

## S.1 R function for Critical values of Two-Hypothesis Testing

```
library(mnormt)

twohypo_func <-function(rho, alpha, ratio){ ##rho is the
  correlation, alpha is the test level and ratio is for the
  allocation of critical values

s1<-matrix(c(1,rho,rho,1),2,2)
Fxy<-sadmvn(lower=c(qnorm(1-alpha),qnorm(1-alpha)),upper=c(
  Inf, Inf),c(0,0),s1)

solve_boundary1<-function(alpha1){
  integrand1<-function(p1) pnorm((qnorm(1-alpha1/p1)-rho*qnorm
    (1-p1))/sqrt(1-rho^2),lower.tail=T)
  t<-alpha-Fxy[1]-integrate(integrand1,lower=alpha1,upper=
    alpha)$value
  return(t)}

solve_boundary2<-function(alpha1){
  integrand1<-function(p1) pnorm((qnorm(1-alpha1/p1)-rho*qnorm
    (1-p1))/sqrt(1-rho^2),lower.tail=T)
  integrand2<-function(p2) pnorm((qnorm(1-alpha1*ratio/p2)-rho
    *qnorm(1-p2))/sqrt(1-rho^2),lower.tail=T)
  t<-alpha-Fxy[1]-integrate(integrand1,lower=alpha1,upper=
    alpha)$value-integrate(integrand2,lower=alpha1*ratio,
    upper=alpha)$value
  return(t)}
```

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solve_boundary3<-function(alpha1){

integrand1<-function(p1) pnorm((qnorm(1-alpha1/p1)-rho*qnorm
(1-p1))/sqrt(1-rho^2),lower.tail=T)

integrand2<-function(p1) pnorm((qnorm(1-alpha)-rho*qnorm(1-
p1))/sqrt(1-rho^2),lower.tail=T)

t<-integrate(integrand1,lower=alpha1/alpha,upper=alpha)$
  value-integrate(integrand1,lower=alpha1,upper=alpha)$
  value+integrate(integrand2,lower=0,upper=alpha1/alpha)$
  value

return(t)}

solve_boundary4<-function(alpha1){

integrand1<-function(p1) pnorm((qnorm(1-alpha1/p1)-rho*qnorm
(1-p1))/sqrt(1-rho^2),lower.tail=T)

integrand2<-function(p2) pnorm((qnorm(1-alpha1*ratio/p2)-rho
*qnorm(1-p2))/sqrt(1-rho^2),lower.tail=T)

integrand3<-function(p1) pnorm((qnorm(1-alpha)-rho*qnorm(1-
p1))/sqrt(1-rho^2),lower.tail=T)

t<-integrate(integrand1,lower=alpha1/alpha,upper=alpha)$
  value-integrate(integrand1,lower=alpha1,upper=alpha)$
  value-integrate(integrand2,lower=alpha1*ratio,upper=alpha
 )$value+integrate(integrand3,lower=0,upper=alpha1/alpha)$
  value

