$W_1(G_\varepsilon) = \arctan^*\left(\frac{\varepsilon \sin x}{\rho(1-\varepsilon) + \varepsilon \cos x}\right)$ and $W_1(F_\mu) = \mu$. The rest of the proof is similar to Theorem 3.1.

b) Let F_μ and δ_x be as in the proof of theorem 6.1. Let G_ε be as in the proof of theorem 2.2. We note the following $W_1(G_\varepsilon) = \arctan^*\left(\frac{\rho(1-\varepsilon)\sin\mu + \varepsilon \sin x}{\rho(1-\varepsilon)\cos\mu + \varepsilon \cos x}\right)$ and $W_1(F_0) = 0$. The rest of the proof is similar to theorem 3.2.

Proof of Theorem 6.3 **parts a), b), c) and d):** Let F_μ and δ_x be as in the proof of theorem 6.1. Let $G_\varepsilon = (1-\varepsilon)F_0 + \varepsilon\delta_x$, $x \in [-\pi, \pi)$ and $0 \le \gamma < 0.5$. Following the similar steps in theorem 4.1, we get $W_\gamma(G_\varepsilon) = \arctan^*\left[\frac{\vartheta_{\gamma,0}(1-\varepsilon)\tan\mu_{\gamma,0} + \varepsilon \sin x}{\vartheta_{\gamma,0}(1-\varepsilon) + \varepsilon \cos x}\right], \theta_1 < x < \theta_2$

and $W_\gamma(F_\mu) = \mu$. Now using Lemma 2, the rest of the proof follows. To prove part d) let $G_\varepsilon = (1-\varepsilon)F_\mu + \varepsilon\delta_x$, $x \in [\mu-\pi, \mu+\pi)$ and $0 \le \gamma < 0.5$. Following the similar steps in theorem 4.2 for $c_1(\mu) < x < c_2(\mu)$, we get $W_\gamma(G_\varepsilon) = (1-\varepsilon)\lambda_\mu + \varepsilon \sin x$ and $W_\gamma(F_0) = 0$. Using Lemma 4, the rest of the proof follows.

Proof of Theorem 7.1 a): Let $G_\varepsilon = (1-\varepsilon)F_0 + \varepsilon\delta_x$ where $F_0 \sim M(\tilde{\boldsymbol{\mu}}_{\mathbf{0}}, \kappa)$ and x is a point on the unit sphere. Then, G_ε can be written as

$$W^*(G_\varepsilon) = \left\|(1-\varepsilon)E_{F_0}(\tilde{\mathbf{X}}) + \varepsilon\tilde{\mathbf{X}}\right\| - \tilde{\boldsymbol{\mu}}_{\mathbf{0}}^{\mathbf{T}}\left((1-\varepsilon)E_{F_0}(\tilde{\mathbf{X}}) + \varepsilon\tilde{\mathbf{X}}\right). \qquad \text{... (7.1)}$$

But since $E_{F_0}(\sin\Theta\cos\Phi) = E_{F_0}(\sin\Theta\sin\Phi) = 0$ and $E_{F_0}(\cos\Theta) = \rho$ we have from (7.1)

$$W^*(G_\varepsilon) = \sqrt{\rho^2(1-\varepsilon^2) + 2\rho\varepsilon(1-\varepsilon)\cos\theta + \varepsilon^2} - (\rho(1-\varepsilon) + \varepsilon\cos\theta). \qquad \text{... (7.2)}$$

Now using the fact that $E_{F_\mu}(\sin\Theta\cos\Phi) = \rho\sin\alpha\cos\beta$, $E_{F_\mu}(\sin\Theta\sin\Phi) = \rho\sin\alpha\sin\beta$, and $E_{F_\mu}(\cos\Theta) = \rho\cos\alpha$ we have

$$W^*(F_\mu) = \left\|E_{F_\mu}(\tilde{\mathbf{X}})\right\| - \tilde{\boldsymbol{\mu}}_{\mathbf{0}}^{\mathbf{T}} E_{F_\mu}(\tilde{\mathbf{X}}) = 2\rho\sin^2\left(\frac{\alpha}{2}\right). \quad \text{... (7.3)}$$

