0}_{d\times T_{2d}} & \mathbf{0}_{d\times d} \\ \mathbf{0}_{T_{2d}\times d} & \mathbf{I}_{T_{2d}} & \mathbf{0}_{T_{2d}\times d} \\ \mathbf{0}_{d\times d} & \mathbf{0}_{d\times T_{2d}} & \mathbf{I}_d \end{bmatrix},
\end{aligned}
$$

we have

$$
\begin{aligned}
\mathbf{A}_T &= \mathbf{K}'_T\mathbf{B}'_{T_d}\mathbf{L}_{T_d}\mathbf{F}_T\mathbf{L}'_T\mathbf{L}_T - \mathbf{I}_T = \mathbf{K}'_T\mathcal{I}_{T_d}\mathbf{F}'_{T_d}\mathcal{I}_{T_{2d}}\mathbf{L}_{T_d}\mathbf{F}_T\mathbf{L}'_T\mathbf{L}_T - \mathbf{I}_T \\
&= \left(\mathbf{K}'_T\mathcal{I}_{T_d}\mathbf{L}_T\right)\mathbf{F}'_T\left(\mathbf{L}'_{T_d}\mathcal{I}_{T_{2d}}\mathbf{L}_{T_d}\right)\mathbf{F}_T\left(\mathbf{L}'_T\mathbf{L}_T\right) - \mathbf{I}_T \\
&= \begin{bmatrix} -\mathbf{I}_d & \mathcal{I}_d\mathbf{F}'_{23}\mathcal{I}_{T_{2d}}\mathbf{F}_{22} & \mathcal{I}_d\mathbf{F}'_{23}\mathcal{I}_{T_{2d}}\mathbf{F}_{23} \\ \mathbf{0}_{T_{2d}\times d} & \mathcal{I}_{T_{2d}}\mathbf{F}'_{22}\mathcal{I}_{T_{2d}}\mathbf{F}_{22} - \mathbf{I}_{T_{2d}} & \mathcal{I}_{T_{2d}}\mathbf{F}'_{22}\mathcal{I}_{T_{2d}}\mathbf{F}_{23} \\ \mathbf{0}_{d\times d} & \mathbf{0}_{d\times T_{2d}} & -\mathbf{I}_d \end{bmatrix} \qquad \text{(S.16)} \\
&= \{\mathbf{A}_{ij}\}, (i, j = 1, 2, 3).
\end{aligned}
$$

Next, we derive the form of each $\mathbf{A}_{ij}$. Using

$$
\mathcal{I}_{T_{2d}}\mathbf{F}_{22} = \begin{bmatrix} 0 & \cdots & 0 & f_{T_dT_d} \\ \vdots & \iddots & \iddots & \vdots \\ 0 & f_{d+2,d+2} & \cdots & f_{d+2,T_d} \\ f_{d+1,d+1} & f_{d+1,d+2} & \cdots & f_{d+1,T_d} \end{bmatrix}, \quad \mathcal{I}_{T_{2d}}\mathbf{F}'_{22} = \begin{bmatrix} f_{d+1,T_d} & f_{d+2,T_d} & \cdots & f_{T_dT_d} \\ \vdots & \vdots & \iddots & 0 \\ f_{d+1,d+2} & f_{d+2,d+2} & & \vdots \\ f_{d+1,d+1} & 0 & \cdots & 0 \end{bmatrix},
$$

$$
\mathcal{I}_{T_{2d}}\mathbf{F}_{23} = \begin{bmatrix} f_{T_d,T_d+1} & f_{T_d,T} \\ \vdots & \vdots \\ f_{d+2,T_d+1} & f_{d+2,T} \\ f_{d+1,T_d+1} & f_{d+1,T} \end{bmatrix}, \quad \mathcal{I}_d\mathbf{F}'_{23} = \begin{bmatrix} f_{d+1,T} & f_{d+2,T} & \cdots & f_{T_d,T} \\ f_{d+1,T_d+1} & f_{d+2,T_d+1} & \cdots & f_{T_d,T_d+1} \end{bmatrix}
$$

we have

$$
\begin{aligned}
\underset{(d\times T_{2d})}{\mathbf{A}_{12}} &= \mathcal{I}_d\mathbf{F}'_{23}\mathcal{I}_{T_{2d}}\mathbf{F}_{22} = \left\{a_{12}^{jk}\right\} \\
&= \begin{bmatrix} f_{T_d,T}f_{d+1,d+1} & f_{T_d-1,T}f_{d+2,d+2} + f_{T_d,T}f_{d+1,d+2} & \cdots & \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T}f_{T_d-\ell+1,T_d} \\ f_{T_d,T_d+1}f_{d+1,d+1} & f_{T_d-1,T_d+1}f_{d+2,d+2} + f_{T_d,T_d+1}f_{d+1,d+2} & \cdots & \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T_d+1}f_{T_d-\ell+1,T_d} \end{bmatrix}
\end{aligned}
$$

with

$$
\begin{aligned}
\left\{a_{12}^{jk}\right\} &= \sum_{\ell=1}^{k} f_{T_d-\ell+1,T-j+1}f_{d+\ell,d+k} = \sum_{\ell=1}^{k-1} f_{T_d-\ell+1,T-j+1}f_{d+\ell,d+k} + f_{T_d-k+1,T-j+1}f_{d+k,d+k} \\
&= \sum_{\ell=1}^{k-1} \frac{4c_{\ell+2}^{\mathcal{T}}c_{T-\ell-1}^{\mathcal{T}}\left(3j-\ell-3\right)\left(2T-3k+\ell-3\right)}{\ell\left(\ell+1\right)\left(T-\ell-2\right)\left(T-\ell-3\right)} - \frac{2c_{k+2}^{\mathcal{T}}c_{T-k-1}^{\mathcal{T}}\left(3j-k-3\right)}{k\left(k+1\right)} \\
&= O\left(\frac{\log T}{T}\right), \quad \text{for } (j = 1, 2; k = 1, ..., T_{2d}).
\end{aligned}
$$

$$
\underset{(d\times d)}{\mathbf{A}_{13}} = \mathcal{I}_d\mathbf{F}'_{23}\mathcal{I}_{T_{2d}}\mathbf{F}_{23} = \left\{a_{13}^{jk}\right\} = \begin{bmatrix} \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T}f_{T_d-\ell+1,T_d+1} & \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T}f_{T_d-\ell+1,T} \\ \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T_d+1}f_{T_d-\ell+1,T_d+1} & \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T_d+1}f_{T_d-\ell+1,T} \end{bmatrix}
$$

with

$$\begin{aligned}\left\{a_{13}^{jk}\right\} &= \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T-j+1} f_{T_d-\ell+1,T_d+k} = \sum_{\ell=1}^{T_{2d}} \frac{4c_{\ell+2}^{\tau} c_{T-\ell-1}^{\tau} \left(3k+\ell-6\right)\left(T-3j-\ell\right)}{\ell\left(\ell+1\right)\left(T-\ell-2\right)\left(T-\ell-3\right)} \\ &= O\left(\frac{\log T}{T}\right) \qquad \text{for } j=1,2; k=1,2.\end{aligned}$$

$$\underset{(T_{2d}\times T_{2d})}{\mathbf{A}_{22}} = \mathcal{I}_{T_{2d}} \mathbf{F}_{22}' \mathcal{I}_{T_{2d}} \mathbf{F}_{22} - \mathbf{I}_{T_{2d}} = \left\{a_{22}^{jk}\right\} = \begin{bmatrix} a_{22}^{11} & a_{22}^{12} & \cdots & a_{22}^{1,T_{2d}} \\ 0 & a_{22}^{22} & \cdots & a_{22}^{2,T_{2d}} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{22}^{T_{2d},T_{2d}} \end{bmatrix}$$

with

$$\begin{aligned}\left\{a_{22}^{jk}\right\} &= \begin{cases} 0 & \text{if } j>k \\ f_{T_d-j+1,T_d-j+1} f_{d+j,d+j} - 1 & \text{if } j=k \\ \sum_{\ell=1}^{k} f_{T_d-\ell+1,T_d-j+1} f_{d+\ell,d+k} & \text{if } j<k \end{cases} \\ &= \begin{cases} 0 & \text{if } j>k \\ c_{T-j-1}^{\tau} c_{j+2}^{\tau} - 1 = O\left(\frac{1}{j}\right) + O\left(\frac{1}{(T-j)}\right) & \text{if } j=k \\ \sum_{\ell=1,\ell>j,\ell<k}^{k} \frac{4c_{\ell+2}^{\tau} c_{T-\ell-1}^{\tau}(3j-\ell+3)(2T-3k+\ell-3)}{\ell(\ell+1)(T-\ell-2)(T-\ell-3)} & \\ -\frac{2c_{j+2}^{\tau} c_{T-j-1}^{\tau}(2T+j-3k-3)}{(T-j-2)(T-j-3)} - \frac{2c_{k+2}^{\tau} c_{T-k-1}^{\tau}(3j-k+3)}{k(k+1)} = O\left(\frac{\log T}{T}\right) & \text{if } j<k \\ (j,k=1,...,T_{2d}) & \end{cases}.\end{aligned}$$

$$\underset{(T_{2d}\times d)}{\mathbf{A}_{23}} = \mathcal{I}_{T_{2d}} \mathbf{F}_{22}' \mathcal{I}_{T_{2d}} \mathbf{F}_{23} = \left\{a_{23}^{jk}\right\} = \begin{bmatrix} \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T_d} f_{T_d-\ell+1,T_d+1} & \sum_{\ell=1}^{T_{2d}} f_{d+\ell,T_d} f_{T_d-\ell+1,T} \\ \vdots & \vdots \\ \sum_{\ell=1}^{2} f_{d+\ell,d+2} f_{T_d-\ell+1,T_d+1} & \sum_{\ell=1}^{2} f_{d+\ell,d+2} f_{T_d-\ell+1,T} \\ \sum_{\ell=1}^{1} f_{d+\ell,d+1} f_{T_d-\ell+1,T_d+1} & \sum_{\ell=1}^{1} f_{d+\ell,d+1} f_{T_d-\ell+1,T} \end{bmatrix}$$

with

$$\begin{aligned}\left\{a_{23}^{jk}\right\} &= \sum_{\ell=1}^{T_{2d}-j+1} f_{d+\ell,T_d-j+1} f_{T_d-\ell+1,T_d+k} = \sum_{\ell=1}^{T_{2d}-j} f_{d+\ell,T_d-j+1} f_{T_d-\ell+1,T_d+k} + f_{T_d-j+1,T_d-j+1} f_{d+j,T_d+k} \\ &= \sum_{\ell=1}^{T_{2d}-j} \frac{4c_{\ell+2}^{\tau} c_{T-\ell-1}^{\tau} \left(3k+\ell-6\right)\left(T-3j-\ell-6\right)}{\ell\left(\ell+1\right)\left(T-\ell-2\right)\left(T-\ell-3\right)} + \frac{2c_{j+2}^{\tau} c_{T-j-1}^{\tau} \left(T-j+3k-9\right)}{\left(T-j-2\right)\left(T-j-3\right)} \\ &= O\left(\frac{\log T}{T}\right) \qquad \text{for } j=1,...,T_{2d}; k=1,2.\end{aligned}$$

We now assess the first term of (S.15). Using $\mathbf{\Xi}_i = (\mathbf{\Xi}_{1i}, \mathbf{\Xi}_{2i}, \mathbf{\Xi}_{3i})'$ where $\mathbf{\Xi}_{1i}$ is $d \times k$, $\mathbf{\Xi}_{2i}$ is $T_{2d} \times k$, and $\mathbf{\Xi}_{3i}$ is $d \times k$, we have

$$\mathbf{S}_i = \mathbf{\Xi}_i' \mathbf{A}_T \mathbf{\Xi}_i = \mathbf{S}_{1i} + \mathbf{S}_{2i} + \mathbf{S}_{3i} + \mathbf{S}_{4i} + \mathbf{S}_{5i} + \mathbf{S}_{6i}$$

where

$$\begin{aligned}\mathbf{S}_{1i} &= -\mathbf{\Xi}_{1i}' \mathbf{\Xi}_{1i}, \quad \mathbf{S}_{2i} = \mathbf{\Xi}_{1i}' \mathbf{A}_{12} \mathbf{\Xi}_{2i}, \quad \mathbf{S}_{3i} = \mathbf{\Xi}_{1i}' \mathbf{A}_{13} \mathbf{\Xi}_{3i}, \quad \mathbf{S}_{4i} = \mathbf{\Xi}_{2i}' \mathbf{A}_{22} \mathbf{\Xi}_{2i}, \\ \mathbf{S}_{5i} &= \mathbf{\Xi}_{2i}' \mathbf{A}_{23} \mathbf{\Xi}_{3i}, \quad \mathbf{S}_{6i} = -\mathbf{\Xi}_{3i}' \mathbf{\Xi}_{3i}.\end{aligned}$$

We now evaluate each term. From the definition of $\boldsymbol{\Xi}_{1i}$, we have

$$E(\mathbf{S}_{1i}) = -E\left(\boldsymbol{\xi}_{i1}\boldsymbol{\xi}_{i1}'\right) - E\left(\boldsymbol{\xi}_{i2}\boldsymbol{\xi}_{i2}'\right) = -2\boldsymbol{\Gamma}_{i0} = O\left(1\right).$$

Since $a_{12}^{1,t-1}$, $a_{12}^{2,t-1}$, a_{13}^{1k} and a_{13}^{2k} are $O\left(\frac{\log T}{T}\right)$ for all t and k, using Assumption 3, we have

$$\begin{aligned}
E(\mathbf{S}_{2i}) &= \sum_{t=2}^{T_d}\left[a_{12}^{1,t-1}E\left(\boldsymbol{\xi}_{i1}\boldsymbol{\xi}_{it}'\right) + a_{12}^{2,t-1}E\left(\boldsymbol{\xi}_{i2}\boldsymbol{\xi}_{it}'\right)\right] \\
&= O\left(\frac{\log T}{T}\right)\sum_{t=2}^{T_1}\boldsymbol{\Gamma}_{i,t-1} + O\left(\frac{\log T}{T}\right)\sum_{t=2}^{T_1}\boldsymbol{\Gamma}_{i,t-2} = O\left(\frac{\log T}{T}\right),
\end{aligned}$$

$$\begin{aligned}
E(\mathbf{S}_{3i}) &= \sum_{t=T-1}^{T}\left[\left(a_{13}^{1,1} + a_{13}^{1,2}\right)E\left(\boldsymbol{\xi}_{i1}\boldsymbol{\xi}_{it}'\right) + \left(a_{13}^{2,1} + a_{13}^{2,2}\right)E\left(\boldsymbol{\xi}_{i2}\boldsymbol{\xi}_{it}'\right)\right] \\
&= O\left(\frac{\log T}{T}\right)\left(\boldsymbol{\Gamma}_{i,T-1} + 2\boldsymbol{\Gamma}_{i,T-2} + \boldsymbol{\Gamma}_{i,T-3}\right) = O\left(\frac{\log T}{T}\right).
\end{aligned}$$

Similarly, using $a_{22}^{t-1,t-1} = O(1/(t+1)) + O(1/(T-t))$ and $a_{22}^{s-1,t-1} = O(\log T/T)$ for all $s \neq t$, we have

$$\begin{aligned}
E(\mathbf{S}_{4i}) &= \sum_{s=3}^{T_d}\sum_{t=3}^{T_d}a_{22}^{s-1,t-1}E\left(\boldsymbol{\xi}_{is}\boldsymbol{\xi}_{it}'\right) = \sum_{t=3}^{T_d}a_{22}^{t-1,t-1}E\left(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{it}'\right) + \sum_{s=3}^{T_d-1}\sum_{t=s+1}^{T_d}a_{22}^{s-1,t-1}E\left(\boldsymbol{\xi}_{is}\boldsymbol{\xi}_{it}'\right) \\
&= \sum_{t=3}^{T_d}\left[O\left(\frac{1}{t+1}\right) + O\left(\frac{1}{T-t}\right)\right]E\left(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{it}'\right) + O\left(\frac{\log T}{T}\right)\sum_{s=3}^{T_d-1}\sum_{t=s+1}^{T_d}\boldsymbol{\Gamma}_{t-s,i} \\
&= O\left(\log T\right).
\end{aligned}$$