return(t)}

```

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##Nested search for stopping boudnary

if (class(try(uniroot(solve_boundary1,c(alpha^2,alpha))))!="
  try-error") {alpha1<-uniroot(solve_boundary1,c(alpha^2,
  alpha),tol=1e-06)} else if (class(try(uniroot(solve_
boundary2,c(alpha^2,alpha/ratio))))!="try-error") {alpha1
<-uniroot(solve_boundary2,c(alpha^2,alpha/ratio),tol=1e
-06)} else if (class(try(uniroot(solve_boundary3,c
(0.00001,alpha^2))))!="try-error") {alpha1<-uniroot(solve
_boundary3,c(0.00001,alpha^2),tol=1e-06)} else if (class(
try(uniroot(solve_boundary4,c(0.00001,alpha^2))))!="try-
error") {alpha1<-uniroot(solve_boundary4,c(0.00001,alpha
^2),tol=1e-06)}

return(alpha1$root)
}

##Example: to obtain the equal critical values given rho=0.5
and alpha=0.025, need to call the function:
twohypo_func(rho=0.5,alpha=0.025,ratio=1)
[1] 0.001823292

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## S.2 R function for Critical values of Three-Hypothesis Testing

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threehypo_func <-function(alpha,rho,ratio){

alpha1<-twohypo_func(rho=rho,alpha=alpha,ratio=ratio)
solve_boundary1<-function(alpha4){
func1<-function(p1,p2)  (1-pnorm((qnorm(1-alpha)-rho/(1+rho)

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*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho)))*1/2
/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+qnorm
(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2))/dnorm(qnorm(1-p1
))/dnorm(qnorm(1-p2))

func2<-function(p1,p2)  (1-pnorm((qnorm(1-alpha1/p1)-rho/(1+
rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho))))*
1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+
qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2))/dnorm(qnorm
(1-p1))/dnorm(qnorm(1-p2))

func22<-function(p1,p2)  (1-pnorm((qnorm(1-alpha1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho))
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

func3<-function(p1,p2)  (1-pnorm((qnorm(1-alpha4/p1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

##Value of pi3_3
pi3_3<-(integrate(function(p1) sapply(p1,function(p1
integrate(func1, lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
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lower=0,upper=alpha4/alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func1, lower=0,upper=
alpha4/alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=
alpha1/alpha)$val)+2*integrate(function(p1) sapply(p1,
function(p1) integrate(func2, lower=0,upper=alpha4/alpha1
,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$val+(integrate(
function(p1) sapply(p1,function(p1) integrate(func3,
lower=alpha4/alpha/p1,upper=1,p1=p1)[[1]]),lower=alpha4/
alpha1,upper=alpha1/alpha)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func3, lower=alpha4/
alpha1,upper=1,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$
val)

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*##Value of pi31\_3*

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pi31_3<-(integrate(function(p1) sapply(p1,function(p1)
integrate(func1,lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
lower=0,upper=alpha4/alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func1,lower=0,upper=
alpha4/alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=
alpha)$val)+integrate(function(p1) sapply(p1,function(p1)
integrate(func22,lower=alpha1/alpha,upper=1,p1=p1)[[1]])
,lower=0,upper=alpha4/alpha1)$val+(integrate(function(p1)
sapply(p1,function(p1) integrate(func3,lower=alpha1/
alpha,upper=1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=
alpha1)$val+integrate(function(p1) sapply(p1,function(p1)
integrate(func3,lower=alpha1/alpha,upper=alpha1/p1,p1=p1

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) [[1]]),lower=alpha1,upper=alpha)$val+integrate(function(
p1) sapply(p1,function(p1) integrate(func3,lower=alpha4/
alpha/p1,upper=alpha1/alpha,p1=p1)[[1]]),lower=alpha4/
alpha1,upper=alpha)$val)

##value of pi123_2

pi123_2<-(integrate(function(p1) sapply(p1,function(p1)
integrate(func1,lower=0,upper=alpha,p1=p1)[[1]]),lower=0,
upper=alpha4/alpha^2)$val+integrate(function(p1) sapply(
p1,function(p1) integrate(func1,lower=0,upper=alpha4/
alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha^2,upper=alpha)$
val)+integrate(function(p1) sapply(p1,function(p1)
integrate(func3,lower=alpha4/alpha/p1,upper=alpha,p1=p1)
[[1]]),lower=alpha4/alpha^2,upper=alpha)$val

t<-3*pi3_3-3*pi31_3+pi123_2-alpha

return(t)}

solve_boundary2<-function(alpha4){

func1<-function(p1,p2) (1-pnorm((qnorm(1-alpha)-rho/(1+rho)
*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho)))))*1/2
/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+qnorm
(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(qnorm(1-p1
))/dnorm(qnorm(1-p2))

func2<-function(p1,p2) (1-pnorm((qnorm(1-alpha1/p1)-rho/(1+
rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho)))))
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*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+
qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(qnorm
(1-p1))/dnorm(qnorm(1-p2))

func22<-function(p1,p2)  (1-pnorm((qnorm(1-alpha1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

func3<-function(p1,p2)  (1-pnorm((qnorm(1-alpha4/p1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

##Value of pi3_3

pi3_3<-(integrate(function(p1) sapply(p1,function(p1)
integrate(func1, lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
lower=0,upper=alpha4/alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func1, lower=0,upper=
alpha4/alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=
alpha1/alpha)$val)+2*integrate(function(p1) sapply(p1,
function(p1) integrate(func2, lower=0,upper=alpha4/alpha1
,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$val+(integrate(
function(p1) sapply(p1,function(p1) integrate(func3,

