Supplementary Material: Surrogate Residuals for Discrete Choice Models

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This file presents the appendices (Appendices A, B and C) and supplementary tables and figures unshown in the manuscript.

Appendix A: Generating Samples from Truncated Multi-dimensional Distribution

In this section, we introduce a Gibbs method to generate samples from truncated multi-dimensional distribution like $\boldsymbol{S}$ in section 3.1. Before the multi-dimensional case, we at first present the method to generate truncated univariate distribution, because the ability to generate univariates is a central building block in the Gibbs method.

1. The univariate case

Let X follow a distribution G truncating on the interval (a, b), where G is a univariate continuous cumulative distribution function, a is the left truncated point and b is the right truncated

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point. The two truncated points are arbitrary but fixed on $\mathbb{R}$ and $-\infty \le a < b \le +\infty$. Then, the distribution function of X is

$$F(x) = \frac{G(x) - G(a)}{G(b) - G(a)} \quad a \le x \le b.$$

Based on the process of *inverse CDF method*, solving the equation $F(x) = u$, $u \in (0, 1)$, we get

$$x = G^{-1}\Big(u\big(G(b) - G(a)\big) + G(a)\Big).$$

Then, we simulate X as follows:

Step 1: Generate a sample u from Unif$(0, 1)$, where Unif$(0, 1)$ denotes the standard uniform distribution.

Step 2: Return $X = G^{-1}\Big(u\big(G(b) - G(a)\big) + G(a)\Big)$. Then, X follows the target distribution.

2. The multivariate case

Now, we propose a Gibbs sampling approach for drawing samples from a truncated multi-dimensional density like $\boldsymbol{S}$ in section 3.1. Gibbs sampling is a Markov chain Monte Carlo (MCMC) method for generation of samples from high dimensional densities by drawing the samples from full conditional densities. In other words, to implement the Gibbs sampling for an J-dimensional vector, we need to find full conditionals of its subvectors.

Let $\boldsymbol{X} = [X_1, \ldots, X_J]^T$ denote a variable that follows a J-dimensional distribution $G(x_1, \ldots, x_J)$ using the support area $\mathcal{D}_k$, where $G(x_1, \ldots, x_J)$ is a continuous cumulative distribution function and $\mathcal{D}_k$ is a support area in $\mathbb{R}^J$ such as $X_k > X_j$, $\forall j \neq k$. Denote by $\boldsymbol{x} = (x_1, x_2, \ldots, x_J)^T$ a sample from $\boldsymbol{X}$. We shall denote the c.d.f of the full conditionals $X_j|X_{-j}$, $j = 1, 2, \ldots, J$, where $X_{-j} = (X_1, X_2, \ldots, X_{j-1}, X_{j+1}, \ldots, X_J)^T$, by $G_{X_j|X_{-j}}(x_j)$. The conditional density $X_j|X_{-j}$ is

$$F_{X_j|X_{-j}}(x_j|x_{-j}) = \frac{G_{X_j|X_{-j}}(x_j|x_{-j}) - G_{X_j|X_{-j}}(a|x_{-j})}{G_{X_j|X_{-j}}(b|x_{-j}) - G_{X_j|X_{-j}}(a|x_{-j})},$$

where

$$\begin{cases} a = \max x_{-j}, b = +\infty & \text{if } j = k \\ a = -\infty, b = x_k & \text{if } j \neq k \end{cases}$$

and

$$G_{X_j|X_{-j}}(x_j|x_{-j}) = \int_{-\infty}^{x_j} \frac{g(x_1, \ldots, x_{j-1}, v_j, x_{j+1}, \ldots, x_J)}{\int_{-\infty}^{+\infty} g(x_1, \ldots, x_{j-1}, v_j, x_{j+1}, \ldots, x_J) dv_j} dv_j.$$