Let $y = \rho(1-\varepsilon) + \varepsilon\cos\theta$ and $c = 2\rho\sin^2\left(\frac{\alpha}{2}\right)$. Then using (7.2) and (7.3) we get

$$\varepsilon_\mu^{**}(W^*) = \inf\left\{\varepsilon > 0 : \sqrt{y^2 + \varepsilon^2\sin^2\theta} = y + c \ \text{ for some } \theta \in [0, \pi]\right\}. \quad \text{... (7.4)}$$

Hence the theorem is established.

Proof of Theorem 7.1 b): Let $G_\varepsilon = (1-\varepsilon)F_\mu + \varepsilon\delta_x$ where $F_\mu \sim M(\tilde{\boldsymbol{\mu}}_{\mathbf{0}}, \kappa)$ and x is a point on the unit sphere. Then, G_ε can be written as

$$W^*(G_\varepsilon) = \left\|(1-\varepsilon)E_{F_\mu}(\tilde{\mathbf{X}}) + \varepsilon\tilde{\mathbf{X}}\right\| - \tilde{\boldsymbol{\mu}}_{\mathbf{0}}^{\mathbf{T}}\left((1-\varepsilon)E_{F_\mu}(\tilde{\mathbf{X}}) + \varepsilon\tilde{\mathbf{X}}\right). \quad \text{… (7.5)}$$

But $\tilde{\boldsymbol{\mu}}_{\mathbf{0}}^{\mathbf{T}}\left((1-\varepsilon)E_{F_\mu}(\tilde{\mathbf{X}}) + \varepsilon\tilde{\mathbf{X}}\right) = \rho(1-\varepsilon)\cos\alpha + \varepsilon\cos\theta$ and

$$\left\|(1-\varepsilon)E_{F_\mu}(\tilde{\mathbf{X}}) + \varepsilon\tilde{\mathbf{X}}\right\| = \sqrt{\rho^2(1-\varepsilon)^2 + 2\rho\varepsilon(1-\varepsilon)\left[\sin\alpha\sin\theta\cos(\phi-\beta) + \cos\alpha\cos\theta\right] + \varepsilon^2}\,.$$

We also have $W^*(F_0) = \left\|E_{F_0}(\tilde{\mathbf{X}})\right\| - \tilde{\boldsymbol{\mu}}_{\mathbf{0}}^{\mathbf{T}} E_{F_0}(\tilde{\mathbf{X}}) = 0$. … (7.6)

Using (7.5) and (7.6) we get,

$$\varepsilon_\mu^*(W^*) = \inf\left\{\begin{array}{l} \varepsilon > 0 : \varepsilon^2\left(\rho^2\sin^2\alpha - 2\rho\sin\alpha\sin\theta\cos(\varphi-\beta) + \sin^2\theta\right) \\ \qquad -2\rho\sin\alpha\left(\sin\theta\cos(\varphi-\beta) - \rho\sin\alpha\right)\varepsilon + \rho^2\sin^2\alpha = 0 \text{ for} \\ \qquad \text{some } \theta \in [0, \pi] \text{ and } \varphi \in [0, 2\pi) \end{array}\right\}.$$

Let $u = \sin\theta, v = \cos(\varphi - \beta)$ and $c = \rho\sin\alpha$. Then the above PBF reduces to

$$\varepsilon_\mu^*(W^*) = \inf\left\{\begin{array}{c}\varepsilon > 0 : (c^2 - 2cuv + u^2)\varepsilon^2 - 2c(uv - c)\varepsilon + c^2 = 0 \\ \text{for some } \theta \in [0, \pi] \text{ and } \varphi \in [0, 2\pi)\end{array}\right\}.$$

The equation $(c^2 - 2cuv + u^2)\varepsilon^2 - 2c(uv - c)\varepsilon + c^2 = 0$ has roots $\varepsilon_1 = \dfrac{c(t_1 - t_2)}{2t_3}$ and $\varepsilon_2 = \dfrac{c(t_1 + t_2)}{2t_3}$, where $t_1 = (uv - c), t_2 = \left(|u|\sqrt{1 - v^2}\right)$ and $t_3 = c^2 - 2cuv + u^2$.

