Pattern search method for accelerating Stope Boundary Optimization problem in underground mining operations

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Supplemental Material

Validation of New Method on Hypothetical Data

Solving process of the proposed new optimization method is shown by implementation on a two- dimensional hypothetical example. The economic block model consists of one hundred blocks and is illustrated in Figure 1. Suppose that the minimum size of stopes along both Y and Z axes is equal to four. In the first step, a warm-start solution is obtained by the Greedy algorithm when the number of stopes in the final solution is assumed to be seven. The Greedy algorithm picks up the first stope which its origin block is located in the 5th column and 3rd row. The value of this decision is 45 units. Second stope is located in the 1st column and 6th row. The value of this decision is 27 units. After the second decision, the value of current boundary is updated i.e., z=45+27=72. Since this algorithm should pick up seven stopes, there remain five decisions yet. So, these mentioned steps have been iterated for the rest five decisions. After the seventh decision, solving process is terminated and this algorithm returns the solution and its value. This solution has been illustrated in Figure 2 and its value is equal to 113 units.

It should be noted to have a better display and in order to distinguish decisions in each iteration, the selected stopes in Figure 2 are placed behind each other.

A simple pattern is used to update the current solution. Figure 3 shows this selected pattern. Suppose that the maximum step size is equal to one (S_{max}=1). So, there is at most five trial points around each stope. For those stopes which are located in the margin of block model, the number of trial points is decreased, because it is not possible to move stopes outside of block model.

In the next step, the set of trial points for all seven stopes is determined and an integer programming is build based on these input parameters. This IP model is presented by the following equations. It tries to improve the solution by

	1	2	3	4	5	6	7	8	9	10
1	-10	9	-9	2	-6	5	-1	3	3	-6
2	-8	-9	9	2	-1	-2	7	-2	6	-4
3	8	1	-4	5	7	9	-6	2	0	7
4	-10	4	-3	-1	6	-5	3	-3	-2	9
5	9	-7	9	-9	6	-9	8	9	-8	7
6	6	1	7	7	3	-1	8	8	-9	-2
7	2	7	-6	-2	-9	9	-2	8	-10	9
8	1	-6	1	-3	-8	-1	-5	-5	1	-1
9	8	2	0	2	-7	-8	0	2	-9	-5
10	-8	-1	-8	-9	-9	7	-6	2	-7	-6

Figure 1: A two-dimensional economic block model.

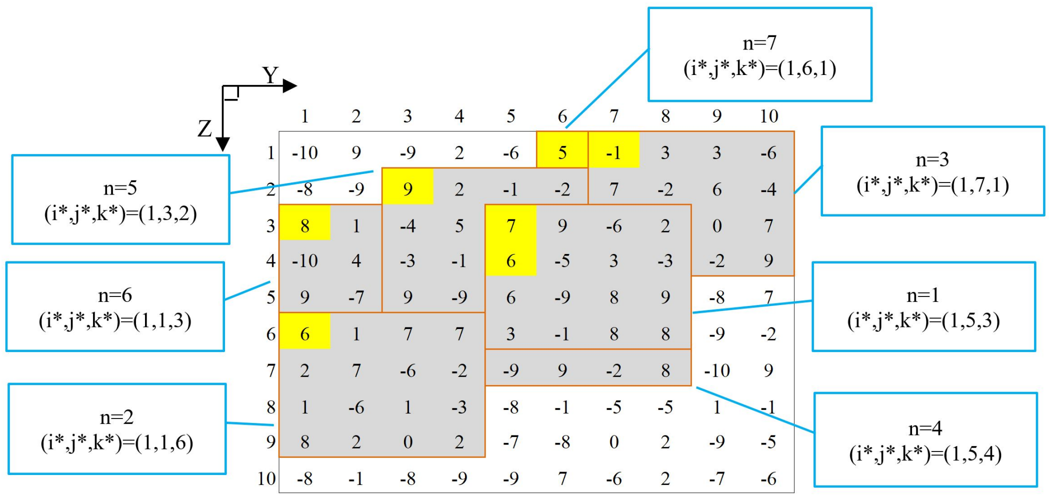

Figure 2: A warm-start solution by the Greedy algorithm when the number of stopes is seven (N=7).

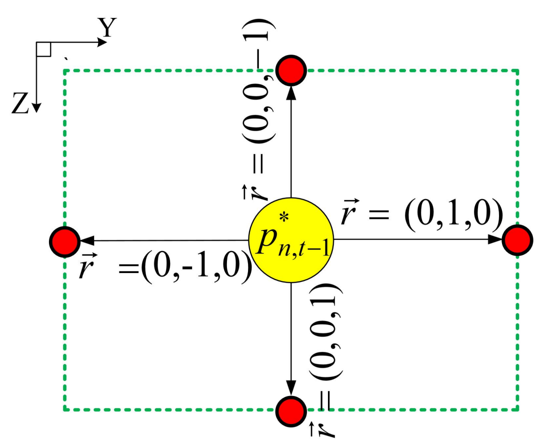

Figure 3: A two-dimensional pattern search to move the selected stopes (S_{max}=1).

some beneficial movement on the current position of stopes. Finally with three movements, the value of boundary is updated to 118 units. Red arrows in Figure 4 show these movements.