Here, $g(.)$ is the corresponding p.d.f of $G(.)$. The distribution above is nothing but a truncated univariate distribution whose samples can be generated by the previous subsection. Now we present the Gibbs sampling scheme that will be used to produce samples from $\boldsymbol{X}$. First, we get initial values $\{x_k^{(0)}\}_{k=2}^J$ from the support area $\mathcal{D}_k$, and then for $i = 1, 2, \ldots, M$ generate samples according to

$$\begin{array}{lcl} X_1|x_2^{(i-1)}, x_3^{(i-1)}, \ldots, x_J^{(i-1)} & \longrightarrow & x_1^{(i)} \\ X_2|x_1^{(i)}, x_3^{(i-1)}, \ldots, x_J^{(i-1)} & \longrightarrow & x_2^{(i)} \\ \ldots & & \\ X_{J-1}|x_1^{(i)}, \ldots, x_{J-2}^{(i)}, x_J^{(i-1)} & \longrightarrow & x_{J-1}^{(i)} \\ X_J|x_1^{(i)}, x_2^{(i)}, \ldots, x_{J-1}^{(i)} & \longrightarrow & x_J^{(i)} \end{array}$$

where the arrows represent sampling from the corresponding distribution, and each x_k is sampled according to the univariate scheme. We should discard the first B samples, where B is a given number in advance, which are called burn-in samples, and the remaining samples are considered as the samples from $\boldsymbol{X}$.

Appendix B: Proofs for Theorems 3 and 4

In this section, we prove Theorems 3 and 4. First, we derive the distribution of $\hat{\boldsymbol{S}}$ that follows the truncated distribution obtained by truncating the distribution of $\hat{\boldsymbol{U}} = \hat{\boldsymbol{V}}(\boldsymbol{X}, \hat{\boldsymbol{\beta}}) + \boldsymbol{\epsilon}$ using the area $\{\hat{U}_y > \hat{U}_j, \forall j \neq y\}$, denoted by $\hat{\mathcal{D}}_y$ henceforth, given $Y = y$. Noting that $\{\hat{U}_y > \hat{U}_j, \forall j \neq y\}$ only represents the order information, we have $\hat{\mathcal{D}}_y \equiv \mathcal{D}_y$, whether $\hat{\boldsymbol{\beta}}$ is a consistent estimate or not. When $\hat{\boldsymbol{\beta}} = \boldsymbol{\beta} + o_p(1)$ and under the assumption that $\boldsymbol{V}(\boldsymbol{X}, \boldsymbol{\beta})$ is a continuous function, we have $\boldsymbol{V}(\boldsymbol{X}, \hat{\boldsymbol{\beta}}) = \boldsymbol{V}(\boldsymbol{X}, \boldsymbol{\beta}) + o_p(1)$, thus $\hat{\boldsymbol{U}} = \boldsymbol{U} + o_p(1)$. Therefore, for all continuity sets $\mathcal{B} \in \mathbb{R}^J$, we have $Pr(\hat{\boldsymbol{U}} \in \mathcal{B}) = Pr(\boldsymbol{U} \in \mathcal{B}) + o(1)$, which implies $Pr(\hat{\boldsymbol{U}} \in \mathcal{C} \cap \mathcal{D}_y) = Pr(\boldsymbol{U} \in \mathcal{C} \cap \mathcal{D}_y) + o(1)$

and $Pr(\hat{\boldsymbol{U}} \in \mathcal{D}_y) = Pr(\boldsymbol{U} \in \mathcal{D}_y) + o(1)$. Now, conditional on $\boldsymbol{X}$, for any arbitrary but fixed $\boldsymbol{c} = [c_1, \ldots, c_J]$,

$$
\begin{aligned}
& |Pr(\hat{\boldsymbol{S}} \le \boldsymbol{c}) - Pr(\boldsymbol{S} \le \boldsymbol{c})| \\
= & \left| \sum_{y=1}^J Pr(\hat{\boldsymbol{S}} \le \boldsymbol{c} | Y = y) Pr(Y = y) - \sum_{y=1}^J Pr(\boldsymbol{S} \le \boldsymbol{c} | Y = y) Pr(Y = y) \right| \\
\le & \sum_{y=1}^J \left| Pr(\hat{\boldsymbol{S}} \le \boldsymbol{c} | Y = y) - Pr(\boldsymbol{S} \le \boldsymbol{c} | Y = y) \right| Pr(Y = y) \\
= & \sum_{y=1}^J \left| \frac{Pr(\hat{\boldsymbol{U}} \in \mathcal{C} \cap \hat{\mathcal{D}}_y)}{Pr(\hat{\boldsymbol{U}} \in \hat{\mathcal{D}}_y)} - \frac{Pr(\boldsymbol{U} \in \mathcal{C} \cap \mathcal{D}_y)}{Pr(\boldsymbol{U} \in \mathcal{D}_y)} \right| Pr(Y = y) \\
= & \sum_{y=1}^J \left| \frac{Pr(\hat{\boldsymbol{U}} \in \mathcal{C} \cap \mathcal{D}_y)}{Pr(\hat{\boldsymbol{U}} \in \hat{\mathcal{D}}_y)} - \frac{Pr(\boldsymbol{U} \in \mathcal{C} \cap \mathcal{D}_y)}{Pr(\boldsymbol{U} \in \mathcal{D}_y)} \right| Pr(Y = y) \\
= & \sum_{y=1}^J \left| \frac{Pr(\boldsymbol{U} \in \mathcal{C} \cap \mathcal{D}_y) + o(1)}{Pr(\boldsymbol{U} \in \mathcal{D}_y) + o(1)} - \frac{Pr(\boldsymbol{U} \in \mathcal{C} \cap \mathcal{D}_y)}{Pr(\boldsymbol{U} \in \mathcal{D}_y)} \right| Pr(Y = y) \\
= & \; o(1).
\end{aligned}
$$