Hence the theorem is established.

Theorem 7.2: Let $G_\varepsilon = (1-\varepsilon)F_0 + \varepsilon\delta_x$ where $F_0 \sim M(\tilde{\boldsymbol{\mu}}_{\mathbf{0}}, \kappa)$ and x is a point on the unit sphere. Then, we can write

$$W_1^*(G_\varepsilon) = \begin{bmatrix}\varepsilon\sin\theta\cos\varphi \\ \varepsilon\sin\theta\sin\varphi \\ (1-\varepsilon)\rho + \varepsilon\cos\theta\end{bmatrix} \text{ and } W_1^*(F_\mu) = E_{F_\mu}(\tilde{\mathbf{X}}) = \begin{bmatrix}\rho\sin\alpha\cos\beta \\ \rho\sin\alpha\sin\beta \\ \rho\cos\alpha\end{bmatrix}.$$

Now, equating $W_1^*(G_\varepsilon) = W_1^*(F_\mu)$ we get the following set of equations

$$\varepsilon\sin\theta\cos\varphi - \rho\sin\alpha\cos\beta = 0 \qquad \ldots (7.7)$$

$$\varepsilon\sin\theta\sin\varphi - \rho\sin\alpha\sin\beta = 0 \qquad \ldots (7.8)$$

$$(1-\varepsilon)\rho + \varepsilon\cos\theta - \rho\cos\alpha = 0. \qquad \ldots (7.9)$$

Dividing equation (7.7) by equation (7.8) we get $\varphi = \beta$ or $\beta + \pi$. Solving equations (7.7) and (7.9) and using the fact that $\varphi = \beta$, we get

$$\sin\theta = \frac{\rho\sin\alpha}{\varepsilon} \text{ and } \cos\theta = \frac{\rho\cos\alpha - (1-\varepsilon)\rho}{\varepsilon}.$$

Using the identity $\sin^2\theta + \cos^2\theta = 1$ and putting $(1-\varepsilon) = t$ and simplifying we get

$$(1-\rho^2)t^2 - 2(1-\rho^2\cos\alpha)t + (1-\rho^2) = 0. \qquad \ldots (7.10)$$

By putting $c_1 = (1-\rho^2\cos\alpha)$ and $d_1 = (1-\rho^2)$, the solution to (7.10) is given by

$t = \dfrac{c_1^2 \pm \sqrt{c_1^2 - d_1^2}}{d_1}$. Thus, $\varepsilon_\mu^{**}(W_1^*) = \min(\varepsilon_1, \varepsilon_2)$ where $\varepsilon_1 = \dfrac{(d_1 - c_1) + \sqrt{c_1^2 - d_1^2}}{d_1}$ and

$\varepsilon_1 = \dfrac{(d_1 + c_1) + \sqrt{c_1^2 - d_1^2}}{d_1}$.

In the case of PBF, let $G_\varepsilon = (1-\varepsilon)F_\mu + \varepsilon\delta_x$ where $F_\mu \sim M(\tilde{\boldsymbol{\mu}}_\mathbf{0}, \kappa)$ and x is a point on the unit sphere. Then, we can write

$$W_1^*(G_\varepsilon) = \begin{bmatrix} \rho(1-\varepsilon)\sin\alpha\cos\beta + \varepsilon\sin\theta\cos\varphi \\ \rho(1-\varepsilon)\sin\alpha\sin\beta + \varepsilon\sin\theta\sin\varphi \\ \rho(1-\varepsilon)\cos\alpha + \varepsilon\cos\theta \end{bmatrix} \text{ and } W_1^*(F_0) = E_{F_0}(\tilde{\mathbf{x}}) = \begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix}.$$

Now, equating $W_1^*(G_\varepsilon) = W_1^*(F_0)$ we get the following set of equations.