$\Gamma^1_{1,1} = \{(5,3),(6,3),(5,2),(4,3),(5,4)\}$
$\Gamma^1_{2,1} = \{(1,6),(2,6),(1,5),(1,7)\}$
$\Gamma^1_{3,1} = \{(7,1),(8,1),(6,1),(7,2)\}$
$\Gamma^1_{4,1} = \{(5,4),(6,4),(5,3),(4,4),(5,5)\}$
$\Gamma^1_{5,1} = \{(3,2),(4,2),(3,1),(2,2),(3,3)\}$
$\Gamma^1_{6,1} = \{(1,3),(2,3),(1,2),(1,4)\}$
$\Gamma^1_{7,1} = \{(6,1),(7,1),(5,1),(6,2)\}$

$$z^1 = \max -10x_{1,1} - 8x_{1,2} + 8x_{1,3} + \ldots - 6x_{10,10}$$

s.t.

$$\text{Constraint 10b}: \begin{cases} x_{1,1} \leq 0 \\ x_{1,2} \leq y^6_{1,2} \\ x_{1,3} \leq y^6_{1,2} + y^6_{1,3} \\ x_{1,4} \leq y^6_{1,2} + y^6_{1,3} + y^6_{1,4} \\ x_{1,5} \leq y^6_{1,2} + y^6_{1,3} + y^6_{1,4} + y^2_{1,5} \\ \vdots \end{cases}$$

$$\text{Constraint 10c}: \begin{cases} 16y^1_{5,3} \leq x_{5,3} + x_{5,4} + \ldots + x_{8,6} \\ 16y^1_{6,3} \leq x_{6,3} + x_{6,4} + \ldots + x_{9,6} \\ \vdots \end{cases}$$

$$\text{Constraint 10d}: \begin{cases} y^1_{5,3} + y^1_{6,3} + y^1_{5,2} + y^1_{5,4} + y^1_{4,3} = 1 \\ y^2_{1,6} + y^2_{2,6} + y^2_{1,5} + y^2_{1,7} = 1 \\ \vdots \end{cases}$$

$$\text{Constraint 10e}: \begin{cases} y^1_{5,3} + y^4_{5,3} \leq 1 \\ y^1_{6,3} \leq 1 \\ \vdots \end{cases}$$

$z^1 = 118,$
$y^1_{5,4} = y^2_{1,6} = y^3_{7,2} = y^4_{4,4} = y^5_{3,2} = y^6_{1,3} = y^7_{6,1} = 1.$

The solution is updated based on the results of the first iteration. Figure 5 shows this solution, a boundary which its value is equal to 118 units. To achieve more benefits, the optimization method continues to the next iteration. Similar to the first iteration, trail points are recognized. Then based on these input parameters, an IP model is built. The following equations present this new model. After solving this model, the obtained value is 118 units. It means that there is not another beneficial replacement, so the solving process finishes. This optimization method returns the final solution with value 118 units.

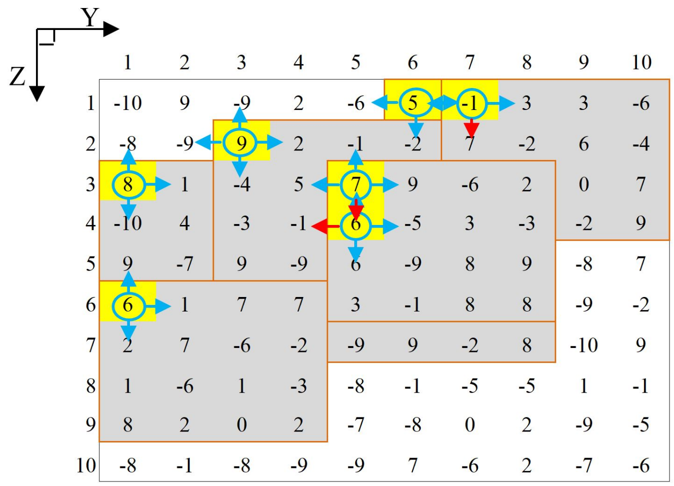

Figure 4: The warm-start solution (arrows show the set of candidates points around current stopes.

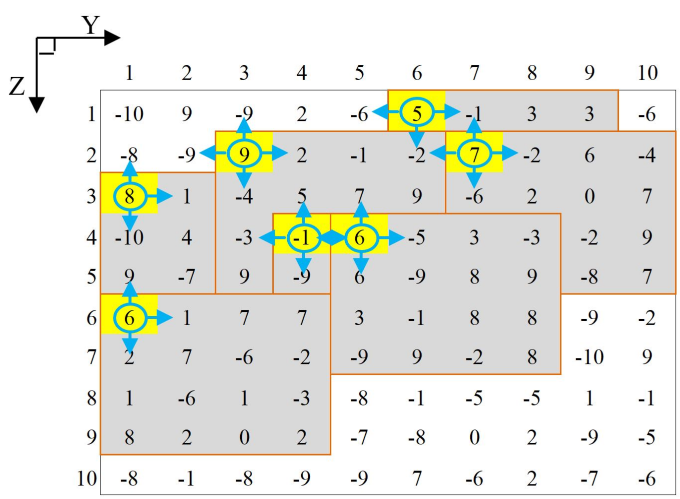

Figure 5: The obtained solution after iteration one.