Finally, using $a_{23}^{t-1,1} = O(\log T/T)$ and $a_{23}^{t-1,2} = O(\log T/T)$ for all t, and the definition of $\boldsymbol{\Xi}_{3i}$, we have

$$\begin{aligned}
E(\mathbf{S}_{5i}) &= \sum_{t=2}^{T_d}a_{23}^{t-1,1}E\left(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{iT_1}'\right) + \sum_{t=2}^{T_d}a_{23}^{t-1,2}E\left(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{iT}'\right) \\
&= \sum_{t=2}^{T_d}O\left(\frac{\log T}{T}\right)\left(\boldsymbol{\Gamma}_{i,T-t-1} + \boldsymbol{\Gamma}_{i,T-t}\right) = O\left(\frac{\log T}{T}\right),
\end{aligned}$$

$$E(\mathbf{S}_{6i}) = -E\left(\boldsymbol{\xi}_{iT}\boldsymbol{\xi}_{iT}'\right) - E\left(\boldsymbol{\xi}_{iT_1}\boldsymbol{\xi}_{iT_1}'\right) = -2\boldsymbol{\Gamma}_{i0} = O\left(1\right).$$

Thus, we have $\mathbf{S}_i = \sum_{l=1}^{6}\mathbf{S}_{li} = O(\log T)$ for all i and obtain

$$\frac{1}{NT}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{A}_T\boldsymbol{\Xi}_i = O_p\left(\frac{\log T}{T}\right). \tag{S.17}$$

Next, we consider the second term of (S.15). Let us define $\mathbf{H}_i = \boldsymbol{\Xi}_i'\mathbf{R}_T\boldsymbol{\Xi}_i$. Then, using (21), we have

$$\begin{aligned}
\mathbf{H}_i &= \frac{2\left(2T+1\right)\boldsymbol{\Xi}_i'\boldsymbol{\iota}_T\boldsymbol{\iota}_T'\boldsymbol{\Xi}_i}{T\left(T-1\right)} + \frac{12\boldsymbol{\Xi}_i'\boldsymbol{\tau}_T\boldsymbol{\tau}_T'\boldsymbol{\Xi}_i}{T\left(T-1\right)\left(T+1\right)} - \frac{6\left(\boldsymbol{\Xi}_i'\boldsymbol{\iota}_T\boldsymbol{\tau}_T'\boldsymbol{\Xi}_i + \boldsymbol{\Xi}_i'\boldsymbol{\tau}_T\boldsymbol{\iota}_T'\boldsymbol{\Xi}_i\right)}{T\left(T-1\right)} \\
&= \mathbf{H}_{1i} + \mathbf{H}_{2i} + \mathbf{H}_{3i}.
\end{aligned}$$

Using Assumption 3 and (54), we have

$$\begin{aligned}
E(\mathbf{H}_{1i}) &= \frac{2\left(2T+1\right)}{T\left(T-1\right)}E\left[\left(\sum_{t=1}^{T}\boldsymbol{\xi}_{it}\right)\left(\sum_{s=1}^{T}\boldsymbol{\xi}_{is}'\right)\right] = O\left(1\right), \\
E(\mathbf{H}_{2i}) &= \frac{12}{T\left(T-1\right)\left(T+1\right)}E\left(\left(\sum_{t=1}^{T}t\boldsymbol{\xi}_{it}\right)\left(\sum_{s=1}^{T}s\boldsymbol{\xi}_{is}'\right)\right) \\
&= \frac{12T}{\left(T-1\right)\left(T+1\right)}\sum_{s=1}^{T}\sum_{t=1}^{T}\left(\frac{t}{T}\right)\left(\frac{s}{T}\right)E(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{is}') = O\left(1\right), \\
E(\mathbf{H}_{3i}) &= \frac{6}{T\left(T-1\right)}\left(\sum_{t=1}^{T}\sum_{s=1}^{T}sE(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{is}') + \sum_{t=1}^{T}\sum_{s=1}^{T}tE(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{is}')\right) \\
&= \frac{6}{\left(T-1\right)}\left(\sum_{s=1}^{T}\sum_{t=1}^{T}\left(\frac{s}{T}\right)E(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{is}') + \sum_{s=1}^{T}\sum_{s=1}^{T}\left(\frac{t}{T}\right)E(\boldsymbol{\xi}_{it}\boldsymbol{\xi}_{is}')\right) = O\left(1\right)
\end{aligned}$$

where we used $0 < t/T \leq 1$ and $0 < s/T \leq 1$ for all s and t. Hence, for each i, we have $E(\mathbf{H}_i) = O(1)$ and obtain

$$\frac{1}{NT}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{R}_T\boldsymbol{\Xi}_i = O_p\left(\frac{1}{T}\right). \tag{S.18}$$

By combining (S.17) and (S.18), we obtain

$$\frac{1}{NT}\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{K}_T'\mathbf{B}_{T_d}'\mathbf{F}_{T_d}\mathbf{L}_T\mathbf{W}_i \quad = \quad \frac{1}{NT}\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{Q}_T\mathbf{W}_i + O_p\left(\frac{\log T}{T}\right).$$

Proof of (b): Using $\mathbf{Q}_T\left(\boldsymbol{\iota}_T, \boldsymbol{\tau}_T\right) = \mathbf{0}$, we have

$$\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{Q}_T\mathbf{v}_i \quad = \quad \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{Q}_T\mathbf{v}_i = \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{v}_i - \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{R}_T\mathbf{v}_i.$$

The first term converges in distribution to $\mathcal{N}(\mathbf{0}, \boldsymbol{\Omega})$ by Assumption 4. To assess the second term, let us define $\mathbf{h}_i = -\boldsymbol{\Xi}_i'\mathbf{R}_T\mathbf{v}_i$. Then, for the trend model, we have

$$\begin{aligned}
\mathbf{h}_i &= -\frac{2\left(2T+1\right)\boldsymbol{\Xi}_i'\boldsymbol{\iota}_T\boldsymbol{\iota}_T'\mathbf{v}_i}{T\left(T-1\right)} - \frac{12\boldsymbol{\Xi}_i'\boldsymbol{\tau}_T\boldsymbol{\tau}_T'\mathbf{v}_i}{T\left(T-1\right)\left(T+1\right)} + \frac{6\left(\boldsymbol{\Xi}_i'\boldsymbol{\iota}_T\boldsymbol{\tau}_T'\mathbf{v}_i + \boldsymbol{\Xi}_i'\boldsymbol{\tau}_T\boldsymbol{\iota}_T'\mathbf{v}_i\right)}{T\left(T-1\right)} \\
&= -\mathbf{h}_{1i} - \mathbf{h}_{2i} + \mathbf{h}_{3i}.
\end{aligned}$$

Using Assumption 3 and (56), we have

$$\begin{aligned}
E(\mathbf{h}_{1i}) &= \frac{2\left(2T+1\right)}{T\left(T-1\right)}E\left[\left(\sum_{t=1}^{T}\boldsymbol{\xi}_{it}\right)\left(\sum_{s=1}^{T}v_{is}\right)\right] = \frac{2\left(2T+1\right)}{T\left(T-1\right)}\sum_{s=1}^{T-1}\sum_{t=s+1}^{T}\boldsymbol{\phi}_{i,t-s} = O\left(1\right), \\
E(\mathbf{h}_{2i}) &= \frac{12}{T\left(T-1\right)\left(T+1\right)}E\left(\left(\sum_{t=1}^{T}t\boldsymbol{\xi}_{it}\right)\left(\sum_{s=1}^{T}sv_{is}\right)\right) \\
&= \frac{12}{\left(T+1\right)}\left(\sum_{s=1}^{T-1}\sum_{t=s+1}^{T}\left(\frac{t}{T}\right)\left(\frac{s}{T-1}\right)\boldsymbol{\phi}_{i,t-s}\right) = O\left(1\right), \\
E(\mathbf{h}_{3i}) &= \frac{6}{T\left(T-1\right)}\left(\sum_{t=1}^{T}\sum_{s=1}^{T}sE(\boldsymbol{\xi}_{it}v_{is}) + \sum_{t=1}^{T}\sum_{s=1}^{T}tE(\boldsymbol{\xi}_{it}v_{is})\right)
\end{aligned}$$

$$= \frac{6}{T}\sum_{s=1}^{T-1}\sum_{t=s+1}^{T}\left(\frac{s}{T-1}\right)\boldsymbol{\phi}_{i,t-s} + \frac{6}{(T-1)}\sum_{s=1}^{T-1}\sum_{s=t+1}^{T}\left(\frac{t}{T}\right)\boldsymbol{\phi}_{i,t-s} = O\left(1\right)$$

where we used $0 < t/T \leq 1$ and $0 < s/(T-1) \leq 1$ for all s and t. Thus, $E(\mathbf{h}_i) = O(1)$ and obtain

$$\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{R}_T\mathbf{v}_i = \sqrt{\frac{N}{T}}\bar{\mathbf{h}}_N = O_p\left(\sqrt{\frac{N}{T}}\right)$$

where $\bar{\mathbf{h}}_N = N^{-1}\sum_{i=1}^{N}\mathbf{h}_i$.

Proof of (c): Noting $\mathbf{B}_{T_d}\mathbf{K}_T\mathbf{C}_T = \mathbf{0}$, we have the following decomposition:

$$\begin{aligned}\frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\mathbf{W}_i'\mathbf{K}_T'\mathbf{B}_{T_d}'\mathbf{F}_{T_d}\mathbf{L}_T\mathbf{v}_i &= \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{K}_T'\mathbf{B}_{T_d}'\mathbf{F}_{T_d}\mathbf{L}_T\mathbf{v}_i \\ &= \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{v}_i + \frac{1}{\sqrt{NT}}\sum_{i=1}^{N}\boldsymbol{\Xi}_i'\mathbf{A}_T\mathbf{v}_i.\end{aligned}$$

The first term converges in distribution to $\mathcal{N}(\mathbf{0},\boldsymbol{\Omega})$ by Assumption 4. To derive the order of the second term, let us define $\mathbf{s}_i = \boldsymbol{\Xi}_i'\mathbf{A}_T\mathbf{v}_i$. Then, using (S.16), and $\mathbf{v}_i = (\mathbf{v}_{i1}', \mathbf{v}_{i2}', \mathbf{v}_{i3}')'$ where $\mathbf{v}_{i1}$ and $\mathbf{v}_{i3}$ are $d \times 1$ and $\mathbf{v}_{i2}$ is $T_{2d} \times 1$, $\mathbf{s}_i$ can be decomposed as

$$\mathbf{s}_i = \mathbf{s}_{1i} + \mathbf{s}_{2i} + \mathbf{s}_{3i} + \mathbf{s}_{4i} + \mathbf{s}_{5i} + \mathbf{s}_{6i}$$

where

$$\begin{aligned}\mathbf{s}_{1i} &= -\boldsymbol{\Xi}_{1i}'\mathbf{v}_{1i}, \quad \mathbf{s}_{2i} = \boldsymbol{\Xi}_{1i}'\mathbf{A}_{12}\mathbf{v}_{2i}, \quad \mathbf{s}_{3i} = \boldsymbol{\Xi}_{1i}'\mathbf{A}_{13}\mathbf{v}_{3i}, \quad \mathbf{s}_{4i} = \boldsymbol{\Xi}_{2i}\mathbf{A}_{22}\mathbf{v}_{2i}, \\ \mathbf{s}_{5i} &= \boldsymbol{\Xi}_{2i}'\mathbf{A}_{23}\mathbf{v}_{3i}, \quad \mathbf{s}_{6i} = -\boldsymbol{\Xi}_{3i}'\mathbf{v}_{3i}.\end{aligned}$$

To derive the variance of $\mathbf{s}_i$, we need to calculate $Var(\mathbf{s}_{ki})$ and $Cov(\mathbf{s}_{ki}, \mathbf{s}_{li}), (k \neq l)$ for $k, l = 1, ..., 6$. We have

$$Var(\mathbf{s}_{1i}) = Var(\sum_{s=1}^{2}\sum_{t=1}^{2}\boldsymbol{\xi}_{it}v_{is}) = Var(\boldsymbol{\xi}_{i1}v_{i1}) + Var(\boldsymbol{\xi}_{i2}v_{i2}) = O(1).$$

Using $a_{12}^{s,t-1} = O\left(\frac{\log T}{T}\right), s =, 2$ for all t and $a_{13}^{jk} = O\left(\frac{\log T}{T}\right)$, we have

$$Var(\mathbf{s}_{2i}) = Var\left(\sum_{s=1}^{2}\sum_{t=3}^{T_{2d}}a_{12}^{j,t-1}\boldsymbol{\xi}_{i1}v_{it}\right) = O\left(\frac{(\log T)^2}{T}\right),$$

$$Var(\mathbf{s}_{3i}) = Var\left(\sum_{s=1}^{2}\sum_{t=1}^{2}a_{13}^{st}\boldsymbol{\xi}_{it}v_{iT_s+1}\right) = \sum_{s=1}^{2}\sum_{t=1}^{2}(a_{13}^{st})^2 Var\left(\boldsymbol{\xi}_{it}v_{iT_s+1}\right) = O\left(\frac{(\log T)^2}{T^2}\right).$$

Similarly, using $a_{22}^{t-1,t-1} = O(1/(t+1)) + O(1/(T-t))$ and $a_{22}^{s-1,t-1} = O(\log T/T)$ for all $s \neq t$, we have

$$Var(\mathbf{s}_{4i}) = Var\left[\sum_{s=3}^{T_d-1}\sum_{t=s}^{T_d}a_{22}^{s-1,t-1}\boldsymbol{\xi}_{is}v_{it}\right] = \sum_{s=3}^{T_d-1}\sum_{t=s}^{T_d}(a_{22}^{s-1,t-1})^2 Var\left(\boldsymbol{\xi}_{is}v_{it}\right)$$

$$
\begin{aligned}
&= \sum_{t=3}^{T_d} (a_{22}^{t-1,t-1})^2 Var\left(\boldsymbol{\xi}_{it} v_{it}\right) + \sum_{s=3}^{T_d-1} \sum_{t=s+1}^{T_d} (a_{22}^{s-1,t-1})^2 Var\left(\boldsymbol{\xi}_{is} v_{it}\right) \\
&= O(1) + O\left((\log T)^2\right).
\end{aligned}
$$

Finally, using $a_{23}^{t-1,s} = O(\log T/T)$, $s = 1, 2$ for all t, and the definition of $\boldsymbol{\Xi}_{3i}$, we have

$$
\begin{aligned}
Var(\mathbf{s}_{5i}) &= Var\left(\sum_{s=1}^{2} \sum_{t=3}^{T_d} a_{23}^{t-1,s} \boldsymbol{\xi}_{it} v_{iT_s+1}\right) \\
&= \sum_{s=1}^{2} \sum_{t=2}^{T_d} (a_{23}^{t-1,s})^2 Var\left(\boldsymbol{\xi}_{it} v_{iT_s+1}\right) = O\left(\frac{(\log T)^2}{T}\right), \\
Var(\mathbf{s}_{6i}) &= Var\left(\sum_{s=1}^{2} \sum_{t=1}^{2} \boldsymbol{\xi}_{iT_t} v_{iT_s}\right) = O(1).
\end{aligned}
$$