Thus, we can establish $Pr(\hat{\boldsymbol{R}}_{\hat{\boldsymbol{\beta}}} \le \boldsymbol{r} | \boldsymbol{X}) = Pr(\boldsymbol{R} \le \boldsymbol{r} | \boldsymbol{X}) + o(1)$. The unconditional distribution of $\hat{\boldsymbol{R}}_{\hat{\boldsymbol{\beta}}}$

$$
Pr(\hat{\boldsymbol{R}}_{\hat{\boldsymbol{\beta}}} \le \boldsymbol{r}) = \int Pr(\hat{\boldsymbol{R}}_{\hat{\boldsymbol{\beta}}} \le \boldsymbol{r} | \boldsymbol{x}) d\mu(\boldsymbol{x}) \to \int Pr(\boldsymbol{R} \le \boldsymbol{r} | \boldsymbol{x}) d\mu(\boldsymbol{x}) = Pr(\boldsymbol{R} \le \boldsymbol{r}),
$$

based on Lebesgue's Dominated Convergence Theorem. Now, Theorem 3 has been proved. Moreover, based on Theorem 3, we have $E(\hat{\boldsymbol{R}}_{\hat{\boldsymbol{\beta}}} | \boldsymbol{X}) = E(\boldsymbol{R} | \boldsymbol{X}) + o(1) = o(1)$ and $Var(\hat{\boldsymbol{R}}_{\hat{\boldsymbol{\beta}}} | \boldsymbol{X}) = Var(\boldsymbol{R} | \boldsymbol{X}) + o(1)$. This completes the proof of Theorem 4.

Appendix C: Proof for $Pr(Z_y > Z_j; \forall j \neq y) = F_a(y; \boldsymbol{X}, \boldsymbol{\beta})$

We now prove that under the assumed model (4.11), $Pr(Z_y > Z_j; \forall j \neq y) = F_a(y; \boldsymbol{X}, \boldsymbol{\beta})$. At first, abbreviating $Pr(Z_y > Z_j; \forall j \neq y)$ as p_y, we have

$$
\begin{aligned}
p_y &= \Pr\big(Z_y > Z_j, \forall j \neq y\big) \\
&= \Pr\big(\mu_y + \epsilon_y > \mu_j + \epsilon_j, \forall j \neq y\big) \\
&= \Pr\big(\epsilon_j < \mu_y - \mu_j + \epsilon_y, \forall j \neq y\big).
\end{aligned}
$$

If ϵ_y is considered given, this equation is the cumulative density function for each ϵ_j evaluated at $\mu_y - \mu_j + \epsilon_y$, according to the c.d.f of standard Gumbel distribution, which is $\exp\big(-\exp(-(\mu_y - \mu_j + \epsilon_y))\big)$. Since all ϵ_y are independent, the cumulative distribution over all $y \neq j$ is the product of the individual cumulative distributions

$$p_y|\epsilon_y = \prod_{j\neq y} \exp\big(- \exp(-(\mu_y - \mu_j + \epsilon_y))\big).$$

But in this question, ϵ_y is unknown, so the probability p_y is the integral of $p_y|\epsilon_y$ weighted by the density of ϵ_y

$$p_y = \int \Big(\prod_{j\neq y} e^{-\exp(-(\mu_y-\mu_j+\epsilon_y))}\Big) e^{-\epsilon_y-\exp(-\epsilon_y)} \mathrm{d}\epsilon_y. \tag{1}$$