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lower=alpha4/alpha/p1,upper=1,p1=p1)[[1]]),lower=alpha4/
alpha1,upper=alpha1/alpha)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func3, lower=alpha4/
alpha1,upper=1,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$
val)

##Value of pi31_2

pi31_2<-(integrate(function(p1) sapply(p1,function(p1)
integrate(func1,lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
lower=0,upper=alpha4/alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func1,lower=0,upper=
alpha4/alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=
alpha)$val)+(integrate(function(p1) sapply(p1,function(p1
) integrate(func22,lower=alpha1/alpha,upper=1,p1=p1)
[[1]]),lower=0,upper=alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func22,lower=alpha1/
alpha,upper=alpha1/p1,p1=p1)[[1]]),lower=alpha1,upper=
alpha4/alpha1)$val)+integrate(function(p1) sapply(p1,
function(p1) integrate(func3,lower=alpha4/alpha/p1,upper=
alpha1/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=alpha)$
val

##value of pi123_2

pi123_2<-(integrate(function(p1) sapply(p1,function(p1)
integrate(func1,lower=0,upper=alpha,p1=p1)[[1]]),lower=0,
upper=alpha4/alpha^2)$val+integrate(function(p1) sapply(

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p1,function(p1) integrate(func1,lower=0,upper=alpha4/
alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha^2,upper=alpha)$
val)+integrate(function(p1) sapply(p1,function(p1)
integrate(func3,lower=alpha4/alpha/p1,upper=alpha,p1=p1)
[[1]]),lower=alpha4/alpha^2,upper=alpha)$val
t<-3*pi3_3-3*pi31_2+pi123_2-alpha
return(t)}

solve_boundary3<-function(alpha4){

func1<-function(p1,p2) (1-pnorm((qnorm(1-alpha)-rho/(1+rho)
*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho)))*1/2
/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+qnorm
(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(qnorm(1-p1
))/dnorm(qnorm(1-p2))

func2<-function(p1,p2) (1-pnorm((qnorm(1-alpha1/p1)-rho/(1+
rho)*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho))))
*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+
qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(qnorm
(1-p1))/dnorm(qnorm(1-p2))

func22<-function(p1,p2) (1-pnorm((qnorm(1-alpha1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

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func3<-function(p1,p2)  (1-pnorm((qnorm(1-alpha4/p1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

##Value of pi3_3

pi3_3<-(integrate(function(p1) sapply(p1,function(p1)
integrate(func1, lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
lower=0,upper=alpha4/alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func1, lower=0,upper=
alpha4/alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=
alpha1/alpha)$val)+2*integrate(function(p1) sapply(p1,
function(p1) integrate(func2, lower=0,upper=alpha4/alpha1
,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$val+(integrate(
function(p1) sapply(p1,function(p1) integrate(func3,
lower=alpha4/alpha/p1,upper=1,p1=p1)[[1]]),lower=alpha4/
alpha1,upper=alpha1/alpha)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func3, lower=alpha4/
alpha1,upper=1,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$
val)

##Value of pi31_2

pi31_2<-(integrate(function(p1) sapply(p1,function(p1)
integrate(func1,lower=0,upper=alpha1/alpha,p1=p1)[[1]]),

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lower=0,upper=alpha4/alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func1,lower=0,upper=
alpha4/alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=
alpha)$val)+(integrate(function(p1) sapply(p1,function(p1
) integrate(func22,lower=alpha1/alpha,upper=1,p1=p1)
[[1]]),lower=0,upper=alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func22,lower=alpha1/
alpha,upper=alpha1/p1,p1=p1)[[1]]),lower=alpha1,upper=
alpha4/alpha1)$val)+integrate(function(p1) sapply(p1,
function(p1) integrate(func3,lower=alpha4/alpha/p1,upper=
alpha1/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=alpha)$
val

##value of pi123_1

s1<-matrix(c(1,rho,rho,rho,1,rho,rho,rho,1),3,3)
pi123_1<-sadmvn(lower=c(qnorm(1-alpha),qnorm(1-alpha),qnorm
(1-alpha)),upper=c(Inf,Inf,Inf),c(0,0,0),s1)[1]

t<-3*pi3_3-3*pi31_2+pi123_1-alpha
return(t)}

solve_boundary4<-function(alpha4){
func1<-function(p1,p2) (1-pnorm((qnorm(1-alpha)-rho/(1+rho)
*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho))))*1/2
/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+qnorm
(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2))/dnorm(qnorm(1-p1

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))/dnorm(qnorm(1-p2))

func2<-function(p1,p2)  (1-pnorm((qnorm(1-alpha1/p1)-rho/(1+
rho)*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho))))
*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+
qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(qnorm
(1-p1))/dnorm(qnorm(1-p2))

func22<-function(p1,p2)  (1-pnorm((qnorm(1-alpha1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

func3<-function(p1,p2)  (1-pnorm((qnorm(1-alpha4/p1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2)))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

##Value of pi3_3

pi3_3<-(integrate(function(p1) sapply(p1,function(p1
integrate(func1, lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
lower=0,upper=alpha4/alpha1)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func1, lower=0,upper=
alpha4/alpha/p1,p1=p1)[[1]]),lower=alpha4/alpha1,upper=

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alpha1/alpha)$val)+2*integrate(function(p1) sapply(p1,
function(p1) integrate(func2, lower=0,upper=alpha4/alpha1
,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$val+(integrate(
function(p1) sapply(p1,function(p1) integrate(func3,
lower=alpha4/alpha/p1,upper=1,p1=p1)[[1]]),lower=alpha4/
alpha1,upper=alpha1/alpha)$val+integrate(function(p1)
sapply(p1,function(p1) integrate(func3, lower=alpha4/
alpha1,upper=1,p1=p1)[[1]]),lower=alpha1/alpha,upper=1)$
val)