For the covariances of trend model,

$$
\begin{aligned}
& Cov(\mathbf{s}_{1i}, \mathbf{s}_{2i}) = Cov(\mathbf{s}_{1i}, \mathbf{s}_{3i}) = Cov(\mathbf{s}_{1i}, \mathbf{s}_{4i}) = Cov(\mathbf{s}_{1i}, \mathbf{s}_{5i}) = Cov(\mathbf{s}_{1i}, \mathbf{s}_{6i}) = Cov(\mathbf{s}_{2i}, \mathbf{s}_{3i}) \\
= \; & Cov(\mathbf{s}_{2i}, \mathbf{s}_{5i}) = Cov(\mathbf{s}_{2i}, \mathbf{s}_{6i}) = Cov(\mathbf{s}_{3i}, \mathbf{s}_{4i}) = Cov(\mathbf{s}_{4i}, \mathbf{s}_{5i}) = Cov(\mathbf{s}_{4i}, \mathbf{s}_{6i}) = \mathbf{0},
\end{aligned}
$$

$$
\begin{aligned}
Cov(\mathbf{s}_{2i}, \mathbf{s}_{4i}) &= E\left[\left(\sum_{s=1}^{2} \sum_{t=2}^{T_d} a_{12}^{s,t-1} \boldsymbol{\xi}_{is} v_{it}\right) \left(\sum_{t=3}^{T_d} a_{22}^{t-1,t-1} \boldsymbol{\xi}_{it} v_{it} + \sum_{s=3}^{T_d-1} \sum_{t=s+1}^{T_d} a_{22}^{s-1,t-1} \boldsymbol{\xi}_{is} v_{it}\right)'\right] \\
&= \sum_{s=1}^{2} \sum_{t_1=3}^{T_d} \sum_{t_2=3}^{T_d} a_{12}^{1,t_1-1} a_{22}^{t_2-1,t_2-1} E(\boldsymbol{\xi}_{is} \boldsymbol{\xi}'_{it_2} v_{it_1} v_{it_2}) \\
&+ \sum_{s_1=1}^{2} \sum_{t_1=3}^{T_d} \sum_{s_2=3}^{T_d-1} \sum_{t_2=s_2+1}^{T_d} a_{12}^{1,t_1-1} a_{22}^{s_2-1,t_2-1} E(\boldsymbol{\xi}_{is_1} \boldsymbol{\xi}'_{is_2} v_{it_1} v_{it_2}) \\
&= \sum_{s=1}^{2} \sum_{t=3}^{T_d} a_{12}^{s,t-1} a_{22}^{t-1,t-1} E(\boldsymbol{\xi}_{is} \boldsymbol{\xi}'_{it} v_{it}^2) + \sum_{s_1=1}^{2} \sum_{s_2=2}^{T_d-1} \sum_{t=s_2+1}^{T_d} a_{12}^{s_1,t-1} a_{22}^{s_2-1,t-1} E(\boldsymbol{\xi}_{is_1} \boldsymbol{\xi}'_{is_2} v_{it}^2) \\
&= O\left(\frac{(\log T)^2}{T}\right) + O\left((\log T)^2\right),
\end{aligned}
$$

$$
Cov(\mathbf{s}_{3i}, \mathbf{s}_{5i}) = \sum_{s_1=1}^{2} \sum_{s_2=1}^{2} \sum_{t=3}^{T_d} a_{13}^{s_1 s_2 1} a_{23}^{t-1,s_2} E(\boldsymbol{\xi}_{is_1} \boldsymbol{\xi}'_{it} v_{iT_{s1}+1}^2) = O\left(\frac{(\log T)^2}{T}\right),
$$

$$
Cov(\mathbf{s}_{3i}, \mathbf{s}_{6i}) = \sum_{s_1=1}^{2} \sum_{s_2=1}^{2} a_{13}^{s_1 s_2} E(\boldsymbol{\xi}_{is_1} \boldsymbol{\xi}'_{iT_{s2}+1} v_{iT_{s_1+1}}^2) = O\left(\frac{\log T}{T}\right),
$$

$$
Cov(\mathbf{s}_{5i}, \mathbf{s}_{6i}) = \sum_{s=1}^{2} \sum_{t=3}^{T_d} a_{23}^{t-1,s} E(\boldsymbol{\xi}_{it} \boldsymbol{\xi}'_{iT_s+1} v_{iT_s+1}^2) = O(\log T).
$$

Therefore, for trend model, we have $Var(\mathbf{s}_i) = O\left((\log T)^2\right)$, and

$$
Var\left(\frac{1}{\sqrt{NT}} \sum_{i=1}^{N} \boldsymbol{\Xi}'_i \mathbf{A}_T \mathbf{v}_i\right) = \frac{1}{NT} \sum_{i=1}^{N} Var(\mathbf{s}_i) = O\left(\frac{(\log T)^2}{T}\right).
$$

Hence, it follows that $\frac{1}{\sqrt{NT}} \sum_{i=1}^{N} \boldsymbol{\Xi}'_i \mathbf{A}_T \mathbf{v}_i = O_p\left(\log T/\sqrt{T}\right) = o_p(1)$.

S.5 Monte Carlo simulation

S.5.1 AR(1) model

In this supplement, we provide the Monte Carlo simulation results for AR(1) model given by

$$\begin{aligned} y_{it} &= \alpha y_{i,t-1} + \eta_i + v_{it}, \qquad (t = 1, ..., T;\ i = 1, ..., N) \\ y_{i0} &= \frac{\eta_i}{1-\alpha} + e_{i0} \end{aligned}$$

where $v_{it} \sim iidN(0, \sigma_v^2)$, $\eta_i \sim iidN(0, \sigma_\eta^2)$, $e_{i0} \sim iidN(0, \sigma_v^2/(1-\alpha^2))$. For parameter values, we consider $\alpha = \{0.4, 0.8\}$, $\sigma_v^2 = 1$, $\sigma_\eta^2 = \{1, 5\}$ and for the sample size, we consider $T = \{10, 20, 50, 100\}$ and $N = \{50, 100, 200, 500\}$. In addition to the estimators considered for a model with weakly exogenous regressor x_{it}, we include the bias-corrected FE estimator due to Hahn and Kuersteiner (2002), which is denoted as "BCFE". The number of replications is 2,000 for all cases.

The results are provided in Tables S.1-S.4. Since a detailed discussion for the FE and IV/GMM estimators is provided in the main body in the context of more general model with weakly exogenous regressor, we focus on the BCFE estimator here. Compared with the FE estimator, the bias of the BCFE estimator is much smaller. However, when compared with the HPJ estimator, the HPJ is more effective in reducing the bias of the FE estimator. In terms of MAE, the BCFE estimator tends to perform better than the HPJ estimator except when $\alpha = 0.8$ and $T = 10$. However, in terms of inference, bot the BCFE estimator has large size distortions when T is not large.

S.5.2 Models with weakly exogenous regressor and time trend

Tables S.5-S.12 provide simulation results for models with weakly exogenous regressors and time trends for the case of $\sigma_\eta^2 = 5$. For the details, see the main body.

References

Amemiya, T. (1985) *Advanced Econometrics*: Harvard university press.

Arellano, M. (2003) *Panel Data Econometrics*, Oxford: Oxford University Press.

Chen, Y. (2013) "A New Algorithm for Computing the Determinant and the Inverse of a Pentadiagonal Toeplitz Matrix," *Engineering*, 5, p. 25.

Dow, M. (2003) "Explicit inverses of Toeplitz and associated matrices," *ANZIAM Journal*, 44, E185–E215.

El-Mikkawy, M. and A. Karawia (2006) "Inversion of General Tridiagonal Matrices," *Applied Mathematics Letters*, 19, 712–720.

Hamilton, J. D. (1994) *Time Series Analysis*: Princeton University Press.

Table S.1: AR(1) model, Bias, IQR, MAE and Size ($\alpha = 0.4,\ \sigma_\eta^2 = 1$)

	Bias				IQR				MAE				Size			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-14.91	-7.51	-2.75	-1.45	5.91	3.94	2.48	1.79	14.91	7.51	2.75	1.48	91.2	68.7	32.3	20.3
100	-14.69	-7.15	-2.82	-1.40	4.29	2.91	1.78	1.27	14.69	7.15	2.82	1.40	99.7	92.8	57.2	34.0
200	-14.77	-7.15	-2.81	-1.40	2.90	1.90	1.17	0.87	14.77	7.15	2.81	1.40	100.0	99.8	86.7	56.8
500	-14.76	-7.19	-2.79	-1.40	1.89	1.32	0.77	0.60	14.76	7.19	2.79	1.40	100.0	100.0	99.8	90.7
	IV-L															
50	0.95	0.02	0.06	0.11	13.95	8.47	4.81	3.27	6.87	4.21	2.40	1.64	3.9	5.6	6.9	5.8
100	0.84	0.10	0.01	0.07	9.56	6.22	3.60	2.42	4.88	3.16	1.80	1.22	4.6	4.7	7.4	6.6
200	0.29	0.21	0.09	-0.08	7.43	4.26	2.47	1.69	3.71	2.13	1.23	0.84	4.6	5.2	5.3	5.6
500	0.09	0.03	-0.01	0.02	4.49	2.93	1.61	1.05	2.23	1.46	0.80	0.53	4.9	5.5	5.6	4.9
	GMM-L1															
50	-1.02	-0.69	-0.23	-0.04	13.38	8.37	4.99	3.32	6.45	4.22	2.40	1.69	4.0	5.7	7.9	5.5
100	-0.39	-0.45	-0.15	-0.01	8.97	6.07	3.59	2.38	4.43	3.03	1.75	1.20	4.8	4.1	7.1	6.1
200	-0.29	-0.17	0.01	-0.10	7.04	4.13	2.44	1.67	3.46	2.09	1.20	0.83	4.8	5.7	5.0	5.8
500	-0.02	-0.04	-0.07	0.01	4.38	2.81	1.56	1.04	2.22	1.42	0.78	0.52	4.5	5.2	5.5	4.8
	GMM-L3															
50	-1.93	-1.17	-0.29	-0.13	9.75	6.28	3.55	2.46	5.04	3.15	1.80	1.22	6.9	4.5	5.4	6.0
100	-1.03	-0.52	-0.23	-0.04	7.22	4.47	2.53	1.75	3.54	2.22	1.31	0.87	4.8	3.9	6.0	6.0
200	-0.47	-0.24	-0.06	-0.05	5.49	3.08	1.69	1.19	2.77	1.54	0.86	0.59	5.4	5.7	4.7	4.5
500	-0.25	-0.05	-0.03	-0.03	3.34	1.96	1.15	0.80	1.66	0.99	0.58	0.40	3.9	4.0	4.4	4.1
	IV-B															
50	0.07	-0.04	0.02	-0.04	10.35	5.92	3.06	1.96	5.19	2.90	1.51	0.98	5.7	4.5	6.8	6.6
100	0.13	-0.05	0.00	0.03	7.93	4.17	2.01	1.43	3.94	2.09	1.01	0.69	4.5	3.7	6.4	6.0
200	-0.26	-0.06	-0.04	-0.03	5.66	2.75	1.43	0.95	2.74	1.38	0.72	0.47	5.1	4.8	5.3	5.9
500	-0.09	0.04	0.03	0.02	3.48	1.80	0.95	0.64	1.74	0.89	0.48	0.33	4.4	4.6	4.6	5.2
	GMM-B1															
50	-0.38	-0.27	-0.02	-0.10	10.69	5.75	3.07	1.98	5.28	2.94	1.54	0.99	6.3	5.0	6.2	7.2
100	-0.34	-0.22	-0.07	-0.02	7.73	4.18	1.96	1.36	3.85	2.07	0.98	0.69	4.8	3.5	5.8	6.2
200	-0.33	-0.13	-0.07	-0.03	5.49	2.65	1.41	0.93	2.79	1.37	0.70	0.47	5.5	4.7	5.9	5.5
500	-0.15	0.04	0.03	0.01	3.57	1.72	0.92	0.64	1.81	0.86	0.46	0.32	4.8	4.5	5.3	5.6
	GMM-B3															
50	-2.15	-0.86	-0.20	-0.15	10.37	5.61	3.05	1.99	5.25	2.95	1.54	0.99	7.0	5.7	6.1	6.7
100	-1.15	-0.45	-0.16	-0.06	7.59	4.18	2.01	1.34	3.86	2.16	0.99	0.69	4.8	4.3	6.0	6.0
200	-0.77	-0.25	-0.11	-0.04	5.33	2.68	1.42	0.92	2.81	1.38	0.71	0.47	5.7	4.6	5.6	5.5
500	-0.32	-0.04	0.02	0.01	3.49	1.75	0.91	0.65	1.78	0.88	0.47	0.32	4.7	4.5	4.8	5.8
	HPJ															
50	0.37	0.19	0.19	0.01	8.46	4.75	2.71	1.83	4.25	2.37	1.36	0.91	8.9	6.6	5.2	6.3
100	0.67	0.51	0.11	0.02	5.69	3.40	1.92	1.30	2.93	1.75	0.97	0.66	8.9	6.3	5.9	5.7
200	0.86	0.41	0.13	0.03	4.17	2.32	1.27	0.92	2.10	1.24	0.63	0.46	8.4	6.6	5.5	5.5
500	0.67	0.38	0.13	0.04	2.75	1.56	0.84	0.59	1.45	0.79	0.43	0.30	7.6	8.7	4.9	6.2
	BOD															
50	2.51	1.97	1.59	0.94	7.16	4.53	2.69	1.85	3.96	2.56	1.79	1.16	8.1	11.7	14.5	13.2
100	2.71	2.37	1.56	1.02	5.01	3.15	1.86	1.29	3.28	2.46	1.60	1.05	11.2	16.4	21.3	19.3
200	2.71	2.42	1.56	1.04	3.60	2.06	1.22	0.90	2.88	2.42	1.56	1.04	18.6	28.9	34.1	31.8
500	2.88	2.43	1.60	1.06	2.29	1.50	0.82	0.62	2.88	2.43	1.60	1.06	38.6	61.7	74.1	69.3
	BCBOD															
50	0.02	-0.43	0.05	-0.12	11.72	7.49	4.68	3.20	5.86	3.74	2.34	1.59	21.8	23.2	26.5	28.1
100	-0.03	0.12	0.06	-0.05	7.46	5.15	3.17	2.37	3.73	2.52	1.57	1.18	19.4	20.6	25.3	25.5
200	0.14	0.18	0.15	0.01	5.88	3.60	2.39	1.63	2.90	1.81	1.19	0.82	21.0	22.6	25.6	25.8
500	0.14	0.11	0.10	0.08	3.56	2.39	1.50	1.08	1.83	1.20	0.77	0.54	21.2	23.1	24.8	26.0
	BCFE															
50	-2.40	-0.89	0.00	-0.07	6.50	4.13	2.53	1.81	3.54	2.16	1.28	0.90	10.9	6.0	7.2	7.1
100	-2.16	-0.51	-0.07	-0.01	4.72	3.05	1.82	1.28	2.67	1.55	0.92	0.64	14.3	5.7	6.3	6.0
200	-2.24	-0.50	-0.07	-0.02	3.19	1.99	1.20	0.88	2.33	1.00	0.61	0.44	19.2	7.4	6.0	5.5
500	-2.23	-0.55	-0.04	-0.01	2.08	1.38	0.79	0.61	2.23	0.82	0.39	0.31	36.2	9.0	4.9	5.6

Table S.2: AR(1) model, Bias, IQR, MAE and Size ($\alpha = 0.4,\ \sigma_\eta^2 = 5$)

