

Test 1 Name:

1. Short answer [2pts each]

(a) Give an example of a Bernoulli equation. What substitution would you use to solve it?

(b) What type(s) of ODE is the following?

$$2x \, dy + (x^2 + x) \, y \, dx = 0$$

2. True/False (circle one for each) [1pt each]

(a) The ODE  $y' = (y^2 - 1)^{1/2}$  has a unique solution for the initial value  $y(0) = 0$ . T / F

(b) The equation  $y(t) = \ln(t)$  is a solution to the ODE  $y' = e^{-y}$ .      T / F

(c)  $y(t) = 2e^t + 6$  is a solution to the IVP  $y' - y = -6$ ;  $y(0) = 6$ .      T / F

(d) The equation  $\frac{dy}{dt} + \cos(y + t) = \left(\frac{dy}{dt}\right)^2$  is an ODE. T / F

3. Match the solution to its differential equation [1pt each]

(a)  $y(t) = t^2 + \sin(t)$   $y' = e^y; y(0) = 0$  \_\_\_\_\_

(b)  $y(t) = -\ln(1 - x)$   $y'' - 9y = 9$  \_\_\_\_\_

(c)  $y(t) = 1/\sin(x)$   $y' + \cos(x)y^2 = 0; y\left(\frac{\pi}{2}\right) = 1$  -----

$$(d) \quad y(t) = e^{-3t} - 1 \qquad y'' + y = t^2 + 2 \quad \text{-----}$$

#### 4. First-order Differential Equations

(a) Classify the following ODE. [2pts]

$$y' + 3x^2y = 6x^2$$

(b) Find a solution for the ODE. [3pts]

(c) Find the same solution using *a different* solution method. [3pts]

## 5. Power Homogeneous and Bernoulli Equations

- (a) Why is this equation power homogeneous? [1pt]  $\frac{dy}{dx} = \frac{2xy + y^2}{x^2}$
- (b) Why is the above equation Bernoulli? [1pt]
- (c) What substitution will you use to solve this ODE? (Hint: one choice is easy; the other is difficult) [1pt]
- (d) Solve the ODE. [3pt]
- (e) Check your solution. [1pt]

## 6. Application

The Alviso Salt Flats are human-made salt evaporating pools in North San José. Suppose a new flat is constructed with maximum volume  $2km^3$  and initially contains  $1km^3$  of salt water, containing 1% salt by volume. Suppose sea water enters the flats at a rate of  $0.1km^3/day$  and that this is 4% salt.

- (a) If the water evaporates from the flats at the a rate of  $0.01km^3/day$ , leaving the salt behind, what is the amount (given in  $km^3$  of salt) in the salt flat when it reaches capacity? [3pts]
  
  
  
  
  
  
  
  
  
  
- (b) What is an expression for the concentration of the salt in the flat at time  $t$ ? [2pt]
  
  
  
  
  
  
  
  
  
  
- (c) BONUS ROUND: When the flat reaches capacity, any additional water spills out, in addition to evaporating. Find an equation for the amount of salt at time  $t$ , after the flat has filled. [2pts Bonus]

Student ID:

Total Marks: \_\_\_\_\_ / 50

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