$$\rho(1-\varepsilon)\sin\alpha\cos\beta + \varepsilon\sin\theta\cos\varphi = 0 \qquad \ldots (7.11)$$

$$\rho(1-\varepsilon)\sin\alpha\sin\beta + \varepsilon\sin\theta\sin\varphi = 0 \qquad \ldots (7.12)$$

$$\rho(1-\varepsilon)\cos\alpha + \varepsilon\cos\theta = 0 \qquad \ldots (7.13)$$

Dividing equation (7.11) by equation (7.12) we get $\varphi = \beta$ or $\beta + \pi$. Solving equations (7.11) and (7.13) and using the fact that $\varphi = \beta$, we get

$$\sin\theta = \frac{-\rho(1-\varepsilon)\sin\alpha}{\varepsilon} \text{ and } \cos\theta = \frac{\rho - \rho(1-\varepsilon)\cos\alpha}{\varepsilon}.$$

Arguing in the similar fashion as in the case of LBF, we get the same equation (7.10). Now, when $\varphi = \beta + \pi$ then we have $\cos\varphi = -\cos\beta$ which also yields the same equation (7.10).

Hence the theorem is established.

Proof of Lemma 1: By definition we have for $0 \le \gamma < 0.5$, $\mu_{\gamma,\mu} = \arctan^*\left[\dfrac{E_{\gamma,F\mu}(\sin\Theta)}{E_{\gamma,F\mu}(\cos\Theta)}\right]$

and $\mu = \arctan^*\left[\dfrac{E_{F\mu}(\sin\Theta)}{E_{F\mu}(\cos\Theta)}\right]$. Then equating $\mu_{\gamma,\mu} = \mu$ we get

$$E_{\gamma,F\mu}(\sin\Theta)E_{F\mu}(\cos\Theta) = E_{\gamma,F\mu}(\cos\Theta)E_{F\mu}(\sin\Theta).$$

Define T_1 to be circular arc having μ as the centre point which satisfies $\int_{T_1} f_\mu(\theta)d\theta = 1-2\gamma$

and let $T_2 = T_1'$. Then, by letting $K = (2\pi I_0(\kappa))^{-1}$ and $C_1 = K\int_{T_2}\cos\theta\, f_\mu(\theta)\,d\theta$ we have

$$\begin{aligned} E_{F\mu}(\cos\Theta) &= \int_{-\pi}^{\pi}\cos\theta\, f_\mu(\theta)d\theta = \left[\int_{T_1}\cos\theta\, f_\mu(\theta)d\theta + \int_{T_2}\cos\theta\, f_\mu(\theta)d\theta\right] \\ &= (1-2\gamma)E_{\gamma,F\mu}(\cos\Theta) + K\int_{T_2}\cos\theta\, f_\mu(\theta)d\theta \\ &= (1-2\gamma)E_{\gamma,F\mu}(\cos\Theta) + C_1. \end{aligned}$$

Similar calculations shows that $E_{F\mu}(\sin\Theta) = (1-2\gamma)E_{\gamma,F\mu}(\sin\Theta) + S_1$ where

$S_1 = K\int_{T_2}\sin\theta\, f_\mu(\theta)\,d\theta$. Writing $T_2 = (\mu+\pi-\alpha_1, \mu+\pi+\alpha_1)$ and using the fact that

$\int_{-\alpha_1}^{\alpha_1}\sin\nu\, e^{\kappa\cos\nu}d\nu = 0$ we get $\dfrac{S_1}{C_1} = \dfrac{\int_{\mu+\pi-\alpha_1}^{\mu+\pi+\alpha_1}\left(\sin(\theta-\mu)\cos\mu + \cos(\theta-\mu)\sin\mu\right)e^{\kappa\cos(\theta-\mu)}d\theta}{\int_{\mu+\pi-\alpha_1}^{\mu+\pi+\alpha_1}\left(\cos(\theta-\mu)\cos\mu - \sin(\theta-\mu)\sin\mu\right)e^{\kappa\cos(\theta-\mu)}d\theta}$.