	Bias				IQR				MAE				Size			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-14.91	-7.51	-2.75	-1.45	5.91	3.94	2.48	1.79	14.91	7.51	2.75	1.48	91.2	68.7	32.3	20.3
100	-14.69	-7.15	-2.82	-1.40	4.29	2.91	1.78	1.27	14.69	7.15	2.82	1.40	99.7	92.8	57.2	34.0
200	-14.77	-7.15	-2.81	-1.40	2.90	1.90	1.17	0.87	14.77	7.15	2.81	1.40	100.0	99.8	86.7	56.8
500	-14.76	-7.19	-2.79	-1.40	1.89	1.32	0.77	0.60	14.76	7.19	2.79	1.40	100.0	100.0	99.8	90.7
	IV-L															
50	2.06	0.42	-0.02	0.06	28.45	17.41	9.55	6.44	14.32	8.54	4.73	3.21	4.5	5.0	5.7	5.3
100	1.40	0.18	-0.12	0.16	19.31	12.68	7.18	4.58	9.68	6.26	3.56	2.32	4.8	4.3	5.7	6.5
200	0.66	0.23	0.06	-0.12	14.18	8.51	4.89	3.28	7.04	4.27	2.45	1.65	5.3	5.7	5.7	5.8
500	0.15	0.08	-0.02	0.05	8.85	5.59	3.08	2.00	4.40	2.77	1.53	1.00	4.8	5.2	6.0	5.3
	GMM-L1															
50	-4.79	-2.36	-1.08	-0.37	24.28	15.26	9.04	6.09	12.17	7.38	4.43	2.89	3.1	4.8	6.2	6.3
100	-2.87	-1.60	-0.75	-0.12	17.19	11.08	7.06	4.37	8.89	5.78	3.46	2.15	4.1	4.6	6.3	6.3
200	-1.51	-0.72	-0.31	-0.23	13.39	8.02	4.79	3.15	6.81	3.94	2.36	1.58	4.9	5.0	6.1	5.8
500	-0.53	-0.30	-0.24	-0.06	8.37	5.39	3.08	1.98	4.13	2.63	1.52	1.00	4.7	5.3	5.9	5.3
	GMM-L3															
50	-2.37	-1.44	-0.24	-0.18	10.91	6.74	3.85	2.61	5.53	3.41	1.91	1.29	7.1	3.9	5.3	5.4
100	-1.21	-0.69	-0.27	-0.12	8.25	4.81	2.71	1.87	4.10	2.47	1.38	0.94	5.2	4.4	5.4	5.5
200	-0.65	-0.32	-0.09	-0.09	5.96	3.35	1.84	1.26	2.89	1.73	0.93	0.63	5.4	5.6	4.2	5.6
500	-0.44	-0.03	-0.03	-0.05	3.72	2.09	1.16	0.82	1.89	1.03	0.59	0.41	4.4	4.3	4.7	5.0
	IV-B															
50	0.07	-0.04	0.02	-0.04	10.35	5.92	3.06	1.96	5.19	2.90	1.51	0.98	5.7	4.5	6.8	6.6
100	0.13	-0.05	0.00	0.03	7.93	4.17	2.01	1.43	3.94	2.09	1.01	0.69	4.5	3.7	6.4	6.0
200	-0.26	-0.06	-0.04	-0.03	5.66	2.75	1.43	0.95	2.74	1.38	0.72	0.47	5.1	4.8	5.3	5.9
500	-0.09	0.04	0.03	0.02	3.48	1.80	0.95	0.64	1.74	0.89	0.48	0.33	4.4	4.6	4.6	5.2
	GMM-B1															
50	-0.38	-0.27	-0.02	-0.10	10.69	5.75	3.07	1.98	5.28	2.94	1.54	0.99	6.3	5.0	6.2	7.2
100	-0.34	-0.22	-0.07	-0.02	7.73	4.18	1.96	1.36	3.85	2.07	0.98	0.69	4.8	3.5	5.8	6.2
200	-0.33	-0.13	-0.07	-0.03	5.49	2.65	1.41	0.93	2.79	1.37	0.70	0.47	5.5	4.7	5.9	5.5
500	-0.15	0.04	0.03	0.01	3.57	1.72	0.92	0.64	1.81	0.86	0.46	0.32	4.8	4.5	5.3	5.6
	GMM-B3															
50	-2.15	-0.86	-0.20	-0.15	10.37	5.61	3.05	1.99	5.25	2.95	1.54	0.99	7.0	5.7	6.1	6.7
100	-1.15	-0.45	-0.16	-0.06	7.59	4.18	2.01	1.34	3.86	2.16	0.99	0.69	4.8	4.3	6.0	6.0
200	-0.77	-0.25	-0.11	-0.04	5.33	2.68	1.42	0.92	2.81	1.38	0.71	0.47	5.7	4.6	5.6	5.5
500	-0.32	-0.04	0.02	0.01	3.49	1.75	0.91	0.65	1.78	0.88	0.47	0.32	4.7	4.5	4.8	5.8
	HPJ															
50	0.37	0.19	0.19	0.01	8.46	4.75	2.71	1.83	4.25	2.37	1.36	0.91	8.9	6.6	5.2	6.3
100	0.67	0.51	0.11	0.02	5.69	3.40	1.92	1.30	2.93	1.75	0.97	0.66	8.9	6.3	5.9	5.7
200	0.86	0.41	0.13	0.03	4.17	2.32	1.27	0.92	2.10	1.24	0.63	0.46	8.4	6.6	5.5	5.5
500	0.67	0.38	0.13	0.04	2.75	1.56	0.84	0.59	1.45	0.79	0.43	0.30	7.6	8.7	4.9	6.2
	BOD															
50	2.87	2.31	1.65	1.00	6.86	4.48	2.67	1.84	4.12	2.73	1.82	1.17	10.5	13.7	14.8	13.6
100	3.05	2.61	1.65	1.06	5.02	3.17	1.87	1.30	3.50	2.64	1.67	1.09	13.3	20.1	23.2	20.4
200	3.10	2.63	1.64	1.07	3.57	2.09	1.22	0.90	3.19	2.63	1.64	1.07	22.2	34.1	36.7	33.4
500	3.28	2.65	1.68	1.09	2.25	1.46	0.80	0.62	3.28	2.65	1.68	1.09	48.4	68.5	77.7	71.6
	BCBOD															
50	-0.20	-0.60	0.10	-0.18	20.54	13.96	8.81	6.17	10.17	7.01	4.41	3.10	47.8	50.2	52.9	55.2
100	-0.32	0.01	0.22	0.00	14.29	9.29	6.05	4.54	7.09	4.68	3.03	2.29	46.4	48.1	53.5	55.0
200	0.11	0.24	0.27	0.03	10.96	6.88	4.55	3.09	5.44	3.43	2.30	1.52	49.7	51.0	55.4	54.6
500	0.09	0.05	0.12	0.05	6.67	4.43	2.86	2.00	3.34	2.21	1.46	1.00	48.3	51.1	55.5	56.3
	BCFE															
50	-2.40	-0.89	0.00	-0.07	6.50	4.13	2.53	1.81	3.54	2.16	1.28	0.90	10.9	6.0	7.2	7.1
100	-2.16	-0.51	-0.07	-0.01	4.72	3.05	1.82	1.28	2.67	1.55	0.92	0.64	14.3	5.7	6.3	6.0
200	-2.24	-0.50	-0.07	-0.02	3.19	1.99	1.20	0.88	2.33	1.00	0.61	0.44	19.2	7.4	6.0	5.5
500	-2.23	-0.55	-0.04	-0.01	2.08	1.38	0.79	0.61	2.23	0.82	0.39	0.31	36.2	9.0	4.9	5.6

Table S.3: AR(1) model, Bias, IQR, MAE and Size ($\alpha = 0.8,\ \sigma_\eta^2 = 1$)

	Bias				IQR				MAE				Size			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-22.08	-10.56	-3.86	-1.91	5.47	3.38	1.91	1.22	22.08	10.56	3.86	1.91	100.0	99.0	83.1	57.0
100	-22.02	-10.33	-3.80	-1.86	3.96	2.41	1.35	0.88	22.02	10.33	3.80	1.86	100.0	100.0	98.1	82.6
200	-21.77	-10.40	-3.86	-1.85	2.80	1.61	0.85	0.61	21.77	10.40	3.86	1.85	100.0	100.0	100.0	98.5
500	-21.81	-10.46	-3.84	-1.86	1.73	1.13	0.54	0.40	21.81	10.46	3.84	1.86	100.0	100.0	100.0	100.0
	IV-L															
50	1.48	0.25	-0.19	0.04	30.81	15.54	7.00	4.15	15.68	7.51	3.33	2.10	1.3	4.1	5.2	6.6
100	1.58	0.25	-0.12	0.14	20.27	11.23	5.18	2.95	10.59	5.60	2.59	1.49	2.3	3.6	5.1	6.9
200	0.62	0.16	0.11	-0.04	15.10	7.31	3.57	2.13	7.57	3.68	1.78	1.06	3.9	5.7	5.6	5.9
500	0.33	0.00	0.02	0.03	9.52	4.88	2.16	1.34	4.77	2.44	1.07	0.67	4.5	5.0	6.3	5.3
	GMM-L1															
50	-9.60	-4.47	-1.52	-0.50	28.63	13.03	6.69	4.03	13.47	6.88	3.26	1.98	2.5	3.1	4.3	5.3
100	-5.42	-2.65	-0.84	-0.24	18.22	9.82	5.12	2.90	9.58	5.12	2.52	1.44	2.4	3.0	5.0	6.9
200	-2.72	-1.27	-0.35	-0.22	13.26	6.89	3.47	2.15	7.15	3.36	1.70	1.05	3.3	3.8	5.7	5.8
500	-1.07	-0.50	-0.18	-0.05	8.86	4.47	2.21	1.33	4.32	2.31	1.08	0.66	4.1	4.3	5.5	5.8
	GMM-L3															
50	-9.92	-4.17	-1.14	-0.45	15.54	8.07	3.80	2.38	10.69	4.92	2.05	1.27	13.0	8.3	7.3	7.0
100	-6.21	-2.20	-0.63	-0.18	12.29	5.81	2.75	1.81	7.53	3.39	1.41	0.95	10.1	6.3	6.1	6.4
200	-3.52	-1.21	-0.33	-0.19	9.57	4.46	1.98	1.21	5.47	2.35	1.01	0.64	7.9	5.7	5.7	6.1
500	-1.48	-0.42	-0.11	-0.04	5.86	2.82	1.36	0.85	3.25	1.39	0.67	0.44	5.4	6.2	4.8	5.0
	IV-B															
50	-0.19	-0.09	0.07	-0.04	21.04	8.30	3.04	1.69	10.48	4.15	1.50	0.83	3.9	4.1	5.9	5.8
100	-0.17	0.06	0.03	-0.02	15.93	5.59	2.11	1.17	8.04	2.83	1.07	0.58	4.0	4.3	5.9	5.0
200	-0.51	-0.08	0.00	-0.02	11.31	3.83	1.47	0.78	5.59	1.90	0.74	0.38	5.1	5.1	5.0	5.3
500	-0.07	0.04	0.03	0.01	7.34	2.57	0.92	0.54	3.71	1.31	0.46	0.27	4.4	5.3	4.6	4.8
	GMM-B1															
50	-5.18	-1.40	-0.13	-0.14	19.05	7.54	2.86	1.70	10.18	3.86	1.40	0.84	6.0	5.3	6.4	6.3
100	-2.77	-0.77	-0.13	-0.05	14.22	5.33	2.04	1.15	7.25	2.70	1.01	0.57	4.9	3.7	5.9	5.3
200	-1.53	-0.38	-0.05	-0.04	10.63	3.64	1.40	0.75	5.34	1.86	0.70	0.38	6.4	4.2	4.9	5.4
500	-0.53	-0.08	0.03	0.00	6.77	2.42	0.89	0.51	3.36	1.21	0.45	0.26	5.3	4.3	5.0	4.7
	GMM-B3															
50	-11.11	-2.96	-0.52	-0.25	17.81	7.34	2.77	1.62	12.14	3.96	1.42	0.82	14.5	8.6	7.2	6.4
100	-6.83	-1.63	-0.24	-0.09	12.83	4.98	1.95	1.14	8.25	2.82	1.00	0.57	10.7	5.9	6.0	5.1
200	-3.89	-0.89	-0.12	-0.07	9.51	3.39	1.38	0.73	5.76	1.89	0.67	0.37	9.2	5.0	5.6	5.6
500	-1.56	-0.29	-0.02	-0.01	6.55	2.34	0.90	0.51	3.35	1.19	0.45	0.26	6.5	4.9	5.0	4.6
	HPJ															
50	-1.17	0.88	0.41	0.10	9.43	4.91	2.20	1.38	4.80	2.55	1.19	0.70	12.0	10.4	7.7	6.2
100	-1.08	1.17	0.55	0.16	6.70	3.65	1.69	0.95	3.52	1.99	0.94	0.49	10.8	13.9	10.8	6.3
200	-0.79	0.93	0.51	0.16	4.72	2.56	1.18	0.66	2.33	1.44	0.69	0.35	10.8	14.8	12.1	6.4
500	-0.71	0.93	0.47	0.14	3.12	1.67	0.73	0.46	1.53	1.06	0.51	0.24	13.6	19.0	17.9	8.9
	BOD															
50	1.95	2.18	1.92	1.34	5.91	3.17	1.84	1.25	3.28	2.46	1.95	1.34	10.8	17.0	31.7	33.0
100	2.17	2.51	1.96	1.39	4.25	2.30	1.38	0.89	2.65	2.51	1.96	1.39	12.6	33.2	53.3	60.0
200	2.44	2.55	1.97	1.43	3.06	1.63	0.85	0.60	2.53	2.55	1.97	1.43	20.1	55.2	83.7	88.3
500	2.50	2.54	1.98	1.42	1.82	1.07	0.54	0.41	2.50	2.54	1.98	1.42	43.8	88.8	99.4	99.8
	BCBOD															
50	0.34	0.24	0.21	0.05	12.46	7.66	4.45	3.33	6.18	3.84	2.25	1.65	31.1	37.7	40.3	42.6
100	0.47	0.32	0.24	0.08	8.16	5.03	3.18	2.37	4.12	2.59	1.59	1.19	28.9	34.4	41.8	43.2
200	0.66	0.43	0.27	0.15	6.05	3.71	2.38	1.58	3.03	1.90	1.20	0.80	32.2	34.5	43.4	42.7
500	0.35	0.26	0.20	0.14	3.64	2.40	1.53	1.06	1.88	1.23	0.76	0.55	32.2	35.8	42.5	45.6
	BCFE															
50	-6.29	-2.09	-0.34	-0.13	6.02	3.55	1.95	1.24	6.30	2.31	0.97	0.65	36.9	18.9	8.7	6.9
100	-6.22	-1.84	-0.28	-0.07	4.35	2.53	1.38	0.89	6.22	1.90	0.74	0.45	58.2	23.3	9.8	6.3
200	-5.95	-1.92	-0.34	-0.07	3.08	1.69	0.86	0.61	5.95	1.92	0.50	0.32	83.2	38.3	9.4	5.2
500	-5.99	-1.99	-0.32	-0.08	1.90	1.19	0.56	0.41	5.99	1.99	0.38	0.21	99.2	72.2	13.9	7.5

Table S.4: AR(1) model, Bias, IQR, MAE and Size ($\alpha = 0.8,\ \sigma_\eta^2 = 5$)

	Bias				IQR				MAE				Size			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-22.08	-10.56	-3.86	-1.91	5.47	3.38	1.91	1.22	22.08	10.56	3.86	1.91	100.0	99.0	83.1	57.0
100	-22.02	-10.33	-3.80	-1.86	3.96	2.41	1.35	0.88	22.02	10.33	3.80	1.86	100.0	100.0	98.1	82.6
200	-21.77	-10.40	-3.86	-1.85	2.80	1.61	0.85	0.61	21.77	10.40	3.86	1.85	100.0	100.0	100.0	98.5
500	-21.81	-10.46	-3.84	-1.86	1.73	1.13	0.54	0.40	21.81	10.46	3.84	1.86	100.0	100.0	100.0	100.0
	IV-L															
50	7.40	3.30	0.07	0.14	64.12	31.88	15.77	8.85	32.77	15.98	7.58	4.43	0.5	3.5	7.8	7.6
100	4.00	0.85	-0.22	0.30	43.93	24.30	11.65	6.24	22.67	11.93	5.51	3.13	0.9	3.2	6.8	6.6
200	1.98	0.64	0.08	-0.07	34.67	16.47	7.66	4.62	17.36	8.28	3.77	2.31	1.8	5.3	6.2	5.6
500	0.60	0.12	-0.04	0.10	20.71	10.44	4.83	2.81	10.36	5.14	2.31	1.41	2.0	3.6	5.7	5.0
	GMM-L1															
50	-22.69	-10.13	-3.38	-1.38	37.49	20.61	10.95	6.87	24.27	11.21	5.06	3.26	4.1	4.2	4.6	5.6
100	-17.24	-8.16	-2.52	-0.94	34.95	18.04	9.48	5.30	19.21	9.71	4.55	2.59	3.2	2.8	3.4	5.6
200	-11.90	-5.03	-1.56	-0.66	28.85	15.02	7.18	4.31	16.21	7.44	3.55	2.13	2.1	2.3	3.9	4.8
500	-5.56	-2.31	-0.89	-0.26	18.81	9.65	4.61	2.67	9.64	4.77	2.29	1.38	1.6	2.3	4.2	5.3
	GMM-L3															
50	-12.36	-4.72	-1.22	-0.52	17.29	8.74	4.10	2.69	12.83	5.74	2.26	1.37	15.7	9.7	8.5	7.0
100	-8.52	-2.75	-0.76	-0.27	13.47	6.47	3.08	2.04	9.36	3.89	1.66	1.02	12.2	7.7	6.1	6.7
200	-4.78	-1.53	-0.36	-0.18	10.97	4.75	2.27	1.40	6.39	2.75	1.13	0.70	8.7	6.8	5.3	5.9
500	-2.29	-0.53	-0.13	-0.05	6.79	3.28	1.45	0.96	3.74	1.63	0.73	0.48	6.4	5.8	4.6	5.7
	IV-B															
50	-0.19	-0.09	0.07	-0.04	21.04	8.30	3.04	1.69	10.48	4.15	1.50	0.83	3.9	4.1	5.9	5.8
100	-0.17	0.06	0.03	-0.02	15.93	5.59	2.11	1.17	8.04	2.83	1.07	0.58	4.0	4.3	5.9	5.0
200	-0.51	-0.08	0.00	-0.02	11.31	3.83	1.47	0.78	5.59	1.90	0.74	0.38	5.1	5.1	5.0	5.3
500	-0.07	0.04	0.03	0.01	7.34	2.57	0.92	0.54	3.71	1.31	0.46	0.27	4.4	5.3	4.6	4.8
	GMM-B1															
50	-5.18	-1.40	-0.13	-0.14	19.05	7.54	2.86	1.70	10.18	3.86	1.40	0.84	6.0	5.3	6.4	6.3
100	-2.77	-0.77	-0.13	-0.05	14.22	5.33	2.04	1.15	7.25	2.70	1.01	0.57	4.9	3.7	5.9	5.3
200	-1.53	-0.38	-0.05	-0.04	10.63	3.64	1.40	0.75	5.34	1.86	0.70	0.38	6.4	4.2	4.9	5.4
500	-0.53	-0.08	0.03	0.00	6.77	2.42	0.89	0.51	3.36	1.21	0.45	0.26	5.3	4.3	5.0	4.7
	GMM-B3															
50	-11.11	-2.96	-0.52	-0.25	17.81	7.34	2.77	1.62	12.14	3.96	1.42	0.82	14.5	8.6	7.2	6.4
100	-6.83	-1.63	-0.24	-0.09	12.83	4.98	1.95	1.14	8.25	2.82	1.00	0.57	10.7	5.9	6.0	5.1
200	-3.89	-0.89	-0.12	-0.07	9.51	3.39	1.38	0.73	5.76	1.89	0.67	0.37	9.2	5.0	5.6	5.6
500	-1.56	-0.29	-0.02	-0.01	6.55	2.34	0.90	0.51	3.35	1.19	0.45	0.26	6.5	4.9	5.0	4.6
	HPJ															
50	-1.17	0.88	0.41	0.10	9.43	4.91	2.20	1.38	4.80	2.55	1.19	0.70	12.0	10.4	7.7	6.2
100	-1.08	1.17	0.55	0.16	6.70	3.65	1.69	0.95	3.52	1.99	0.94	0.49	10.8	13.9	10.8	6.3
200	-0.79	0.93	0.51	0.16	4.72	2.56	1.18	0.66	2.33	1.44	0.69	0.35	10.8	14.8	12.1	6.4
500	-0.71	0.93	0.47	0.14	3.12	1.67	0.73	0.46	1.53	1.06	0.51	0.24	13.6	19.0	17.9	8.9
	BOD															
50	2.13	2.37	1.99	1.36	5.93	3.15	1.86	1.22	3.28	2.51	2.01	1.37	11.5	18.8	33.7	34.5
100	2.33	2.63	2.03	1.41	4.27	2.33	1.39	0.89	2.80	2.64	2.03	1.41	13.9	36.5	56.3	62.0
200	2.56	2.70	2.03	1.45	3.06	1.60	0.86	0.60	2.65	2.70	2.03	1.45	22.7	59.2	85.5	89.8
500	2.66	2.68	2.04	1.45	1.80	1.05	0.55	0.40	2.66	2.68	2.04	1.45	48.7	92.1	99.5	99.8
	BCBOD															
50	0.82	-0.03	0.39	-0.13	23.05	15.01	8.97	6.80	11.42	7.39	4.49	3.41	57.5	62.6	65.7	72.8
100	0.54	0.21	0.36	-0.02	15.22	9.92	6.45	4.87	7.75	4.95	3.23	2.44	57.3	59.6	67.8	70.9
200	1.03	0.30	0.30	0.15	11.63	7.38	4.79	3.27	5.80	3.79	2.37	1.66	61.3	63.2	66.0	68.0
500	0.56	0.20	0.19	0.19	7.20	4.82	2.96	2.21	3.64	2.41	1.53	1.09	58.2	63.9	68.0	69.8
	BCFE															
50	-6.29	-2.09	-0.34	-0.13	6.02	3.55	1.95	1.24	6.30	2.31	0.97	0.65	36.9	18.9	8.7	6.9
100	-6.22	-1.84	-0.28	-0.07	4.35	2.53	1.38	0.89	6.22	1.90	0.74	0.45	58.2	23.3	9.8	6.3
200	-5.95	-1.92	-0.34	-0.07	3.08	1.69	0.86	0.61	5.95	1.92	0.50	0.32	83.2	38.3	9.4	5.2
500	-5.99	-1.99	-0.32	-0.08	1.90	1.19	0.56	0.41	5.99	1.99	0.38	0.21	99.2	72.2	13.9	7.5