Mark ϵ_y as t and note that $\mu_y - \mu_y = 0$. Thus, collecting the terms in the exponent of e, we have

$$\begin{aligned} p_y &= \textstyle\int_{t=-\infty}^{+\infty} \Big(\prod_j e^{-\exp(-(\mu_y-\mu_j-t))}\Big) e^{-t}\mathrm{d}t \\ &= \textstyle\int_{t=-\infty}^{+\infty} \exp\Big(- \sum_j e^{-\mu_y+\mu_j+t}\Big) e^{-t}\mathrm{d}t \\ &= \textstyle\int_{t=-\infty}^{+\infty} \exp\Big(- e^{-t} \sum_j e^{-\mu_y+\mu_j}\Big) e^{-t}\mathrm{d}t. \end{aligned}$$

Define $s = e^{-t}$ such that $e^{-t}\mathrm{d}t = -\mathrm{d}s$, using the new term, we have

$$\begin{aligned} p_j &= \textstyle\int_0^{+\infty} \exp\Big(- s \sum_j e^{-\mu_y+\mu_j}\Big) \mathrm{d}t = \left(\frac{\exp\left(-s\sum_j e^{-\mu_y+\mu_j}\right)}{-\sum_j e^{-\mu_y+\mu_j}} \right)_0^{+\infty} \\ &= \frac{\exp(\mu_y)}{\sum_{j=1}^J \exp(\mu_j)} = \frac{F_a(y;\boldsymbol{X},\boldsymbol{\beta})}{\sum_{j=1}^J F_a(j;\boldsymbol{X},\boldsymbol{\beta})} \\ &= F_a(y;\boldsymbol{X},\boldsymbol{\beta}). \end{aligned}$$

This completes the proof.

Supplementary Tables and Figures

Table 1: Comparison of Pearson's residual ($r_{ij}^{(P)}$), deviance residual ($r_{ij}^{(D)}$), normalized surrogate residual ($r_{ij}^{(N)}$), for Scenario I.

