Name: ______________________ **Section:** ______________

1. What is the definition of an **antiderivative** for a function $f(x)$?

2. **Warmup**: By reversing our basic derivative rules for common functions, we get antiderivative rules for most of the common functions. State the (most general) antiderivative $F(x)$ of each for the following functions. (Here, $a \neq 0$ is a constant and n is also a constant).

   (a) $f(x) = x^n$, $n \neq -1$

   (b) $f(x) = \dfrac{1}{x}$

   (c) $f(x) = e^{ax}$

   (d) $f(x) = \cos(ax)$

   (e) $f(x) = \sin(ax)$

   (f) $f(x) = \sec^2(x)$

   (g) $f(x) = \sec(x)\tan(x)$

   (h) $f(x) = \dfrac{1}{1+x^2}$

   (i) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$

   (j) $f(x) = \dfrac{1}{x\sqrt{x^2-1}}$

3. For each of the functions below, find the *unique* antiderivative $F(x)$ that satisfies the given intial condition.

   (a) $f(x) = \cos(x) + x^5, \quad F(0) = 1$

(b) $f(x) = 3x^2 - 4x + 2, \quad F(0) = 2,235,141$

(c) $f(x) = \dfrac{2}{x} + \dfrac{1}{1+x^2} - e^{6x}, \quad F(1) = 0$

(d) $f(x) = \dfrac{2x}{x^2+1}, \quad F(0) = 1$. (Hint: Notice that the numerator of $f(x)$ is the derivative of the denominator, i.e., $f(x) = \dfrac{g'(x)}{g(x)}$ for $g(x) = x^2 + 1$. What kind of function $F(x)$ would give a derivative of the form $\dfrac{g'(x)}{g(x)}$?)

4. Sheeraz is walking on the moon. He jumps off of a cliff 20 meters high at an initial velocity of 2 meters per second. Fortunately, the acceleration due to gravity on the moon is -1.6 meters per second (only 1/6 that of Earth's!).

   (a) Find a formula for $v(t)$, Sheeraz's velocity at a given time t. (Velocity is the antiderivative of acceleration).

   (b) After 3 seconds what is Sheeraz's velocity?

   (c) Find a formula for $r(t)$, Sheeraz's position at a given time t.

(d) At what time does Sheeraz land on the ground? (You may use a calculator to find this value)

5. A company estimates that the marginal cost (in dollars per item) of producing x items is $1.92 - 0.002x$. If the cost of producting one item is \$562, find the cost of producing 100 items.

6. **Challenge**: Consider the situation of someone throwing a ball through the air. If the only force acting on the ball is the force of gravity, then this type of motion is called **projectile motion**. The ball moves in a parabolic arc and so has two different positions, its horizontal position and its vertical position (relative to the starting position).

   Suppose that the initial velocity of the ball in the horizontal direction is $v_x(0) = 20$ feet per second and in the vertical direction is $v_y(0) = 16$ feet per second. The acceleration due to gravity only affects the ball vertically and is $a_y(t) = -32$ feet per second (that is $a_x(t) = 0$).

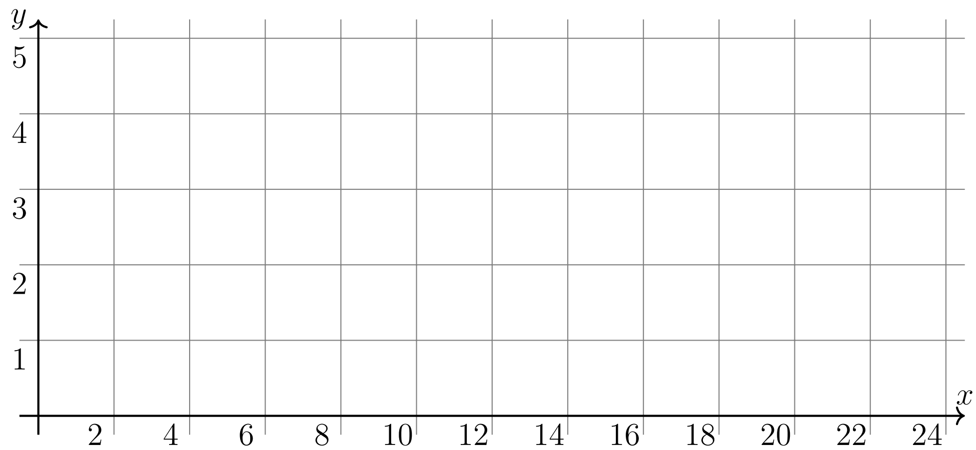
   (a) Find a formula for the position of the ball in both the horizontal direction $s_x(t)$ and in the vertical direction $s_y(t)$.

   (b) At what time does the ball hit the ground after being thrown?

   (c) How far does the ball travel horizontally before it hits the ground?

(d) How high does the ball travel vertically? (Hint: $s_y(t)$ is a parabola, use the t value corresponding to its vertex)

(e) Note that if we graph the position of the ball the graph would consist of the points $(s_x(t), s_y(t))$. Graph the position of the ball (Note that time starts at $t = 0$ and ends when the ball hits the ground)



7. **Challenge**: Since raindrops grow as they fall, their surface area increases and therefore the resistance to their falling increases. A raindrop has an initial downward velocity of 10 meters per second and its downward acceleration is

$$a(t) = \begin{cases} 9 - 0.9t & \text{if } 0 \leq t \leq 10 \\ 0 & \text{if } t > 10 \end{cases}$$

If the raindrop is initially 500 meters above the ground, how long does it take to fall?