After some simplification, it can be shown that $\dfrac{S_1}{C_1} = \tan\mu$.

Hence the lemma is established.

***Proof of Lemma 2*:** Suppose $\theta_1 = F_0^{-1}\left(\frac{\gamma}{1-\varepsilon}\right)$ and $\theta_2 = F_0^{-1}\left(1-\frac{\gamma}{1-\varepsilon}\right)$. Note that since F_0 is symmetric about zero we have $\theta_1 = -\theta_2$. Let $\varepsilon < \min(\gamma, 1-\gamma)$, $\lambda = F_0^{-1}\left(\frac{\gamma-\varepsilon}{1-\varepsilon}\right)$ and

$\psi = F_0^{-1}\left(\frac{1-\gamma}{1-\varepsilon}\right)$. When $x \le \theta_1$, $E_{\gamma,(1-\varepsilon)F_0+\varepsilon\delta_x}(\sin\Theta) = (1-\varepsilon)\tilde{E}_{\gamma,F_0}(\sin\Theta) = \frac{1-\varepsilon}{1-2\gamma}\int_\lambda^{\theta_2}\sin\theta dF_0$ and

$E_{\gamma,(1-\varepsilon)F_0+\varepsilon\delta_x}(\cos\Theta) = (1-\varepsilon)\tilde{E}_{\gamma,F_0}(\cos\Theta) = \frac{1-\varepsilon}{1-2\gamma}\int_\lambda^{\theta_2}\cos\theta dF_0$. Therefore, by using Lemma 1, we get $W_\gamma(G_\varepsilon) = \arctan^*\left[\frac{\tilde{E}_{\gamma,F_0}(\sin\Theta)}{\tilde{E}_{\gamma,F_0}(\cos\Theta)}\right]$ and $W_\gamma(F_\mu) = \arctan^*\left[\frac{E_{\gamma,F_\mu}(\sin\Theta)}{E_{\gamma,F_\mu}(\cos\Theta)}\right] = \mu$. But,

$$\tilde{E}_{\gamma,F_0}(\sin\Theta) = (1-2\gamma)^{-1}\int_\lambda^{\theta_2}\sin\theta dF_0 = (1-2\gamma)^{-1}\left(\int_\lambda^{-\theta_2}\sin\theta dF_0 + \int_{-\theta_2}^{\theta_2}\sin\theta dF_0\right)$$
$$= (1-2\gamma)^{-1}\int_\lambda^{-\theta_2}\sin\theta dF_0$$

and

$$\tilde{E}_{\gamma,F_0}(\cos\Theta) = (1-2\gamma)^{-1}\int_\lambda^{\theta_2}\cos\theta dF_0 = (1-2\gamma)^{-1}\left[\int_{-\theta_2}^{\theta_2}\cos\theta dF_0 + \int_\lambda^{-\theta_2}\cos\theta dF_0\right]$$
$$= (1-2\gamma)^{-1}\left[2\int_0^{\theta_2}\cos\theta dF_0 + \int_\lambda^{-\theta_2}\cos\theta dF_0\right]$$
$$= (1-2\gamma)^{-1}\left[A(\kappa) - 2\int_{\theta_2}^{\pi}\cos\theta dF_0 + \int_\lambda^{-\theta_2}\cos\theta dF_0\right]$$
$$= (1-2\gamma)^{-1}\left[A(\kappa) - \int_{\theta_2}^{\pi}\cos\theta dF_0 + \int_{-\pi}^{\lambda}\cos\theta dF_0\right].$$

Therefore, $\frac{\tilde{E}_{\gamma,F_0}(\sin\Theta)}{\tilde{E}_{\gamma,F_0}(\cos\Theta)} = \frac{\int_\lambda^{-\theta_2}\sin\theta dF_0}{A(\kappa) - \int_{\theta_2}^{\pi}\cos\theta dF_0 + \int_{-\pi}^{\lambda}\cos\theta dF_0} \neq 0$, since $\int_\lambda^{-\theta_2}\sin\theta dF_0 \neq 0$.