```

*##Value of pi31\_1*

```

pi31_1<-integrate(function(p1) sapply(p1,function(p1)
integrate(func1,lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
lower=0,upper=alpha)$val+(integrate(function(p1) sapply(
p1,function(p1) integrate(func22,lower=alpha1/alpha,upper
=1,p1=p1)[[1]]),lower=0,upper=alpha1)$val+integrate(
function(p1) sapply(p1,function(p1) integrate(func22,
lower=alpha1/alpha,upper=alpha1/p1,p1=p1)[[1]]),lower=
alpha1,upper=alpha)$val)

```

*##value of pi123\_1*

```

s1<-matrix(c(1,rho ,rho ,rho ,1,rho ,rho ,rho ,1),3,3)
pi123_1<-sadmvn(lower=c(qnorm(1-alpha),qnorm(1-alpha),qnorm
(1-alpha)),upper=c(Inf ,Inf ,Inf ),c(0,0,0),s1)[1]

```

```
t<-3*pi3_3-3*pi31_1+pi123_1-alpha
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```

return(t)}

solve_boundary5<-function(alpha4){

func1<-function(p1,p2)  (1-pnorm((qnorm(1-alpha)-rho/(1+rho)
*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho))))*1/2
/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+qnorm
(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(qnorm(1-p1
))/dnorm(qnorm(1-p2))

func2<-function(p1,p2)  (1-pnorm((qnorm(1-alpha1/p1)-rho/(1+
rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho))))*
1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)^2+
qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(qnorm
(1-p1))/dnorm(qnorm(1-p2))

func22<-function(p1,p2)  (1-pnorm((qnorm(1-alpha1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho))
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))

func3<-function(p1,p2)  (1-pnorm((qnorm(1-alpha4/p1/p2)-rho/
(1+rho)*(qnorm(1-p1)+qnorm(1-p2))/sqrt(1-2*rho^2/(1+rho)
))*1/2/pi/sqrt(1-rho^2)*exp(-1/2/(1-rho^2)*(qnorm(1-p1)
^2+qnorm(1-p2)^2-2*rho*qnorm(1-p1)*qnorm(1-p2)))/dnorm(
qnorm(1-p1))/dnorm(qnorm(1-p2))
}