j	i	mean			variance			skewness			kurtosis			K-S test		
		$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$
	1	0.02	-0.06	-0.00	1.01	1.23	1.02	0.59	0.47	0.04	1.93	1.45	3.01	0.230(0.000)	0.284(0.000)	0.009(0.456)
	2	-0.03	-0.17	-0.02	0.72	0.52	0.97	3.09	2.29	-0.01	13.06	7.37	3.09	0.400(0.000)	0.400(0.000)	0.018(0.003)
	3	-0.00	-0.15	-0.02	0.77	0.56	1.00	2.85	2.15	0.02	11.11	6.56	2.90	0.388(0.000)	0.388(0.000)	0.014(0.039)
	4	-0.02	-0.03	-0.02	0.99	1.26	1.00	0.07	0.06	-0.01	1.71	1.28	2.98	0.170(0.000)	0.226(0.000)	0.009(0.430)
	5	-0.00	-0.02	0.00	1.01	1.28	1.01	0.13	0.11	-0.03	1.67	1.27	2.95	0.180(0.000)	0.238(0.000)	0.009(0.453)
	6	-0.00	-0.06	-0.00	1.02	1.25	1.00	0.52	0.42	-0.03	1.86	1.41	2.93	0.230(0.000)	0.285(0.000)	0.006(0.894)
	7	-0.02	-0.00	-0.00	0.99	1.23	1.01	-0.15	-0.10	0.05	1.89	1.33	2.90	0.151(0.000)	0.206(0.000)	0.009(0.401)
	8	0.01	-0.14	0.01	0.89	0.78	0.99	1.99	1.59	-0.03	6.05	4.05	2.96	0.332(0.000)	0.332(0.000)	0.010(0.233)
	9	0.02	-0.12	0.02	0.95	0.91	1.02	1.60	1.29	0.03	4.47	3.07	2.97	0.292(0.000)	0.308(0.000)	0.016(0.016)
1	10	0.01	-0.09	0.01	1.01	1.17	0.99	0.85	0.70	0.01	2.35	1.74	3.00	0.247(0.000)	0.294(0.000)	0.009(0.366)
	11	-0.05	-0.18	-0.01	0.65	0.45	0.96	3.47	2.51	0.01	16.14	8.79	2.93	0.415(0.000)	0.415(0.000)	0.008(0.519)
	12	-0.02	-0.16	-0.00	0.84	0.73	0.99	2.13	1.70	-0.01	6.74	4.48	3.00	0.346(0.000)	0.346(0.000)	0.006(0.912)
	13	0.01	-0.07	0.00	1.01	1.22	0.99	0.66	0.54	-0.01	2.02	1.52	2.92	0.240(0.000)	0.293(0.000)	0.008(0.598)
	14	0.01	-0.13	0.01	0.97	0.99	1.02	1.38	1.12	0.05	3.69	2.59	2.98	0.275(0.000)	0.305(0.000)	0.009(0.427)
	15	0.00	0.06	-0.01	0.93	1.09	0.99	-0.63	-0.40	0.01	2.95	1.66	3.06	0.168(0.000)	0.211(0.000)	0.010(0.323)
	16	0.00	0.01	-0.01	1.01	1.26	1.00	-0.02	-0.03	-0.01	1.84	1.30	2.94	0.156(0.000)	0.213(0.000)	0.012(0.124)
	17	0.00	0.04	0.01	0.98	1.20	0.98	-0.31	-0.22	0.00	2.12	1.41	2.99	0.165(0.000)	0.218(0.000)	0.008(0.607)
	18	-0.04	-0.17	-0.00	0.68	0.47	0.98	3.38	2.44	-0.01	15.40	8.32	2.97	0.410(0.000)	0.410(0.000)	0.010(0.271)
	19	0.02	-0.13	0.01	0.93	0.84	1.00	1.82	1.45	0.00	5.26	3.58	2.99	0.314(0.000)	0.314(0.000)	0.005(0.924)
	20	-0.04	-0.17	-0.02	0.72	0.54	1.00	2.92	2.21	0.02	11.69	6.96	2.96	0.395(0.000)	0.395(0.000)	0.011(0.189)
	1	-0.00	-0.13	-0.00	0.96	1.01	0.99	1.32	1.06	0.01	3.60	2.47	3.01	0.273(0.000)	0.307(0.000)	0.007(0.767)
	2	0.00	0.06	-0.00	0.99	1.21	0.96	-0.51	-0.38	-0.01	2.11	1.45	3.02	0.200(0.000)	0.252(0.000)	0.007(0.678)
	3	-0.03	0.02	-0.01	0.99	1.22	1.00	-0.41	-0.29	0.03	2.03	1.39	3.03	0.183(0.000)	0.235(0.000)	0.013(0.070)
	4	-0.00	-0.15	0.01	0.88	0.75	0.99	2.17	1.62	-0.00	7.34	4.27	2.92	0.336(0.000)	0.336(0.000)	0.008(0.620)
	5	0.00	-0.14	0.00	0.89	0.81	1.03	1.88	1.47	-0.02	5.70	3.69	3.01	0.317(0.000)	0.319(0.000)	0.009(0.339)
	6	0.01	-0.12	0.02	0.96	0.97	0.98	1.41	1.12	0.03	3.88	2.63	2.96	0.275(0.000)	0.303(0.000)	0.009(0.361)
	7	-0.04	-0.18	-0.02	0.78	0.63	0.99	2.54	1.92	-0.04	9.39	5.54	2.94	0.370(0.000)	0.370(0.000)	0.010(0.285)
	8	-0.00	-0.01	0.00	1.02	1.31	0.99	0.03	0.04	-0.02	1.52	1.21	3.01	0.191(0.000)	0.248(0.000)	0.006(0.865)
	9	0.00	-0.04	0.00	1.00	1.28	1.01	0.32	0.25	0.02	1.59	1.26	2.98	0.214(0.000)	0.271(0.000)	0.006(0.818)