Similar computations shows that $W_{\gamma}(G_{\varepsilon}) \neq \mu$ when $x \geq \theta_2$.

Hence the lemma is established.

Proof of Lemma 3: Suppose $\theta^* < \pi/2$. Then $\cos\theta > 0 \ \forall\, 0 \leq \theta \leq \theta^* \Rightarrow C_{\gamma} > 0$. When $\theta^* > \pi/2$, let $\theta^* = \pi/2 + \delta$ and $\beta^* = \pi/2 - \delta$. Then $\cos\beta = -\cos\alpha$ and hence

$e^{\kappa\cos\alpha} > e^{\kappa\cos\beta}$. Thus, $\int_{\beta^*}^{\pi/2} \cos\alpha\, e^{\kappa\cos\alpha} d\alpha > \left| \int_{\pi/2}^{\theta^*} \cos\beta\, e^{\kappa\cos\beta} d\beta \right|$. Therefore, we get

$$\int_0^{\theta^*} \cos\alpha\, e^{\kappa\cos\alpha} d\alpha = \int_0^{\beta^*} \cos\alpha\, e^{\kappa\cos\alpha} d\alpha + \int_{\beta^*}^{\pi/2} \cos\alpha\, e^{\kappa\cos\alpha} d\alpha + \int_{\pi/2}^{\theta^*} \cos\alpha\, e^{\kappa\cos\alpha} d\alpha > 0.$$

Proof of Lemma 4: Suppose $\theta^* < \pi/2$. Then $\cos\theta > 0 \ \forall\, 0 \leq \theta \leq \theta^* \Rightarrow C_{\gamma} > 0$. When $\theta^* > \pi/2$, let $\theta^* = \pi/2 + \delta$ and $\beta^* = \pi/2 - \delta$. Since $f_0(\alpha) > f_0(\beta)$ (see Stadje, 1984 with $\alpha = 2\pi$, $j = 1$ and $\theta = 0$) we have , $\int_{\beta^*}^{\pi/2} \cos\alpha\, f_0(\alpha) d\alpha > \left| \int_{\pi/2}^{\theta^*} \cos\beta\, f_0(\beta) d\beta \right|$. Therefore, we get

$$\int_0^{\theta^*} \cos\alpha\, f_0(\alpha) d\alpha = \int_0^{\beta^*} \cos\alpha\, f_0(\alpha) d\alpha + \int_{\beta^*}^{\pi/2} \cos\alpha\, f_0(\alpha) d\alpha + \int_{\pi/2}^{\theta^*} \cos\alpha\, f_0(\alpha) d\alpha > 0.$$

Hence the lemma is established.

Reference

Stadje, W. (1984), *Wrapped Distributions and Measurement Errors*, Metrika, vol. 31, 303-317.

Tables and Graphs

Table A PBP for different values of κ				
Test Functional	$\kappa=1$	$\kappa=2$	$\kappa=4$	$\kappa=10$
W	0.31	0.41	0.46	0.49
W_1	0.31	0.41	0.46	0.49

Table B: Size of CM and γ-CTM tests when observations come from $(1-\varepsilon)CN(0,\kappa)+\varepsilon\delta_x$ for different values of ε. ($n=100$, $\kappa=1$ and $\gamma=0.05$)						
X	CM ($\varepsilon=0.01$)	γ-CTM ($\varepsilon=0.01$)	CM ($\varepsilon=0.05$)	γ-CTM ($\varepsilon=0.05$)	CM ($\varepsilon=0.1$)	γ-CTM ($\varepsilon=0.1$)
$-\pi/4$	0.05	0.05	0.07	0.08	0.15	0.17
$-\pi/6$	0.05	0.05	0.05	0.05	0.07	0.08
$-\pi/8$	0.05	0.04	0.04	0.05	0.05	0.05
$-\pi/50$	0.04	0.05	0.03	0.04	0.02	0.02
$\pi/8$	0.05	0.05	0.04	0.05	0.05	0.07
$\pi/6$	0.05	0.05	0.05	0.05	0.07	0.09
$\pi/4$	0.05	0.05	0.07	0.08	0.15	0.17