Table S.5: Fixed effects, Bias and IQR ($\alpha = 0.4$, $\beta = 1$, $\sigma_\eta^2 = 5$)

	Bias								IQR							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-11.43	-5.54	-2.21	-1.05	3.67	2.77	1.18	0.67	5.44	3.35	2.03	1.47	14.16	9.67	5.95	4.01
100	-11.59	-5.60	-2.20	-1.10	3.23	2.54	1.08	0.73	3.92	2.52	1.56	1.13	10.42	6.58	4.22	2.97
200	-11.61	-5.51	-2.17	-1.06	3.46	2.47	1.15	0.60	2.61	1.78	1.14	0.77	7.06	4.57	2.85	1.98
500	-11.50	-5.52	-2.15	-1.07	3.71	2.63	1.24	0.61	1.72	1.13	0.74	0.51	4.76	2.88	1.79	1.29
1000	-11.41	-5.51	-2.16	-1.07	3.67	2.55	1.23	0.67	1.21	0.78	0.47	0.36	3.11	2.03	1.26	0.88
	IV-L															
50	0.45	0.10	0.10	0.26	2.54	0.82	-0.25	0.32	35.71	21.84	12.07	8.24	93.05	44.35	22.75	14.34
100	0.57	-0.42	-0.15	-0.09	1.98	-0.76	-0.02	-0.15	26.88	14.64	7.88	5.82	64.08	31.60	14.33	9.70
200	-1.54	-0.18	0.01	-0.12	-2.73	-0.42	-0.03	-0.04	18.55	9.93	5.86	3.92	42.91	21.54	10.00	6.70
500	-0.65	0.14	0.21	0.05	0.01	0.07	0.22	-0.02	11.27	6.75	3.74	2.59	28.40	14.12	6.63	4.55
1000	-0.18	0.17	0.11	0.04	-0.42	0.28	0.23	0.12	8.09	4.52	2.76	1.91	20.30	9.54	4.83	3.29
	GMM-L1															
50	-5.17	-1.64	-0.77	-0.42	-9.82	-2.14	-1.57	-0.31	16.46	11.24	6.38	4.84	44.85	22.96	12.98	8.93
100	-3.90	-2.04	-0.74	-0.36	-8.20	-3.66	-1.07	-0.78	15.62	9.81	5.73	4.37	37.99	20.35	10.53	7.48
200	-3.02	-1.18	-0.42	-0.30	-6.34	-2.66	-1.00	-0.47	12.26	7.65	4.39	3.24	29.27	15.24	7.89	5.54
500	-1.77	-0.58	-0.09	-0.11	-3.98	-1.14	-0.45	-0.21	8.85	5.25	3.18	2.40	21.54	11.18	5.87	4.05
1000	-0.91	-0.24	-0.16	-0.01	-1.99	-0.28	-0.03	-0.04	6.45	3.73	2.47	1.65	15.70	8.40	4.37	2.86
	GMM-L3															
50	-3.68	-1.42	-0.50	-0.20	-3.28	-0.87	-0.12	0.01	8.23	5.00	2.77	2.03	22.21	12.70	6.81	4.95
100	-2.23	-0.94	-0.31	-0.12	-2.06	-0.98	-0.29	-0.02	6.80	3.69	2.06	1.60	17.33	9.34	5.10	3.57
200	-1.11	-0.51	-0.10	-0.03	-1.39	-0.46	-0.23	-0.05	4.56	2.99	1.53	1.13	12.77	7.09	3.82	2.56
500	-0.48	-0.19	-0.11	-0.03	-0.64	-0.24	-0.19	-0.04	3.27	1.79	1.03	0.76	8.18	4.37	2.30	1.57
1000	-0.20	-0.11	-0.04	-0.02	-0.22	-0.24	-0.05	0.05	2.27	1.37	0.74	0.51	5.55	3.12	1.68	1.06
	IV-B															
50	-0.38	-0.13	-0.01	0.02	2.27	0.36	0.09	-0.03	9.86	5.17	2.37	1.58	27.53	13.94	6.48	4.23
100	-0.48	-0.02	-0.03	-0.02	0.50	-0.07	-0.12	0.12	6.67	3.41	1.74	1.24	21.01	9.51	4.68	3.14
200	-0.05	-0.09	0.02	0.02	-0.14	-0.08	-0.14	0.02	4.66	2.62	1.33	0.86	13.84	6.94	3.51	2.27
500	0.01	-0.05	0.00	-0.01	0.11	0.04	-0.07	-0.03	3.36	1.54	0.84	0.56	9.23	4.20	2.13	1.47
1000	0.09	0.00	0.00	0.00	0.14	-0.13	-0.03	0.04	2.22	1.23	0.57	0.39	6.21	2.98	1.62	0.95
	GMM-B1															
50	-1.64	-0.51	-0.14	-0.05	0.64	0.10	0.11	0.06	9.60	4.88	2.36	1.58	26.26	13.12	6.42	4.24
100	-1.04	-0.32	-0.13	-0.04	-0.03	-0.30	-0.04	0.13	6.80	3.41	1.74	1.25	19.76	9.26	4.75	3.10
200	-0.39	-0.17	0.01	0.00	0.00	-0.18	-0.11	0.00	4.56	2.54	1.33	0.85	13.35	6.86	3.50	2.24
500	-0.15	-0.11	0.00	-0.01	0.19	-0.03	-0.02	-0.02	3.24	1.51	0.81	0.55	8.87	4.10	2.19	1.43
1000	-0.03	-0.02	-0.02	-0.01	0.03	-0.09	-0.05	0.06	2.10	1.15	0.58	0.39	5.94	3.08	1.58	0.95
	GMM-B3															
50	-3.90	-1.23	-0.34	-0.12	-0.39	0.04	0.11	0.05	8.78	4.79	2.35	1.53	24.82	12.97	6.34	4.17
100	-2.18	-0.69	-0.24	-0.08	-1.03	-0.35	0.04	0.10	6.57	3.35	1.74	1.22	18.10	9.16	4.68	3.16
200	-0.99	-0.35	-0.07	-0.02	-0.49	-0.26	-0.12	0.03	4.46	2.48	1.27	0.85	13.47	6.87	3.43	2.27
500	-0.36	-0.17	-0.04	-0.02	0.14	-0.04	-0.03	-0.01	3.14	1.50	0.82	0.56	8.74	4.22	2.20	1.42
1000	-0.14	-0.07	-0.03	-0.01	-0.20	-0.09	-0.06	0.03	2.06	1.16	0.57	0.40	5.96	3.06	1.57	0.93
	HPJ															
50	1.24	0.55	0.06	0.03	5.29	1.91	0.11	-0.01	7.69	4.10	2.34	1.52	17.78	10.28	6.03	3.96
100	1.29	0.37	0.04	0.01	5.58	1.83	0.27	0.15	5.23	2.91	1.62	1.17	12.91	6.94	4.26	3.01
200	1.06	0.36	0.03	0.04	5.21	1.51	0.20	0.02	3.59	1.99	1.17	0.78	8.16	4.98	2.87	1.96
500	1.22	0.40	0.09	0.02	5.58	1.58	0.27	0.05	2.27	1.39	0.79	0.51	5.55	3.07	1.84	1.32
1000	1.33	0.43	0.08	0.02	5.64	1.54	0.26	0.08	1.59	0.94	0.51	0.38	3.86	2.20	1.23	0.90
	BOD-OLS															
50	-0.21	0.23	0.29	0.26	11.71	8.41	4.61	2.96	6.21	3.71	2.14	1.51	16.85	10.27	6.13	3.97
100	-0.22	0.28	0.31	0.26	11.57	8.32	4.76	3.07	4.59	2.63	1.68	1.16	11.34	7.13	4.29	3.09
200	-0.11	0.39	0.36	0.30	11.67	8.30	4.66	2.95	3.13	1.91	1.17	0.80	8.03	5.00	3.03	2.06
500	0.05	0.41	0.41	0.29	12.01	8.31	4.76	2.97	2.03	1.27	0.75	0.51	4.95	3.08	1.88	1.31
1000	0.11	0.43	0.38	0.29	12.04	8.39	4.77	2.99	1.43	0.86	0.50	0.37	3.42	2.23	1.33	0.92
	BCBOD-OLS															
50	-0.19	-0.08	0.16	0.13	1.35	1.12	0.88	1.31	19.51	11.85	7.23	5.30	42.03	29.19	19.09	13.56
100	-0.74	-0.01	0.03	0.00	0.20	0.76	0.37	0.69	13.06	8.71	4.96	3.75	32.76	20.75	13.38	9.40
200	-0.60	-0.17	0.05	0.07	0.13	0.30	0.39	0.33	9.47	6.03	3.57	2.49	22.80	15.22	9.37	6.36
500	-0.51	-0.27	0.01	-0.02	0.88	0.61	0.21	0.04	5.76	3.65	2.40	1.67	14.04	8.99	5.87	4.24
1000	-0.44	-0.14	-0.03	-0.01	1.26	1.03	0.25	0.17	3.63	2.74	1.60	1.26	9.87	6.67	3.94	3.03

Table S.6: Fixed effects model, MAE and Size ($\alpha = 0.4$, $\beta = 1$, $\sigma_\eta^2 = 5$)