2	10	0.01	-0.11	0.03	0.99	1.11	1.00	1.04	0.83	-0.00	2.79	1.97	2.99	0.255(0.000)	0.298(0.000)	0.013(0.061)
	11	0.02	0.08	0.00	0.97	1.15	0.99	-0.62	-0.47	0.03	2.57	1.60	3.03	0.205(0.000)	0.250(0.000)	0.008(0.544)
	12	0.00	0.00	0.02	1.02	1.31	1.01	-0.01	-0.00	0.05	1.54	1.21	2.98	0.185(0.000)	0.242(0.000)	0.011(0.152)
	13	0.01	-0.12	0.01	0.98	1.03	0.99	1.25	1.00	0.04	3.37	2.33	2.97	0.267(0.000)	0.302(0.000)	0.006(0.866)
	14	0.01	-0.06	-0.00	1.01	1.25	0.98	0.50	0.42	0.01	1.78	1.39	3.04	0.226(0.000)	0.281(0.000)	0.010(0.254)
	15	-0.03	-0.16	-0.00	0.71	0.49	0.99	3.33	2.31	-0.03	15.53	7.74	2.95	0.403(0.000)	0.403(0.000)	0.005(0.951)
	16	-0.01	-0.15	-0.00	0.86	0.73	1.01	2.19	1.67	-0.01	7.41	4.46	2.96	0.344(0.000)	0.344(0.000)	0.006(0.908)
	17	-0.01	-0.15	0.00	0.82	0.62	1.00	2.71	1.93	-0.01	10.76	5.61	2.99	0.369(0.000)	0.369(0.000)	0.008(0.546)
	18	0.01	0.07	-0.00	0.97	1.17	0.99	-0.61	-0.44	-0.00	2.40	1.55	3.08	0.202(0.000)	0.249(0.000)	0.010(0.278)
	19	-0.02	-0.05	-0.02	1.00	1.29	1.00	0.18	0.16	0.02	1.52	1.23	2.93	0.213(0.000)	0.270(0.000)	0.012(0.117)
	20	0.01	0.06	-0.01	0.99	1.22	1.03	-0.44	-0.33	0.01	2.04	1.40	3.05	0.198(0.000)	0.252(0.000)	0.009(0.333)
	1	-0.01	-0.08	-0.01	1.00	1.23	0.98	0.59	0.49	0.01	1.94	1.46	2.99	0.244(0.000)	0.297(0.000)	0.014(0.050)
	2	0.01	-0.10	-0.00	0.98	1.07	1.00	1.09	0.80	-0.04	3.28	2.02	2.98	0.246(0.000)	0.283(0.000)	0.007(0.731)
	3	0.01	-0.09	0.01	0.98	1.09	1.00	1.03	0.76	0.04	3.13	1.94	2.98	0.240(0.000)	0.278(0.000)	0.008(0.606)
	4	0.02	-0.06	0.02	1.01	1.18	1.01	0.78	0.57	-0.03	2.48	1.63	2.99	0.223(0.000)	0.266(0.000)	0.018(0.002)
	5	-0.01	-0.09	0.01	1.00	1.18	1.01	0.76	0.59	-0.02	2.39	1.64	2.94	0.237(0.000)	0.284(0.000)	0.010(0.239)
	6	-0.01	-0.08	-0.01	1.00	1.23	1.01	0.61	0.49	0.00	2.01	1.48	2.99	0.239(0.000)	0.292(0.000)	0.007(0.666)
	7	0.04	-0.05	0.02	0.99	1.13	1.00	0.85	0.61	-0.04	2.75	1.73	3.04	0.213(0.000)	0.251(0.000)	0.012(0.103)
	8	-0.01	-0.10	-0.01	1.02	1.21	1.00	0.79	0.62	0.00	2.37	1.63	2.96	0.254(0.000)	0.302(0.000)	0.008(0.532)
	9	-0.02	-0.10	-0.02	0.99	1.22	1.00	0.66	0.56	0.01	1.98	1.53	3.03	0.257(0.000)	0.309(0.000)	0.010(0.317)
3	10	-0.01	-0.08	0.00	1.01	1.25	1.01	0.55	0.46	0.01	1.85	1.42	3.04	0.248(0.000)	0.301(0.000)	0.005(0.940)
	11	0.00	-0.10	0.00	0.96	1.04	0.99	1.16	0.84	-0.02	3.70	2.15	3.00	0.246(0.000)	0.277(0.000)	0.006(0.877)
	12	0.00	-0.09	0.00	1.01	1.20	1.02	0.78	0.61	-0.06	2.33	1.62	2.95	0.249(0.000)	0.295(0.000)	0.009(0.403)
	13	-0.01	-0.08	-0.00	1.02	1.25	0.99	0.59	0.47	0.00	1.95	1.44	3.01	0.243(0.000)	0.298(0.000)	0.004(0.989)
	14	-0.01	-0.09	-0.00	1.00	1.24	1.01	0.60	0.50	0.02	1.89	1.45	3.01	0.255(0.000)	0.308(0.000)	0.005(0.948)
	15	0.00	-0.10	-0.01	0.90	0.97	1.00	1.20	0.80	-0.01	4.24	2.22	2.99	0.222(0.000)	0.246(0.000)	0.007(0.696)
	16	-0.01	-0.10	-0.02	0.98	1.14	0.99	0.84	0.64	-0.01	2.65	1.75	2.90	0.236(0.000)	0.278(0.000)	0.012(0.123)
	17	-0.01	-0.11	-0.00	0.96	1.07	0.99	1.02	0.74	0.01	3.32	1.97	3.05	0.233(0.000)	0.269(0.000)	0.009(0.352)
	18	0.00	-0.10	-0.01	0.94	1.04	1.02	1.12	0.82	0.03	3.49	2.10	3.04	0.245(0.000)	0.278(0.000)	0.011(0.182)
	19	0.00	-0.08	-0.00	0.99	1.21	1.00	0.68	0.56	0.04	2.04	1.53	2.94	0.247(0.000)	0.296(0.000)	0.008(0.527)
	20	0.01	-0.10	-0.01	0.99	1.10	1.00	1.07	0.77	0.03	3.44	1.94	2.96	0.246(0.000)	0.284(0.000)	0.009(0.445)