```

```

##value of pi3_2

pi11<-integrate(function(p1) sapply(p1,function(p1)
  integrate(func1,lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
  lower=0,upper=alpha1/alpha)$val

pi121<-integrate(function(p1) sapply(p1,function(p1)
  integrate(func2,lower=0,upper=p1,p1=p1)[[1]]),lower=
  alpha1/alpha,upper=alpha4/alpha1)$val
pi122<-integrate(function(p1) sapply(p1,function(p1)
  integrate(func2,lower=0,upper=alpha4/alpha1,p1=p1)[[1]]),
  lower=alpha4/alpha1,upper=1)$val
pi12<-pi121+pi122

pi13<-integrate(function(p1) sapply(p1,function(p1)
  integrate(func3,lower=alpha4/alpha1,upper=1,p1=p1)[[1]]),
  lower=alpha4/alpha1,upper=1)$val
pi3_2=pi11+2*pi12+pi13

##Value of pi31_1

pi31_1<-integrate(function(p1) sapply(p1,function(p1)
  integrate(func1,lower=0,upper=alpha1/alpha,p1=p1)[[1]]),
  lower=0,upper=alpha)$val+(integrate(function(p1) sapply(
  p1,function(p1) integrate(func22,lower=alpha1/alpha,upper
  =1,p1=p1)[[1]]),lower=0,upper=alpha1)$val+integrate(

```

```

function(p1) sapply(p1,function(p1) integrate(func22 ,
lower=alpha1/alpha ,upper=alpha1/p1 ,p1=p1)[[1]]),lower=
alpha1 ,upper=alpha )$val)

##value of pi123_1

s1<-matrix(c(1,rho ,rho ,rho ,1,rho ,rho ,rho ,1) ,3,3)
pi123_1<-sadmvn(lower=c(qnorm(1-alpha ),qnorm(1-alpha ),qnorm
(1-alpha )),upper=c(Inf ,Inf ,Inf ),c(0,0,0),s1)[1]

t<-3*pi3_2-3*pi31_1+pi123_1-alpha
return(t)}

```

  

```

if (class(try(uniroot(solve_boundary1 ,c(0+0.000001,min(
alpha1^2, alpha^3)-0.000001)),silent=T))!="try-error"){
alpha4<-uniroot(solve_boundary1 ,c(0+ 0.000001,min(alpha1
^2, alpha^3)-0.000001),tol=1e-06)$root} else if (class(try
(uniroot(solve_boundary2 ,c(min(alpha1^2, alpha^3)+
0.000001,max(alpha1^2, alpha^3)-0.000001)),silent=T))!="
try-error"){alpha4<-uniroot(solve_boundary2 ,c(min(alpha1
^2, alpha^3)+ 0.000001,max(alpha1^2, alpha^3)-0.000001)$
root,tol=1e-06)} else if (class(try(uniroot(solve_
boundary3 ,c(max(alpha1^2, alpha^3)+ 0.000001,alpha1*alpha
-0.000001)),silent=T))!="try-error"){alpha4<-uniroot(
solve_boundary3 ,c(max(alpha1^2, alpha^3)+ 0.000001,alpha1*
alpha-0.000001),tol=1e-06)$root} else if (class(try(

```

```

uniroot(solve_boundary4,c(alpha1*alpha+ 0.000001,alpha1^2
/alpha-0.000001),silent=T))!="try-error"){alpha4<-
uniroot(solve_boundary4,c(alpha1*alpha+ 0.000001,alpha1^2
/alpha-0.000001),tol=1e-06)$root} else if (class(try(
uniroot(solve_boundary5,c(alpha1^2/alpha+ 0.000001,alpha1
-0.000001)),silent=T))!="try-error"){alpha4<-uniroot(
solve_boundary5,c(alpha1^2/alpha+ 0.000001,alpha1
-0.000001),tol=1e-06)$root}else {alpha4<-1}

return(c(alpha1,alpha4))
}

##Example: to obtain the critical values given rho=0.5 and
##alpha=0.025, need to call the function:
threehypo_func(alpha=0.025,rho=0.5,ratio=1)
[1] 0.001823292    0.001079318
##0.001823 is for alpha_1 and 0.00108 is for alpha_4.

```