Test 2 Name:

1. Short answer

   (a) What is the solution to $ay'' + by' + cy + d = 0$, where $a, b, c, \& \ d$ are real numbers with $b^2 = 4ac$? [2pts]

   (b) Create an IVP with solution $y(t) = e^t + e^{2t} + e^{3t}$ (hint: think second-order linear). [2pts]

2. True/False (circle one for each) [1pt each]

   (a) $y(x) = 1/x$ is a solution to $x^2y'' + 3xy' + y = 0$. T / F

   (b) The spring-mass system described by $mx'' + cx' + kx = 0$ oscilates for all values of $m, c, k > 0$. T / F

   (c) $y(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!}$ is a solution to $y'' - y = 0$ T / F

   (d) The particular solution to $y'' + 3y' + 2y = \cos(t)$ is of the form $y_p(t) = A\cos(t)$ T / F

3. Second-order linear ordinary differential equations.

(a) Solve $y'' - y = e^{-t}$. [5pts]

(b) Find the solution to the corresponding homogeneous equation of the above using power series methods. [4pts]

4. Laplace Transforms

Solve the following IVP using Laplace transform methods. [5 pt]

$$y'' + 3y' + 2y = \sin(t),\ y(0) = 0,\ y'(0) = 0$$

5. Application

(a) Suppose you have a spring-mass system, where the mass is $1kg$, the spring constant is $k = 5$ and the drag coefficient – which acts proportional to the velocity of the mass – is $\mu = 2$. Derive a differential equation for this system. [2pts]

(b) Solve the differential equation with the initial conditions $y(0) = 1$, and $y'(0) = 0$. [2pts]

(c) Now suppose I improve the spring-mass system by eliminating drag entirely. I also now apply a periodic force to this system of constant amplitude. What would the amplitude and frequency of this force need to be for the spring/mass system to experience resonance? (Resonance, in this context, is defined as a solution $y_r(t)$ to the ODE model of this system that becomes unbounded; ie. $\lim_{t\to\infty} y_r(t) = \infty$). If you couldn't solve (a) and (b) above, use the IVP

$$y'' + 5y = f(t),\ y(0) = 1,\ y'(0) = 0,$$

where $f(t)$ is the forcing term. [2pts]

(d) If we re-introduce the drag force, as in the original setup, and still apply a periodic force of constant amplitude, can the spring-mass system ever experience the unbounded resonance you found in part (c)? What is the long-term behaviour of this system? [2pt]

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