Note: The numbers in parentheses represent the p-value of the K-S test for r_{ij}.

Table 2: Comparison of Pearson's residual ($r_{ij}^{(P)}$), deviance residual ($r_{ij}^{(D)}$), normalized surrogate residual ($r_{ij}^{(N)}$), for Scenario II.

j	i	mean			variance			skewness			kurtosis			K-S test		
		$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$
1	1	-0.02	0.03	-0.01	1.01	1.26	1.01	-0.41	-0.32	0.00	1.74	1.33	3.10	0.206(0.000)	0.262(0.000)	0.010(0.293)
	2	-0.07	-0.08	-0.03	0.98	1.25	1.01	0.09	0.11	-0.02	1.65	1.26	3.00	0.198(0.000)	0.255(0.000)	0.014(0.046)
	3	-0.05	0.00	-0.02	1.02	1.27	0.99	-0.41	-0.31	0.02	1.72	1.31	3.07	0.195(0.000)	0.250(0.000)	0.012(0.128)
	4	0.01	-0.07	0.01	1.00	1.19	0.99	0.72	0.54	0.03	2.37	1.59	3.00	0.225(0.000)	0.274(0.000)	0.006(0.800)
	5	0.05	-0.04	0.03	1.04	1.20	0.99	0.74	0.52	0.00	2.47	1.57	3.00	0.214(0.000)	0.261(0.000)	0.014(0.043)
	6	-0.04	0.01	-0.00	1.03	1.28	1.02	-0.38	-0.27	0.02	1.77	1.30	2.94	0.195(0.000)	0.251(0.000)	0.009(0.399)
	7	0.04	-0.07	0.02	0.99	1.08	1.03	1.08	0.77	-0.03	3.29	1.97	2.99	0.238(0.000)	0.270(0.000)	0.017(0.006)
	8	-0.02	0.03	-0.02	1.01	1.26	1.01	-0.39	-0.31	0.00	1.73	1.32	2.89	0.204(0.000)	0.260(0.000)	0.012(0.096)
	9	0.06	-0.03	0.05	1.06	1.17	1.01	0.91	0.60	0.02	3.15	1.69	2.98	0.218(0.000)	0.259(0.000)	0.021(0.000)
	10	0.08	0.09	0.04	0.95	1.15	1.02	-0.01	-0.08	-0.04	2.25	1.47	2.94	0.144(0.000)	0.200(0.000)	0.025(0.000)
	11	-0.02	-0.06	-0.01	1.00	1.25	0.98	0.37	0.31	-0.01	1.78	1.35	3.04	0.205(0.000)	0.261(0.000)	0.007(0.647)
	12	-0.04	-0.08	-0.02	0.99	1.24	0.99	0.39	0.32	-0.00	1.92	1.37	2.99	0.215(0.000)	0.270(0.000)	0.014(0.053)
	13	0.05	0.06	0.03	0.98	1.21	1.02	-0.13	-0.14	-0.05	2.02	1.38	2.97	0.164(0.000)	0.220(0.000)	0.018(0.004)
	14	-0.04	0.01	-0.00	1.00	1.26	0.99	-0.35	-0.28	0.00	1.68	1.30	2.96	0.196(0.000)	0.251(0.000)	0.009(0.397)
	15	-0.06	-0.09	-0.03	0.99	1.25	0.99	0.29	0.26	0.03	1.75	1.33	3.03	0.212(0.000)	0.268(0.000)	0.019(0.001)
	16	0.04	0.05	0.01	0.95	1.18	1.00	-0.04	-0.08	-0.02	2.08	1.40	3.04	0.146(0.000)	0.201(0.000)	0.008(0.602)
	17	0.05	-0.04	0.02	1.03	1.15	1.01	0.94	0.65	-0.01	3.00	1.75	3.00	0.223(0.000)	0.263(0.000)	0.015(0.017)