	MAE								Size							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	11.43	5.54	2.21	1.11	7.69	5.24	3.11	2.06	84.1	56.6	25.6	15.2	9.8	10.0	7.3	5.6
100	11.59	5.60	2.20	1.10	5.56	3.96	2.23	1.54	98.0	85.8	48.0	28.0	11.6	9.6	7.5	6.9
200	11.61	5.51	2.17	1.06	4.35	3.01	1.65	1.10	100.0	99.0	74.7	46.9	11.3	12.9	9.2	6.1
500	11.50	5.52	2.15	1.07	3.83	2.66	1.32	0.78	100.0	100.0	98.9	81.8	19.7	21.7	15.1	11.0
1000	11.41	5.51	2.16	1.07	3.68	2.55	1.24	0.70	100.0	100.0	100.0	98.9	33.7	39.0	27.1	18.5
	IV-L															
50	17.81	10.81	6.04	4.06	46.20	21.92	11.33	7.00	1.0	3.0	3.1	4.6	1.3	2.4	3.2	4.5
100	13.10	7.38	3.99	2.87	31.39	15.72	7.17	4.78	1.4	4.5	4.2	5.1	1.5	3.3	3.9	4.1
200	9.20	5.08	2.93	1.95	22.46	10.78	5.02	3.44	2.0	4.7	5.3	5.7	2.1	3.9	3.9	5.1
500	5.47	3.37	1.88	1.30	14.33	7.03	3.40	2.26	3.4	3.8	4.5	5.7	3.3	3.2	4.2	5.3
1000	3.98	2.25	1.37	0.96	10.00	4.72	2.40	1.62	5.2	4.1	4.5	5.9	5.2	3.6	4.8	5.8
	GMM-L1															
50	9.13	5.84	3.30	2.41	23.44	12.19	6.43	4.34	5.3	5.9	6.0	5.9	5.1	6.8	5.8	6.1
100	7.86	4.77	2.95	2.17	20.11	10.33	5.37	3.67	4.4	6.0	4.9	5.8	4.2	5.4	4.9	5.0
200	6.41	4.02	2.19	1.63	15.13	7.95	3.94	2.82	5.0	4.5	4.8	5.5	5.0	4.0	4.5	5.0
500	4.54	2.66	1.61	1.18	10.75	5.60	2.95	1.99	5.5	4.1	4.4	5.1	5.6	4.0	3.6	4.5
1000	3.35	1.96	1.22	0.82	7.89	4.17	2.14	1.43	5.4	4.7	5.6	5.4	5.5	3.8	5.8	4.5
	GMM-L3															
50	5.02	2.72	1.48	1.03	11.30	6.46	3.41	2.42	8.1	6.0	5.8	6.4	5.7	6.8	7.1	6.4
100	3.60	2.04	1.06	0.80	8.74	4.72	2.57	1.77	8.0	6.4	6.4	7.3	7.9	7.5	5.7	5.7
200	2.48	1.48	0.78	0.56	6.43	3.61	1.89	1.29	5.7	5.2	5.7	6.0	5.7	6.4	5.5	5.7
500	1.64	0.93	0.53	0.38	4.11	2.22	1.18	0.77	4.6	5.6	5.6	5.8	6.0	5.2	5.0	5.9
1000	1.10	0.68	0.36	0.26	2.75	1.59	0.84	0.53	4.7	5.4	4.2	4.8	5.6	5.5	5.9	5.0
	IV-B															
50	4.90	2.59	1.18	0.78	14.03	6.99	3.26	2.12	6.2	4.7	5.7	5.4	5.9	6.6	6.5	6.4
100	3.40	1.71	0.88	0.63	10.30	4.76	2.33	1.56	5.7	4.9	5.7	5.2	5.1	7.1	6.4	5.5
200	2.33	1.30	0.67	0.43	6.93	3.47	1.75	1.13	4.5	4.5	5.8	3.8	5.0	5.8	4.7	5.1
500	1.69	0.78	0.42	0.28	4.60	2.09	1.06	0.73	4.8	4.1	4.8	6.1	5.6	5.0	5.4	5.6
1000	1.11	0.61	0.29	0.19	3.11	1.50	0.80	0.47	3.8	6.1	5.2	5.3	4.2	4.8	5.3	6.4
	GMM-B1															
50	5.03	2.46	1.17	0.78	12.82	6.62	3.18	2.08	7.1	4.6	5.4	5.6	6.4	6.5	6.8	6.2
100	3.52	1.76	0.87	0.63	9.88	4.60	2.35	1.56	6.3	5.2	5.8	5.2	5.9	6.3	6.0	5.1
200	2.31	1.24	0.65	0.42	6.68	3.41	1.73	1.12	5.1	4.9	5.6	4.6	5.3	5.9	5.0	4.7
500	1.64	0.75	0.41	0.28	4.43	2.03	1.09	0.71	4.6	3.8	4.7	5.7	5.3	5.1	5.7	6.1
1000	1.04	0.58	0.29	0.20	3.00	1.53	0.80	0.47	3.7	6.0	4.5	5.5	4.3	4.9	5.3	6.3
	GMM-B3															
50	5.28	2.50	1.17	0.76	12.44	6.48	3.15	2.08	9.2	5.3	5.2	5.8	6.7	7.6	6.7	5.9
100	3.68	1.75	0.88	0.61	9.27	4.53	2.34	1.58	7.7	5.9	5.9	5.2	5.6	6.1	6.0	5.6
200	2.24	1.29	0.65	0.43	6.86	3.47	1.72	1.13	6.4	4.5	5.9	5.2	4.9	5.3	4.7	4.7
500	1.60	0.77	0.40	0.28	4.36	2.10	1.09	0.71	4.7	3.9	4.7	5.7	6.1	5.2	6.0	6.0
1000	1.03	0.58	0.29	0.19	2.99	1.51	0.78	0.47	3.6	6.5	4.7	5.8	4.6	5.2	5.7	6.3
	HPJ															
50	3.92	2.05	1.16	0.76	9.50	5.24	3.10	1.98	8.1	5.7	5.3	4.6	9.0	6.3	4.6	4.8
100	2.76	1.44	0.82	0.59	7.56	3.92	2.14	1.50	8.4	5.5	5.5	5.5	10.3	5.4	5.1	4.1
200	1.99	1.06	0.58	0.39	5.92	2.77	1.46	0.99	9.1	5.2	5.5	5.9	13.2	7.0	4.5	4.0
500	1.51	0.74	0.41	0.26	5.62	1.91	0.96	0.65	12.0	7.2	5.3	6.1	29.6	8.6	4.6	5.9
1000	1.38	0.57	0.26	0.19	5.64	1.63	0.65	0.45	20.8	10.4	4.8	5.0	52.0	13.1	5.3	5.4
	BOD-OLS															
50	3.12	1.83	1.13	0.74	12.54	8.72	4.79	3.06	6.3	4.8	5.7	6.2	20.6	24.9	22.0	19.5
100	2.33	1.33	0.84	0.61	11.60	8.33	4.76	3.07	5.6	6.0	6.7	7.1	29.3	37.1	36.1	32.2
200	1.54	1.01	0.62	0.45	11.67	8.30	4.66	2.95	5.3	5.6	7.5	8.3	52.1	62.8	60.1	51.1
500	1.01	0.69	0.49	0.35	12.01	8.31	4.76	2.97	5.1	6.6	11.8	14.2	90.3	95.5	94.0	87.3
1000	0.71	0.56	0.40	0.30	12.04	8.39	4.77	2.99	4.2	9.4	17.2	20.0	99.6	99.9	99.8	99.1
	BCBOD-OLS															
50	9.58	5.93	3.60	2.65	20.89	14.43	9.67	6.77	51.6	50.2	52.6	55.7	43.5	48.8	53.6	54.5
100	6.53	4.36	2.48	1.86	16.28	10.20	6.59	4.72	49.2	52.2	51.4	54.2	48.7	49.2	53.2	56.1
200	4.70	3.00	1.79	1.26	11.41	7.53	4.69	3.25	50.2	51.4	52.0	52.8	49.2	50.9	53.7	54.9
500	2.94	1.86	1.18	0.83	7.01	4.43	2.93	2.09	50.5	50.9	53.8	55.5	48.1	48.4	54.2	55.4
1000	1.91	1.36	0.80	0.63	5.10	3.52	2.01	1.50	45.2	52.1	52.5	56.6	48.8	53.0	52.0	55.8

Table S.7: Fixed effects model, Bias and IQR ($\alpha = 0.8$, $\beta = 1$, $\sigma_\eta^2 = 5$)

	Bias								IQR							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-15.05	-6.91	-2.57	-1.20	0.57	2.30	1.54	0.86	4.65	2.71	1.57	0.97	13.95	9.60	5.77	3.92
100	-15.14	-6.85	-2.52	-1.23	0.11	2.22	1.39	0.94	3.46	1.98	1.07	0.67	10.37	6.65	4.06	2.94
200	-15.08	-6.91	-2.50	-1.21	0.26	2.09	1.45	0.79	2.33	1.38	0.74	0.48	7.12	4.49	2.84	1.97
500	-15.01	-6.88	-2.48	-1.19	0.69	2.21	1.54	0.84	1.55	0.83	0.48	0.32	4.75	2.92	1.77	1.27
1000	-14.92	-6.86	-2.48	-1.20	0.58	2.22	1.49	0.89	1.06	0.63	0.31	0.22	3.15	2.00	1.23	0.87
	IV-L															
50	1.58	4.30	1.63	0.70	2.98	7.03	1.81	0.86	53.22	31.07	17.89	11.58	105.31	53.17	24.56	14.34
100	3.63	1.53	0.19	-0.15	7.80	2.13	0.29	-0.31	46.23	27.00	11.86	8.05	87.34	43.08	16.25	9.58
200	-0.27	0.58	0.13	-0.14	-0.80	0.81	-0.03	-0.11	38.00	18.44	8.60	5.41	73.81	31.03	11.33	6.77
500	-0.24	0.32	0.36	0.05	0.48	0.27	0.28	0.00	25.98	12.20	5.66	3.57	50.10	19.79	7.35	4.38
1000	-0.14	0.27	0.18	0.09	-0.06	0.23	0.25	0.14	18.54	8.29	4.00	2.56	35.55	13.58	5.33	3.13
	GMM-L1															
50	-12.31	-4.67	-1.92	-0.94	-21.83	-7.56	-2.17	-1.05	19.18	11.38	5.73	4.14	41.30	20.50	8.89	6.25
100	-11.57	-4.62	-1.83	-0.95	-19.02	-7.11	-2.58	-1.09	18.76	10.73	5.37	3.89	37.74	18.82	8.12	5.32
200	-9.66	-3.93	-1.61	-0.77	-18.43	-6.77	-2.14	-0.95	18.04	9.16	4.94	3.35	34.32	15.42	6.84	3.99
500	-6.41	-2.39	-0.74	-0.49	-12.74	-4.25	-1.11	-0.55	15.11	7.21	3.66	2.50	29.33	11.50	5.26	3.08
1000	-4.30	-1.15	-0.40	-0.16	-8.34	-2.05	-0.56	-0.21	11.80	5.85	2.89	1.94	23.34	9.83	4.13	2.37
	GMM-L3															
50	-10.64	-3.96	-1.26	-0.48	-14.63	-4.11	-1.06	-0.32	10.03	5.60	2.65	1.72	25.13	12.84	6.56	4.40
100	-7.62	-2.91	-0.84	-0.34	-11.50	-3.40	-0.91	-0.16	9.26	4.43	2.17	1.55	21.55	9.96	4.91	3.16
200	-5.42	-1.70	-0.49	-0.17	-8.37	-2.52	-0.66	-0.22	7.40	3.73	1.70	1.07	16.32	8.03	3.70	2.30
500	-2.75	-0.76	-0.17	-0.07	-4.41	-1.06	-0.22	-0.06	5.00	2.46	1.18	0.76	11.14	4.94	2.36	1.51
1000	-1.39	-0.41	-0.10	-0.06	-2.27	-0.58	-0.13	0.02	3.91	1.87	0.84	0.52	8.68	3.59	1.67	1.06
	IV-B															
50	-0.69	0.27	-0.01	-0.03	0.88	0.56	-0.06	-0.04	18.89	6.26	2.15	1.23	41.27	15.56	6.57	4.12
100	-1.06	0.07	-0.04	-0.05	-0.50	0.11	-0.03	0.07	12.95	4.58	1.65	0.92	30.19	10.98	4.66	3.01
200	-0.10	0.02	0.02	-0.01	-0.40	-0.31	-0.11	0.04	9.15	3.17	1.15	0.64	20.41	7.42	3.45	2.19
500	-0.03	0.01	0.02	0.01	0.38	0.07	-0.02	-0.03	5.98	1.96	0.72	0.43	13.74	4.73	2.18	1.42
1000	0.07	0.02	0.01	0.00	0.29	0.01	-0.03	0.03	4.06	1.50	0.48	0.31	9.37	3.44	1.64	0.98
	GMM-B1															
50	-5.97	-1.14	-0.26	-0.12	-6.75	-0.77	0.16	0.01	14.61	5.64	2.03	1.17	32.25	14.79	6.47	4.10
100	-3.70	-0.63	-0.13	-0.09	-4.97	-0.85	-0.07	0.08	10.65	4.37	1.50	0.89	24.32	9.99	4.78	3.08
200	-1.76	-0.31	-0.05	-0.03	-2.06	-0.66	-0.16	0.03	8.14	3.07	1.09	0.61	18.14	7.44	3.42	2.19
500	-0.83	-0.18	0.01	0.00	-1.08	-0.13	0.00	-0.03	5.05	1.91	0.70	0.40	12.31	4.60	2.19	1.39
1000	-0.40	-0.05	0.00	-0.01	-0.45	-0.10	-0.02	0.05	3.67	1.41	0.48	0.29	8.54	3.47	1.61	0.98
	GMM-B3															
50	-10.63	-2.98	-0.61	-0.27	-11.36	-2.03	0.03	0.07	11.55	4.73	2.10	1.12	28.72	13.49	6.39	4.14
100	-7.20	-1.76	-0.37	-0.17	-9.54	-1.55	-0.14	0.12	9.37	3.86	1.48	0.86	22.23	9.93	4.57	3.07
200	-4.31	-0.96	-0.19	-0.07	-5.72	-1.31	-0.21	0.07	6.99	2.98	1.05	0.62	17.21	7.27	3.30	2.15
500	-1.97	-0.40	-0.03	-0.03	-2.64	-0.36	-0.02	-0.02	4.77	1.81	0.71	0.40	11.13	4.68	2.12	1.40
1000	-0.95	-0.21	-0.02	-0.02	-1.60	-0.21	-0.03	0.04	3.63	1.38	0.48	0.30	8.23	3.30	1.63	0.97
	HPJ															
50	2.04	1.21	0.30	0.09	7.50	4.21	0.75	0.21	8.05	4.21	1.91	1.04	18.86	10.27	5.96	3.93
100	1.93	1.21	0.36	0.09	7.98	4.08	0.82	0.33	5.35	2.85	1.28	0.73	13.53	7.23	4.15	2.95
200	1.87	1.27	0.33	0.09	7.51	3.82	0.80	0.21	3.68	1.97	0.90	0.53	8.60	5.08	2.82	1.96
500	2.00	1.20	0.37	0.11	7.87	3.88	0.84	0.21	2.42	1.30	0.59	0.34	5.80	3.05	1.84	1.29
1000	1.93	1.17	0.36	0.10	8.02	3.87	0.84	0.24	1.71	0.94	0.38	0.24	4.04	2.28	1.24	0.88
	BOD-OLS															
50	0.35	0.81	0.64	0.53	8.69	6.92	4.01	2.61	4.99	2.50	1.51	0.95	16.08	10.06	5.95	3.98
100	0.51	0.89	0.76	0.52	8.23	6.53	4.06	2.71	3.52	2.00	1.05	0.71	11.29	7.02	4.26	2.99
200	0.53	0.84	0.75	0.56	8.10	6.52	4.03	2.60	2.43	1.39	0.71	0.48	8.00	4.75	2.89	1.99
500	0.77	0.95	0.79	0.59	8.60	6.60	4.09	2.63	1.69	0.86	0.50	0.32	4.86	2.99	1.82	1.32
1000	0.82	0.94	0.79	0.58	8.49	6.63	4.08	2.66	1.12	0.60	0.34	0.22	3.45	2.22	1.31	0.88
	BCBOD-OLS															
50	0.88	0.22	0.07	0.11	2.29	1.74	1.05	1.28	19.37	12.75	7.58	5.58	41.88	27.42	18.14	12.32
100	-0.94	0.23	0.24	0.04	0.46	1.20	0.48	0.68	13.91	9.14	5.61	3.92	31.58	19.33	12.28	8.79
200	-0.39	-0.19	0.07	0.14	0.59	0.61	0.52	0.39	10.03	6.51	3.69	2.67	22.80	14.21	8.66	5.92
500	-0.58	-0.31	-0.01	0.04	1.26	1.10	0.43	0.28	5.85	4.26	2.54	1.80	13.24	8.58	5.43	3.94
1000	-0.55	-0.12	0.04	0.03	1.48	1.44	0.54	0.29	4.33	3.11	1.76	1.30	9.81	6.54	3.74	2.85

Table S.8: Fixed effects model, MAE and Size ($\alpha = 0.8,\ \ \beta = 1,\ \ \sigma_\eta^2 = 5$)