$\Gamma^1_{1,2} = \{(5,4),(6,4),(5,3),(4,4),(5,5)\}$
$\Gamma^1_{2,2} = \{(1,6),(2,6),(1,5),(1,7)\}$
$\Gamma^1_{3,2} = \{(7,2),(8,2),(7,1),(6,2),(7,3)\}$
$\Gamma^1_{4,2} = \{(4,4),(5,4),(4,3),(3,4),(3,5)\}$
$\Gamma^1_{5,2} = \{(3,2),(4,2),(3,1),(2,2),(3,3)\}$
$\Gamma^1_{6,2} = \{(1,3),(2,3),(1,2),(1,4)\}$
$\Gamma^1_{7,2} = \{(6,1),(7,1),(5,1),(6,2)\}$

$$z^2 = \max -10x_{1,1} - 8x_{1,2} + 8x_{1,3} + \ldots - 6x_{10,10}$$

s.t.

$$\text{Constraint 10b}: \begin{cases} x_{1,1} \le 0 \\ x_{1,2} \le y^6_{1,2} \\ x_{1,3} \le y^6_{1,2} + y^6_{1,3} \\ x_{1,4} \le y^6_{1,2} + y^6_{1,3} + y^6_{1,4} \\ x_{1,5} \le y^6_{1,2} + y^6_{1,3} + y^6_{1,4} + y^2_{1,5} \\ \vdots \end{cases}$$

$$\text{Constraint 10c}: \begin{cases} 16y^1_{5,4} \le x_{5,4} + x_{5,4} + \ldots + x_{8,7} \\ 16y^1_{6,4} \le x_{6,4} + x_{6,5} + \ldots + x_{9,7} \\ \vdots \end{cases}$$

$$\text{Constraint 10d}: \begin{cases} y^1_{5,4} + y^1_{6,4} + y^1_{5,3} + y^1_{4,4} + y^1_{5,5} = 1 \\ y^2_{1,6} + y^2_{2,6} + y^2_{1,5} + y^2_{1,7} = 1 \\ \vdots \end{cases}$$

$$\text{Constraint 10e}: \begin{cases} y^1_{5,4} + y^4_{5,4} \le 1 \\ y^1_{6,4} \le 1 \\ \vdots \end{cases}$$

$z^2 = 118$,
$y^1_{5,4} = y^2_{1,6} = y^3_{7,2} = y^4_{4,4} = y^5_{3,2} = y^6_{1,3} = y^7_{6,1} = 1.$

As has been mentioned earlier, the quality of solution is so dependent on the selected pattern and the step size. Hence, for the second pattern when the step size varies between one to four i.e., $S_{max} \in \{1,2,3,4\}$, analysis has been repeated. Finally, the solution upgraded to 120 units (see Table 1).

The obtained solution by the original mathematical model has been illustrated in Figure 6. Its value is 121 units. It means that, both greedy algorithm and pattern search optimization method failed to find the optimum stope boundary on this example. However, these results prove that the pattern search optimization method could find a near optimum solution. Other advantage of the pattern search is accelerating the solving process in the SBO problem which can be useful in solving large scale problems.

Table 1: Generated results by the Greedy, Pattern Search 1, Pattern Search 2 and IP for the hypothetical example.

Optimization Method	**Step Size (S_{max})**	**Value**	**Optimality Gap (%)**
Greedy	-	113	6.61
Pattern Search (pattern 1)	1	118	2.48
Pattern Search (pattern 2)	1	118	2.48
Pattern Search (pattern 2)	2	120	0.83
Pattern Search (pattern 2)	3	120	0.83
Pattern Search (pattern 2)	4	120	0.83
IP	-	121	-

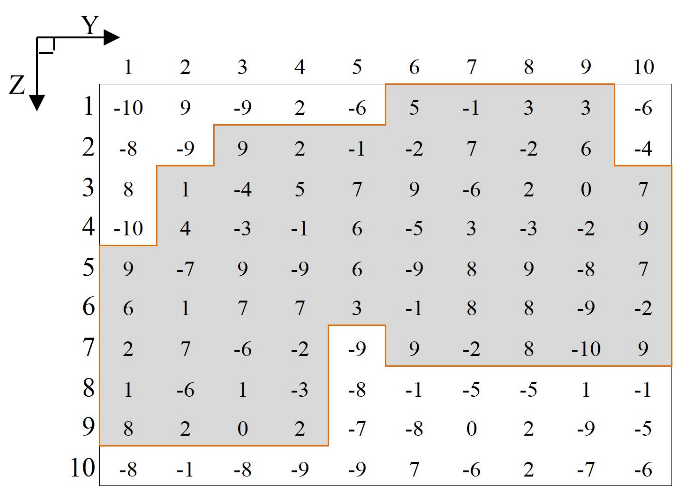

Figure 6: Optimum stope boundary obtained by the original integer programming model.