Table C: Size of CM and γ-CTM tests when observations come from $(1-\varepsilon)CN(0,\kappa)+\varepsilon\delta_x$ for different values of n. ($\kappa=1$, $\varepsilon=0.01$ and $\gamma=0.05$)						
X	CM ($n=25$)	γ-CTM ($n=25$)	CM ($n=50$)	γ-CTM ($n=50$)	CM ($n=100$)	γ-CTM ($n=100$)
$-\pi/4$	0.05	0.05	0.05	0.04	0.05	0.05
$-\pi/6$	0.04	0.05	0.05	0.05	0.05	0.05
$-\pi/8$	0.05	0.05	0.05	0.05	0.05	0.04
$-\pi/50$	0.04	0.05	0.04	0.05	0.04	0.05
$\pi/8$	0.05	0.05	0.05	0.05	0.05	0.05
$\pi/6$	0.05	0.05	0.05	0.05	0.05	0.05
$\pi/4$	0.04	0.05	0.05	0.05	0.05	0.05

Table D: Power values of CM and γ-CTM when observations come from $(1-\varepsilon)CN(0,\kappa)+\varepsilon\delta_x$ for different values of n. $(\kappa=1, \varepsilon=0.01, \mu_a=-\pi/2$ and $\gamma=0.05)$						
μ_a	CM $(n=25)$	γ-CTM $(n=25)$	CM $(n=50)$	γ-CTM $(n=50)$	CM $(n=100)$	γ-CTM $(n=100)$
$\pi/4$	0.68	0.69	0.93	0.93	0.99	0.99
$\pi/8$	0.22	0.23	0.40	0.42	0.68	0.70
$\pi/12$	0.13	0.13	0.21	0.23	0.36	0.40
$\pi/25$	0.06	0.06	0.06	0.10	0.11	0.14
$\pi/50$	0.06	0.06	0.06	0.06	0.06	0.06
$\pi/100$	0.06	0.06	0.06	0.06	0.06	0.06

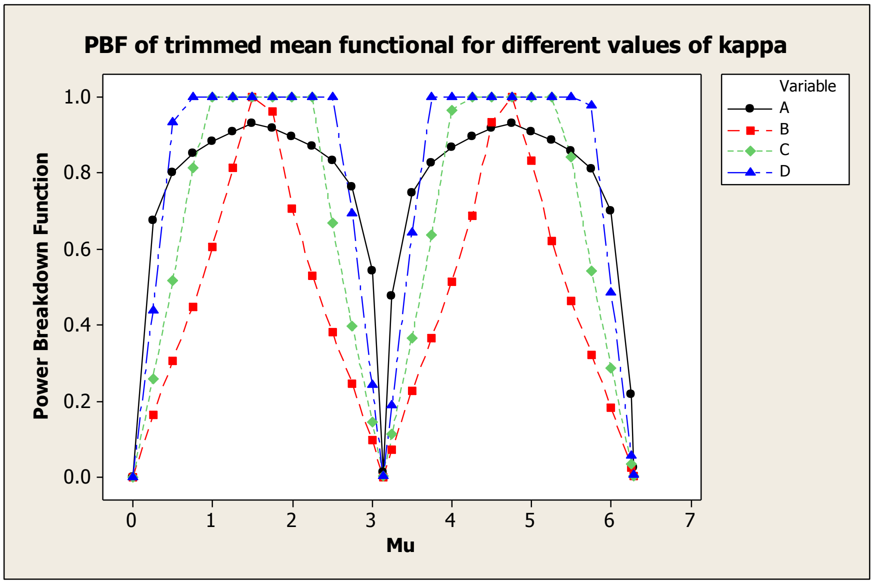

Figure A [Legends: A- W_γ $(\kappa=1)$ **, B-** W_γ $(\kappa=2)$ **, C-** W_γ $(\kappa=4)$ **, D-** W_γ $(\kappa=10)$ **].**