	MAE								Size							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	15.05	6.91	2.57	1.20	7.00	5.01	3.03	2.02	99.3	95.2	65.0	40.3	8.4	9.6	8.5	6.6
100	15.14	6.85	2.52	1.23	5.17	3.72	2.31	1.59	100.0	99.9	91.2	67.4	7.1	8.9	9.0	8.1
200	15.08	6.91	2.50	1.21	3.48	2.87	1.76	1.15	100.0	100.0	99.7	90.4	6.5	11.2	11.1	9.3
500	15.01	6.88	2.48	1.19	2.32	2.31	1.56	0.91	100.0	100.0	100.0	99.9	5.5	18.0	21.5	15.6
1000	14.92	6.86	2.48	1.20	1.68	2.24	1.49	0.90	100.0	100.0	100.0	100.0	5.6	30.1	37.7	29.3
	IV-L															
50	26.23	16.62	9.08	5.63	53.40	26.80	12.81	7.10	0.2	0.8	3.0	4.9	0.7	0.8	2.4	4.3
100	23.22	13.45	5.73	4.03	45.27	21.64	8.18	4.73	0.2	1.6	3.6	4.6	0.2	1.1	2.8	3.4
200	19.34	9.31	4.19	2.73	36.94	15.54	5.70	3.34	0.2	1.7	4.3	4.6	0.1	1.8	2.9	4.0
500	12.95	6.11	2.86	1.79	24.79	9.91	3.72	2.18	0.6	2.0	4.7	4.8	0.8	1.8	4.0	4.2
1000	9.24	4.14	1.99	1.28	17.49	6.79	2.70	1.56	1.2	2.7	4.5	5.6	1.2	2.2	4.2	4.4
	GMM-L1															
50	13.18	6.06	3.11	2.10	25.43	10.97	4.85	3.08	9.1	6.9	5.8	6.1	8.2	6.7	6.5	6.6
100	12.65	6.27	3.06	1.96	22.79	11.12	4.54	2.65	6.8	4.9	6.1	5.1	5.6	4.6	4.7	5.1
200	11.23	5.17	2.66	1.67	21.41	9.30	3.80	2.16	5.9	4.7	5.6	5.4	5.6	4.3	5.9	4.7
500	8.54	3.94	1.93	1.26	16.27	6.51	2.77	1.53	6.2	3.4	4.3	4.5	5.7	3.7	3.8	5.1
1000	6.59	3.00	1.47	0.94	12.85	4.84	2.04	1.16	5.9	4.5	5.3	4.1	5.3	3.7	4.9	4.1
	GMM-L3															
50	10.70	4.13	1.66	0.93	16.77	7.12	3.41	2.20	22.8	13.9	8.1	8.5	12.2	8.5	5.9	6.4
100	7.78	3.13	1.25	0.78	13.31	5.56	2.48	1.61	19.4	9.9	8.3	7.5	13.6	9.0	5.5	6.2
200	5.65	2.21	0.92	0.55	9.90	4.11	1.93	1.15	13.7	8.7	5.8	6.2	9.8	8.2	4.9	5.4
500	3.31	1.31	0.61	0.39	6.55	2.63	1.18	0.76	9.0	7.4	6.2	5.3	8.5	5.8	5.8	5.9
1000	2.20	0.97	0.42	0.26	4.47	1.83	0.85	0.53	7.8	6.5	5.1	6.3	6.5	6.2	6.8	5.7
	IV-B															
50	9.42	3.15	1.07	0.61	20.56	7.62	3.27	2.06	2.9	4.8	5.2	5.9	2.7	6.7	6.6	5.6
100	6.50	2.29	0.83	0.47	15.09	5.51	2.33	1.50	4.1	4.7	5.8	4.8	4.2	6.4	6.4	5.4
200	4.65	1.56	0.58	0.32	10.19	3.68	1.72	1.10	4.1	4.3	5.6	5.2	5.6	5.8	4.8	5.2
500	2.99	0.98	0.35	0.22	6.88	2.36	1.08	0.71	5.1	4.2	5.0	5.6	6.0	4.3	5.1	6.4
1000	2.02	0.75	0.24	0.15	4.58	1.73	0.82	0.49	4.5	6.1	4.8	4.8	4.5	4.8	6.1	6.1
	GMM-B1															
50	8.34	2.85	1.05	0.58	16.46	7.31	3.23	2.05	9.4	5.3	4.6	6.7	5.8	6.9	5.9	6.3
100	6.23	2.13	0.77	0.45	12.68	4.97	2.33	1.55	7.2	4.9	6.0	4.9	4.9	6.6	6.4	5.2
200	4.21	1.47	0.55	0.30	9.18	3.73	1.70	1.10	5.1	4.0	5.8	5.6	5.4	5.7	5.1	5.0
500	2.65	0.96	0.35	0.20	6.15	2.37	1.08	0.70	5.0	4.6	4.8	5.8	6.2	4.3	5.6	6.3
1000	1.84	0.70	0.24	0.15	4.29	1.75	0.80	0.48	4.9	6.0	5.2	4.9	4.5	5.6	6.0	6.0
	GMM-B3															
50	10.77	3.33	1.12	0.58	16.37	6.92	3.19	2.04	23.2	10.5	6.3	6.6	8.4	7.7	6.0	6.4
100	7.47	2.22	0.81	0.45	12.97	4.96	2.31	1.53	18.6	9.4	6.7	5.1	10.2	6.3	5.8	5.6
200	4.77	1.57	0.57	0.31	9.16	3.63	1.65	1.08	12.3	7.2	5.9	5.4	8.4	5.8	4.5	5.0
500	2.82	0.93	0.34	0.19	5.70	2.39	1.05	0.70	7.2	5.6	5.3	5.7	6.7	4.5	6.0	6.1
1000	1.97	0.70	0.24	0.15	4.20	1.69	0.82	0.49	6.6	6.0	5.2	5.1	5.0	6.2	5.3	6.1
	HPJ															
50	4.14	2.11	0.99	0.53	10.47	6.28	2.95	1.92	10.5	9.5	7.5	5.2	11.9	9.2	4.9	4.6
100	3.13	1.56	0.68	0.37	8.92	4.75	2.17	1.51	13.0	13.5	7.8	5.6	16.3	12.0	5.5	4.3
200	2.42	1.38	0.51	0.28	7.68	4.05	1.50	1.02	16.8	17.9	9.7	7.3	23.3	16.7	6.5	4.6
500	2.04	1.22	0.41	0.19	7.87	3.89	1.08	0.65	26.5	33.8	15.5	7.4	49.5	35.4	9.0	6.1
1000	1.93	1.17	0.37	0.14	8.02	3.87	0.95	0.49	43.3	51.1	23.4	9.0	78.9	61.3	15.4	6.4
	BOD-OLS															
50	2.55	1.45	0.88	0.62	9.93	7.51	4.27	2.79	6.8	7.1	12.4	12.2	15.5	19.4	19.2	16.9
100	1.87	1.16	0.81	0.54	8.63	6.61	4.08	2.75	5.9	8.1	17.2	21.9	18.6	26.4	28.0	26.9
200	1.29	0.96	0.76	0.56	8.13	6.52	4.03	2.60	7.2	13.7	27.6	34.7	31.7	45.5	48.5	44.5
500	1.00	0.95	0.79	0.59	8.60	6.60	4.09	2.63	10.6	30.6	59.3	70.0	65.4	83.5	85.3	79.5
1000	0.86	0.94	0.79	0.58	8.49	6.63	4.08	2.66	17.4	52.2	88.1	93.4	92.7	99.2	98.8	98.1
	BCBOD-OLS															
50	9.59	6.37	3.81	2.78	20.89	13.81	9.01	6.19	58.6	64.8	66.6	71.4	44.4	47.3	50.6	52.0
100	6.89	4.60	2.82	2.00	15.92	9.58	6.17	4.41	60.1	65.8	68.7	71.2	48.7	47.8	51.9	52.8
200	5.07	3.22	1.86	1.35	11.73	7.15	4.43	3.02	60.2	65.9	67.5	69.6	50.2	50.6	51.4	51.9
500	3.07	2.05	1.27	0.89	6.96	4.30	2.73	1.99	58.7	65.4	71.3	73.1	48.7	48.3	51.9	53.3
1000	2.14	1.55	0.89	0.65	5.14	3.41	1.91	1.43	59.1	68.9	69.5	72.6	50.3	53.0	50.8	54.5

Table S.9: Trend model, Bias and IQR ($\alpha = 0.4,\ \ \beta = 1,\ \ \sigma_\eta^2 = 5$)

	Bias								IQR							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-24.78	-11.44	-4.38	-2.14	1.58	4.12	2.15	1.18	6.01	3.74	2.02	1.48	16.83	10.53	5.89	4.22
100	-24.84	-11.51	-4.45	-2.21	1.21	3.71	2.12	1.31	4.37	2.66	1.49	1.15	12.56	7.07	4.32	2.96
200	-24.79	-11.51	-4.38	-2.15	0.89	3.53	2.06	1.18	2.92	1.89	1.17	0.78	8.54	4.98	3.13	2.01
500	-24.51	-11.46	-4.37	-2.16	0.99	3.56	2.18	1.17	1.98	1.23	0.74	0.53	5.25	3.10	1.88	1.31
1000	-24.59	-11.44	-4.39	-2.17	1.04	3.59	2.19	1.24	1.32	0.87	0.50	0.36	3.57	2.13	1.35	0.90
	IV-L															
50	-4.65	-0.80	2.38	7.21	-15.35	0.01	8.10	13.39	72.0	67.9	69.8	68.3	367.3	231.0	161.3	135.9
100	-0.87	-1.06	1.12	7.47	-2.04	0.34	5.16	14.86	62.2	68.6	65.9	68.3	305.2	239.0	164.5	142.4
200	-2.82	-0.98	2.02	5.63	-10.16	1.03	4.93	12.22	58.4	61.2	64.1	63.6	268.1	213.4	159.4	130.7
500	-0.92	1.41	2.54	6.31	-3.62	5.69	6.60	13.38	44.1	47.0	49.7	56.6	217.6	156.6	120.8	114.9
1000	0.06	1.94	1.10	1.06	-1.13	5.74	3.09	3.20	28.2	31.6	38.2	48.3	137.3	111.6	94.8	99.1
	GMM-L1															
50	-9.77	-4.24	-1.94	-1.22	-20.98	-10.73	-4.27	-2.30	20.31	11.50	6.75	5.23	78.06	40.01	17.06	10.59
100	-7.80	-4.16	-1.59	-1.03	-21.75	-12.19	-3.40	-2.06	17.61	9.79	6.24	4.63	73.60	37.17	15.28	10.48
200	-6.31	-3.38	-1.28	-0.60	-21.90	-10.92	-3.10	-1.34	14.69	9.47	5.39	4.43	67.37	34.71	14.24	9.34
500	-4.33	-2.25	-0.62	-0.50	-20.95	-7.99	-1.96	-0.99	12.37	7.79	5.03	3.61	60.02	28.45	12.49	7.53
1000	-2.40	-1.41	-0.46	-0.45	-11.68	-4.90	-1.33	-0.88	10.94	6.44	4.06	3.00	51.88	24.32	10.58	6.44
	GMM-L3															
50	-11.35	-4.71	-1.60	-0.61	-18.49	-8.34	-1.83	-0.55	12.35	6.96	3.69	2.37	46.53	23.65	10.01	5.76
100	-8.04	-3.49	-0.94	-0.34	-19.82	-8.44	-1.38	-0.37	10.07	6.19	2.98	1.90	43.59	22.00	8.00	4.25
200	-6.08	-2.56	-0.48	-0.11	-18.17	-6.17	-0.99	-0.16	8.71	4.95	2.24	1.33	43.40	19.29	6.04	3.28
500	-4.04	-1.46	-0.28	-0.08	-17.91	-4.43	-0.57	-0.21	7.20	3.84	1.51	0.90	42.07	14.59	3.94	2.15
1000	-2.85	-1.00	-0.15	-0.05	-13.73	-3.75	-0.37	-0.01	6.60	2.97	1.03	0.59	34.33	12.41	3.17	1.52
	IV-B															
50	-2.06	-0.33	0.06	0.02	5.39	-2.12	-0.05	-0.24	49.81	12.86	3.96	2.03	277.31	51.73	10.99	5.99
100	-0.74	-0.18	-0.01	-0.01	3.89	-0.17	-0.12	-0.05	38.35	9.12	2.70	1.56	206.76	36.34	8.25	4.27
200	0.47	0.03	-0.02	0.05	5.26	-1.01	0.05	-0.07	25.08	5.98	1.95	1.13	146.81	23.71	5.79	2.85
500	0.22	-0.15	-0.04	0.00	0.82	-0.19	0.01	-0.02	15.36	4.06	1.29	0.70	94.88	16.11	3.55	1.89
1000	-0.36	-0.17	-0.03	0.00	0.49	-0.33	-0.07	0.03	10.78	2.81	0.82	0.52	66.57	11.21	2.59	1.32
	GMM-B1															
50	-14.03	-3.27	-0.41	-0.16	-13.31	-4.60	-0.04	-0.12	26.54	9.58	3.63	2.04	93.71	34.48	9.91	5.30
100	-8.72	-2.11	-0.29	-0.07	-13.92	-3.19	-0.20	0.00	20.17	7.86	2.66	1.47	87.69	28.26	7.62	3.98
200	-4.94	-0.98	-0.13	0.01	-11.93	-2.74	-0.08	-0.04	16.87	5.41	1.88	1.12	81.60	20.67	5.43	2.76
500	-2.81	-0.42	-0.09	0.00	-8.74	-1.33	-0.07	0.01	11.79	3.80	1.19	0.69	67.26	14.91	3.44	1.80
1000	-1.77	-0.32	-0.04	-0.01	-6.36	-0.96	0.00	0.03	9.49	2.62	0.80	0.49	53.67	10.60	2.49	1.29
	GMM-B3															
50	-23.78	-6.24	-1.28	-0.48	-16.78	-4.62	-0.09	-0.07	19.41	8.41	3.48	2.02	62.90	26.13	9.30	5.20
100	-17.67	-4.25	-0.69	-0.23	-17.53	-5.24	-0.33	0.03	15.79	6.60	2.50	1.48	58.41	21.25	7.06	3.92
200	-11.22	-2.35	-0.36	-0.05	-17.82	-4.45	-0.22	-0.06	14.22	5.26	1.87	1.08	54.55	17.72	5.31	2.75
500	-6.55	-1.01	-0.18	-0.03	-15.62	-2.07	-0.11	0.00	10.22	3.75	1.19	0.69	51.58	13.83	3.36	1.87
1000	-4.10	-0.65	-0.09	-0.03	-13.86	-1.57	-0.07	0.02	8.21	2.60	0.77	0.49	46.30	10.08	2.42	1.28
	HPJ															
50	4.17	1.80	0.28	0.08	17.95	6.51	1.00	0.21	10.60	5.21	2.50	1.60	24.23	12.59	5.94	4.42
100	4.08	1.85	0.23	0.03	18.96	5.85	0.96	0.27	7.65	3.75	1.83	1.14	17.16	8.84	4.50	2.97
200	4.36	1.70	0.28	0.08	18.89	5.81	0.97	0.25	5.19	2.67	1.32	0.82	12.03	5.76	3.09	2.05
500	4.63	1.74	0.30	0.06	18.97	6.08	0.96	0.20	3.33	1.68	0.83	0.54	8.01	3.78	1.93	1.31
1000	4.46	1.73	0.26	0.06	18.85	6.17	1.03	0.27	2.25	1.20	0.57	0.38	5.39	2.61	1.43	0.91

Table S.10: Trend model, MAE and Size ($\alpha = 0.4,\ \ \beta = 1,\ \ \sigma_\eta^2 = 5$)

	MAE								Size							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	24.78	11.44	4.38	2.14	8.28	6.07	3.39	2.09	100.0	98.6	77.5	48.0	8.0	11.3	10.6	8.2
100	24.84	11.51	4.45	2.21	6.22	4.46	2.58	1.71	100.0	100.0	96.5	75.0	7.1	12.3	12.4	10.0
200	24.79	11.51	4.38	2.15	4.39	3.84	2.19	1.30	100.0	100.0	100.0	96.2	6.7	16.5	19.3	13.3
500	24.51	11.46	4.37	2.16	2.71	3.56	2.19	1.19	100.0	100.0	100.0	100.0	6.3	31.9	37.5	26.0
1000	24.59	11.44	4.39	2.17	1.92	3.59	2.19	1.24	100.0	100.0	100.0	100.0	7.1	57.8	62.9	48.0
	IV-L															
50	36.57	34.51	34.84	36.46	184.7	115.3	82.0	71.2	0.2	0.1	0.2	0.3	0.2	0.1	0.2	0.3
100	30.96	33.88	33.06	36.45	152.6	119.6	80.2	73.5	0.2	0.2	0.2	0.2	0.0	0.0	0.3	0.0
200	29.44	30.85	32.05	31.66	135.1	108.3	78.0	65.2	0.2	0.0	0.1	0.3	0.2	0.2	0.1	0.3
500	21.78	23.20	24.60	29.39	106.1	78.4	60.5	59.6	0.2	0.3	0.2	0.0	0.1	0.1	0.2	0.1
1000	14.17	15.79	19.62	24.47	68.9	56.4	47.7	49.6	0.4	1.0	0.5	0.3	0.3	0.5	0.5	0.3
	GMM-L1															
50	11.92	6.31	3.42	2.67	42.00	21.06	8.67	5.66	9.7	7.3	5.4	5.5	3.0	5.8	4.5	5.4
100	10.00	5.79	3.29	2.35	40.19	20.20	8.33	5.36	7.4	5.3	5.4	5.8	3.8	4.4	4.4	4.8
200	8.63	5.52	2.81	2.15	36.63	18.81	7.43	4.84	6.0	4.5	6.1	5.6	3.2	3.9	5.5	4.9
500	6.87	4.29	2.49	1.81	33.27	15.71	6.08	3.75	3.8	5.1	5.9	4.9	3.3	4.5	5.5	5.3
1000	5.69	3.31	2.09	1.59	27.82	12.06	5.25	3.37	4.0	5.2	6.1	4.1	4.3	4.5	5.8	3.8
	GMM-L3															
50	11.43	5.04	2.14	1.25	28.20	13.19	5.07	2.88	23.9	15.2	10.1	6.7	8.3	7.7	7.4	6.6
100	8.37	4.05	1.62	0.93	26.72	12.77	3.99	2.14	17.5	13.4	7.4	6.9	8.8	8.6	6.1	6.0
200	6.50	3.07	1.13	0.67	26.06	10.74	3.15	1.64	13.6	10.2	6.6	6.1	7.8	7.2	4.6	5.9
500	4.73	2.19	0.79	0.47	24.04	7.97	2.06	1.08	10.5	6.6	5.6	6.0	8.3	6.8	5.0	4.9
1000	3.80	1.62	0.55	0.30	19.84	6.74	1.56	0.75	7.9	6.3	4.5	5.3	7.2	5.3	5.0	5.1
	IV-B															
50	25.62	6.38	1.98	1.02	140.79	26.20	5.42	3.01	1.5	3.5	5.5	5.5	0.1	2.4	5.4	6.4
100	19.32	4.66	1.35	0.78	103.11	18.36	4.09	2.16	1.2	4.6	6.0	7.0	0.0	4.7	5.9	6.0
200	12.55	2.99	0.99	0.56	73.95	11.89	2.92	1.43	2.5	4.6	5.0	4.9	0.5	4.7	4.0	5.6
500	7.74	2.03	0.63	0.35	46.75	7.97	1.77	0.95	3.1	5.3	5.1	5.5	1.6	4.6	4.6	5.4
1000	5.38	1.42	0.41	0.26	33.09	5.61	1.30	0.66	4.1	4.9	4.8	4.8	3.3	5.6	5.7	6.9
	GMM-B1															
50	17.52	5.35	1.79	1.04	47.33	17.82	4.97	2.65	12.6	8.2	5.6	6.2	3.7	4.8	5.4	6.0
100	12.82	4.26	1.27	0.73	45.43	14.09	3.82	2.00	9.3	7.0	6.1	5.5	3.8	5.6	5.0	5.4
200	8.62	2.66	0.94	0.56	41.76	10.51	2.72	1.38	6.8	5.2	4.4	5.3	2.4	3.7	4.7	4.6
500	6.33	1.93	0.60	0.34	34.60	7.40	1.69	0.90	5.8	5.8	4.6	5.3	3.3	5.3	4.7	4.7
1000	4.94	1.36	0.39	0.25	28.73	5.33	1.25	0.64	4.8	6.0	4.5	4.8	4.0	5.7	5.6	6.7
	GMM-B3															
50	23.78	6.39	1.86	1.05	33.65	13.73	4.64	2.62	38.2	19.3	8.1	6.5	7.3	6.3	5.9	6.6
100	17.76	4.73	1.35	0.76	31.67	11.13	3.56	1.97	29.8	12.4	7.3	6.8	6.9	6.1	5.9	6.6
200	11.45	2.94	0.96	0.55	28.68	9.42	2.60	1.36	19.2	10.2	5.5	5.5	6.2	4.2	4.4	4.9
500	7.19	1.97	0.60	0.34	26.64	6.97	1.69	0.92	13.1	7.6	4.8	5.2	6.9	4.7	5.3	4.7
1000	5.25	1.40	0.41	0.25	24.81	5.20	1.27	0.64	10.5	7.4	5.0	5.1	6.9	5.9	5.6	6.2
	HPJ															
50	6.06	2.99	1.24	0.80	18.94	8.24	3.03	2.14	–	–	–	–	–	–	–	–
100	4.87	2.35	0.90	0.56	19.04	6.43	2.23	1.52	–	–	–	–	–	–	–	–
200	4.46	1.83	0.69	0.41	18.89	5.84	1.70	1.07	–	–	–	–	–	–	–	–
500	4.63	1.75	0.50	0.28	18.97	6.08	1.21	0.65	–	–	–	–	–	–	–	–
1000	4.46	1.73	0.33	0.19	18.85	6.17	1.09	0.50	–	–	–	–	–	–	–	–

Table S.11: Trend model, Bias and IQR ($\alpha = 0.8,\ \beta = 1,\ \sigma_\eta^2 = 5,\ \phi_y = 1,\ \phi_x = 1$)

	Bias								IQR							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	-36.57	-15.89	-5.41	-2.53	-9.34	0.44	2.21	1.51	6.01	3.46	1.61	0.98	15.72	10.40	5.77	4.02
100	-36.72	-15.83	-5.42	-2.55	-10.19	0.25	2.05	1.59	4.73	2.34	1.13	0.70	12.41	6.99	4.24	2.89
200	-36.46	-15.80	-5.35	-2.51	-10.05	0.06	2.05	1.46	3.20	1.78	0.81	0.51	8.34	4.93	3.06	2.03
500	-36.43	-15.79	-5.36	-2.49	-10.06	0.07	2.15	1.47	2.05	1.13	0.54	0.33	5.47	3.09	1.86	1.30
1000	-36.42	-15.82	-5.37	-2.50	-10.03	0.10	2.15	1.53	1.45	0.74	0.37	0.24	3.49	2.15	1.32	0.87
	IV-L															
50	-7.44	-0.57	1.81	6.11	4.78	8.27	11.60	12.70	126.6	69.6	53.0	44.2	370.6	225.7	109.5	71.5
100	-9.26	-0.35	0.83	6.16	8.89	8.82	10.00	11.75	111.6	79.8	54.4	47.1	312.2	219.5	103.3	74.1
200	-8.13	-2.80	2.23	6.34	-11.35	9.19	10.98	10.51	104.0	85.9	58.1	42.0	283.1	240.2	105.2	67.2
500	-3.06	-3.79	5.30	5.33	0.60	3.38	12.30	9.94	99.8	81.8	49.1	42.4	249.4	203.2	103.1	69.1
1000	-1.66	0.31	0.91	4.08	-0.17	11.67	6.01	7.32	75.2	65.9	51.9	41.6	216.0	180.7	104.5	68.7
	GMM-L1															
50	-40.88	-15.29	-4.44	-2.17	-12.77	-9.37	-7.34	-3.24	33.06	16.90	6.54	4.21	79.76	48.26	17.89	8.21
100	-38.45	-13.07	-4.17	-2.08	-21.50	-16.51	-8.37	-3.44	31.63	13.67	6.52	3.95	80.79	43.44	15.64	8.09
200	-38.24	-11.73	-3.85	-1.80	-31.55	-21.54	-8.58	-3.11	33.12	14.36	6.13	3.94	84.05	42.88	15.10	7.94
500	-32.27	-9.35	-2.98	-1.53	-41.91	-19.78	-6.37	-2.68	30.96	12.86	6.03	3.45	76.61	32.49	14.28	6.70
1000	-27.37	-8.30	-2.63	-1.05	-44.74	-20.12	-6.12	-1.90	25.69	12.12	5.46	3.08	66.35	32.15	12.55	5.64
	GMM-L3															
50	-29.64	-11.59	-3.79	-1.66	-37.83	-17.81	-6.22	-2.23	18.02	8.38	3.63	2.41	46.58	24.80	11.16	5.90
100	-25.14	-10.63	-3.23	-1.34	-43.79	-22.59	-6.95	-2.19	15.05	8.05	3.79	2.09	45.97	22.20	10.77	5.29
200	-22.33	-9.50	-2.74	-0.88	-47.89	-22.84	-6.89	-1.73	14.71	7.46	3.14	1.69	45.95	22.46	8.85	4.23
500	-19.46	-8.34	-2.33	-0.57	-52.66	-22.44	-5.92	-1.09	13.82	6.84	2.94	1.37	43.30	20.21	7.75	3.23
1000	-17.41	-7.55	-1.86	-0.33	-48.88	-21.62	-4.59	-0.56	13.37	6.71	2.64	1.03	41.21	19.68	7.09	2.42
	IV-B															
50	-19.51	-3.56	0.29	0.01	-51.73	-6.81	1.49	-0.14	114.14	62.35	8.56	2.51	349.71	175.79	19.57	6.74
100	-18.24	0.55	0.02	-0.04	-54.67	2.38	0.47	-0.14	119.73	50.53	5.88	1.84	346.86	139.54	13.48	4.72
200	-13.70	0.49	0.01	0.03	-34.03	3.29	0.10	-0.04	116.13	37.30	3.80	1.40	341.30	103.02	9.21	3.26
500	-14.28	-0.52	0.00	0.01	-38.17	0.57	0.08	0.02	116.66	22.43	2.53	0.87	343.51	61.14	5.50	2.07
1000	-12.40	-0.47	0.03	0.00	-34.44	-0.08	-0.02	0.01	103.19	16.93	1.75	0.61	321.56	48.01	4.20	1.51
	GMM-B1															
50	-36.42	-11.90	-2.47	-0.55	-37.35	-21.64	-3.77	-0.46	36.27	14.82	5.00	2.18	90.48	41.46	13.26	5.98
100	-32.26	-10.25	-1.63	-0.35	-45.73	-22.88	-2.58	-0.34	32.77	14.49	4.25	1.64	85.08	40.40	10.51	4.34
200	-26.34	-8.89	-1.04	-0.13	-49.56	-21.64	-1.85	-0.08	30.91	14.13	3.38	1.24	87.77	38.75	7.83	2.99
500	-24.15	-6.82	-0.45	-0.06	-58.82	-16.30	-0.80	-0.03	29.22	12.36	2.21	0.78	83.18	34.48	4.84	1.99
1000	-21.66	-5.79	-0.21	-0.03	-61.91	-14.44	-0.43	0.02	27.85	10.63	1.55	0.55	84.44	29.89	3.79	1.41
	GMM-B3															
50	-47.30	-15.07	-3.59	-1.11	-35.20	-17.09	-3.70	-0.91	24.45	9.29	3.92	1.87	55.85	26.82	10.78	5.65
100	-42.70	-13.33	-2.86	-0.76	-41.91	-21.75	-4.34	-0.57	23.13	9.23	3.25	1.55	57.91	23.95	8.40	3.99
200	-36.42	-11.74	-2.02	-0.38	-52.68	-23.63	-3.22	-0.29	22.95	9.15	2.72	1.10	58.75	23.41	7.45	2.97
500	-31.02	-9.81	-1.06	-0.14	-60.89	-22.68	-1.89	-0.16	20.10	8.43	2.26	0.75	54.22	22.52	4.93	1.98
1000	-27.53	-8.75	-0.61	-0.09	-64.76	-21.23	-1.05	-0.05	19.46	8.03	1.46	0.54	57.29	21.68	3.59	1.44
	HPJ															
50	4.72	4.98	1.35	0.40	15.14	10.48	2.96	0.82	12.12	6.30	2.51	1.26	25.94	13.91	6.01	4.21
100	4.84	4.85	1.36	0.35	14.30	9.87	2.92	0.92	9.20	4.40	1.68	0.83	18.42	9.46	4.41	2.92
200	5.16	4.93	1.39	0.36	15.11	10.11	2.92	0.82	6.14	3.05	1.21	0.63	11.95	6.28	3.14	2.05
500	5.41	4.90	1.37	0.39	15.45	10.05	2.95	0.82	4.01	1.85	0.75	0.40	8.41	3.91	1.97	1.29
1000	5.37	4.85	1.36	0.38	15.11	10.28	2.99	0.88	2.75	1.30	0.50	0.27	5.58	2.71	1.46	0.91

Table S.12: Trend model, MAE and Size ($\alpha = 0.8$, $\beta = 1$, $\sigma_\eta^2 = 5$, $\phi_y = 1$, $\phi_x = 1$)

	MAE								Size							
	α				β				α				β			
N/T	10	20	50	100	10	20	50	100	10	20	50	100	10	20	50	100
	FE															
50	36.57	15.89	5.41	2.53	11.04	5.19	3.29	2.14	100.0	100.0	99.7	90.9	15.6	8.2	10.7	10.2
100	36.72	15.83	5.42	2.55	10.41	3.44	2.52	1.82	100.0	100.0	100.0	99.8	25.1	6.9	12.7	12.0
200	36.46	15.80	5.35	2.51	10.05	2.46	2.15	1.53	100.0	100.0	100.0	100.0	41.7	6.6	19.6	19.6
500	36.43	15.79	5.36	2.49	10.06	1.54	2.15	1.47	100.0	100.0	100.0	100.0	75.4	5.5	36.6	38.1
1000	36.42	15.82	5.37	2.50	10.03	1.08	2.15	1.53	100.0	100.0	100.0	100.0	96.4	4.6	62.5	65.3
	IV-L															
50	63.64	34.81	27.06	23.88	187.06	111.23	57.18	35.37	0.3	0.0	0.5	0.1	0.1	0.0	0.4	0.9
100	57.68	39.90	27.00	25.05	156.86	107.05	51.70	39.71	0.0	0.2	0.0	0.1	0.1	0.0	0.5	1.4
200	52.15	43.16	28.26	21.41	142.75	119.26	52.87	34.17	0.0	0.0	0.0	0.2	0.0	0.3	0.7	0.0
500	49.40	41.09	25.37	22.55	124.35	100.66	50.84	36.78	0.1	0.2	0.1	0.1	0.0	0.1	0.5	0.2
1000	37.71	33.00	26.38	21.98	108.19	90.18	51.82	35.81	0.0	0.1	0.1	0.4	0.2	0.1	0.3	0.4
	GMM-L1															
50	40.88	15.32	4.74	2.67	41.23	24.77	10.75	4.80	28.8	16.6	10.7	8.2	3.6	5.3	7.7	8.0
100	38.45	13.07	4.55	2.53	41.42	25.65	9.97	4.72	25.6	12.2	9.9	7.4	3.3	4.9	7.1	6.1
200	38.24	11.86	4.20	2.38	46.62	25.71	9.79	4.17	24.0	11.9	7.7	7.1	4.2	5.8	6.0	6.1
500	32.32	9.56	3.57	2.10	49.03	22.42	8.31	3.85	19.2	9.7	7.7	6.0	5.4	6.1	7.0	4.4
1000	27.59	8.57	3.26	1.68	49.22	21.69	7.53	2.97	18.0	9.8	7.5	5.7	7.8	6.4	7.6	5.4
	GMM-L3															
50	29.64	11.59	3.79	1.74	39.09	18.96	7.42	3.55	65.6	47.4	26.3	15.2	21.1	18.8	11.8	9.2
100	25.14	10.63	3.25	1.44	44.19	22.69	7.51	2.98	58.7	45.6	24.4	12.6	26.4	25.3	15.6	7.7
200	22.33	9.50	2.81	1.06	48.01	22.91	7.16	2.43	53.1	40.3	19.4	10.7	35.1	29.3	15.4	7.9
500	19.46	8.34	2.41	0.81	52.66	22.45	6.33	1.78	46.6	34.8	15.3	8.9	38.2	29.4	15.4	6.2
1000	17.41	7.56	1.97	0.57	48.88	21.67	4.92	1.28	41.6	32.7	13.5	6.0	36.3	29.9	12.4	5.8
	IV-B															
50	62.84	31.39	4.30	1.27	176.87	87.96	9.80	3.34	0.6	0.4	3.4	4.0	0.2	0.2	4.0	6.1
100	61.55	25.51	2.93	0.91	176.61	70.24	6.74	2.35	1.0	0.5	4.5	5.4	0.1	0.2	4.0	5.0
200	59.70	18.70	1.90	0.69	172.40	51.27	4.64	1.63	0.6	0.5	4.4	4.1	0.1	0.5	4.3	4.9
500	60.40	11.21	1.26	0.43	177.08	30.65	2.75	1.04	0.6	0.7	4.9	5.0	0.2	0.8	4.8	5.1
1000	53.79	8.33	0.87	0.31	163.57	23.84	2.10	0.77	0.7	1.3	5.5	5.4	0.3	1.0	5.3	5.9
	GMM-B1															
50	36.81	12.11	3.10	1.10	52.45	25.89	6.87	2.94	28.7	20.0	8.1	6.4	6.3	11.1	7.0	6.5
100	33.05	10.67	2.39	0.86	56.97	26.57	5.57	2.12	24.4	18.4	8.0	6.4	8.7	13.8	5.5	6.7
200	27.34	9.40	1.81	0.62	57.79	24.24	4.16	1.51	19.2	16.0	7.7	5.7	9.9	11.6	6.6	5.2
500	25.50	7.82	1.17	0.39	62.51	20.59	2.51	0.99	19.3	12.4	5.7	5.9	13.4	10.0	5.9	4.3
1000	22.96	6.85	0.78	0.27	65.55	18.17	1.86	0.70	15.6	10.0	5.2	5.1	12.2	9.6	5.2	6.1
	GMM-B3															
50	47.30	15.07	3.61	1.23	39.94	18.50	5.65	2.84	76.9	57.0	25.7	14.0	11.6	14.6	9.9	7.2
100	42.70	13.33	2.91	0.92	45.70	22.45	5.47	2.07	72.0	51.2	19.6	9.9	17.0	20.3	9.7	7.0
200	36.42	11.74	2.10	0.60	53.38	23.66	4.04	1.50	63.6	44.9	16.1	8.1	21.9	26.0	10.9	5.6
500	31.02	9.83	1.36	0.38	60.97	23.05	2.85	0.99	55.2	39.3	10.6	7.0	32.2	28.3	7.9	5.2
1000	27.53	8.75	0.86	0.29	64.76	21.29	1.96	0.73	50.9	36.5	8.7	5.7	38.4	29.7	6.8	6.0
	HPJ															
50	7.08	5.15	1.56	0.68	16.88	11.11	3.84	2.12	–	–	–	–	–	–	–	–
100	5.87	4.85	1.41	0.51	14.83	9.92	3.16	1.56	–	–	–	–	–	–	–	–
200	5.30	4.93	1.39	0.41	15.11	10.11	2.94	1.18	–	–	–	–	–	–	–	–
500	5.41	4.90	1.37	0.40	15.45	10.05	2.95	0.88	–	–	–	–	–	–	–	–
1000	5.37	4.85	1.36	0.38	15.11	10.28	2.99	0.88	–	–	–	–	–	–	–	–