	18	0.05	-0.06	0.02	1.02	1.08	1.03	1.13	0.78	-0.02	3.59	2.00	2.96	0.235(0.000)	0.266(0.000)	0.015(0.023)
	19	0.02	0.04	0.01	1.02	1.26	1.02	-0.22	-0.19	0.02	1.90	1.34	3.01	0.178(0.000)	0.234(0.000)	0.010(0.300)
	20	-0.06	-0.00	-0.02	1.03	1.28	1.02	-0.39	-0.29	0.02	1.69	1.30	3.02	0.193(0.000)	0.248(0.000)	0.012(0.093)
2	1	-0.03	-0.15	0.00	0.60	0.47	0.98	2.81	2.28	-0.02	10.12	7.49	2.98	0.408(0.000)	0.408(0.000)	0.007(0.649)
	2	0.08	0.01	0.05	0.96	1.17	1.03	0.51	0.39	-0.03	1.87	1.46	3.04	0.156(0.000)	0.202(0.000)	0.029(0.000)
	3	0.01	-0.12	0.03	0.74	0.64	1.01	2.11	1.74	-0.00	6.37	4.81	3.02	0.362(0.000)	0.362(0.000)	0.016(0.014)
	4	0.00	0.03	-0.00	1.00	1.26	1.01	-0.23	-0.18	-0.00	1.76	1.30	3.00	0.184(0.000)	0.238(0.000)	0.005(0.939)
	5	-0.03	0.00	-0.01	1.02	1.26	0.99	-0.28	-0.19	0.01	1.84	1.31	3.07	0.179(0.000)	0.232(0.000)	0.010(0.238)
	6	-0.03	-0.14	-0.01	0.56	0.43	0.99	3.06	2.46	-0.03	11.78	8.52	2.91	0.418(0.000)	0.418(0.000)	0.007(0.729)
	7	-0.03	0.04	-0.02	0.99	1.18	1.01	-0.62	-0.45	-0.01	2.27	1.54	2.94	0.195(0.000)	0.239(0.000)	0.011(0.214)
	8	-0.03	-0.15	-0.02	0.58	0.44	1.00	3.00	2.43	0.01	11.31	8.36	3.04	0.417(0.000)	0.417(0.000)	0.011(0.150)
	9	-0.06	-0.01	-0.02	1.03	1.24	1.00	-0.44	-0.28	-0.01	2.06	1.38	2.97	0.177(0.000)	0.226(0.000)	0.017(0.007)
	10	-0.04	-0.12	-0.01	0.34	0.18	0.98	5.93	4.21	-0.05	41.79	23.79	2.97	0.471(0.000)	0.471(0.000)	0.006(0.890)
	11	0.03	0.01	0.02	1.02	1.28	1.02	0.15	0.10	0.01	1.68	1.28	2.97	0.163(0.000)	0.220(0.000)	0.011(0.204)
	12	0.05	0.02	0.04	1.01	1.26	0.99	0.17	0.12	-0.01	1.69	1.28	2.96	0.162(0.000)	0.219(0.000)	0.019(0.002)
	13	-0.05	-0.14	-0.02	0.37	0.22	0.97	5.22	3.78	-0.07	32.64	19.42	2.99	0.464(0.000)	0.464(0.000)	0.009(0.437)
	14	-0.03	-0.15	-0.00	0.58	0.45	0.98	2.94	2.38	-0.03	10.93	8.04	2.89	0.414(0.000)	0.414(0.000)	0.007(0.694)
	15	0.06	0.03	0.06	1.00	1.25	1.01	0.26	0.18	-0.02	1.73	1.31	2.99	0.161(0.000)	0.217(0.000)	0.028(0.000)
	16	-0.05	-0.13	0.00	0.35	0.20	0.97	5.51	3.95	0.00	36.15	21.07	3.02	0.467(0.000)	0.467(0.000)	0.010(0.272)
	17	-0.04	0.01	-0.04	1.00	1.21	1.02	-0.48	-0.34	0.01	2.05	1.42	2.99	0.185(0.000)	0.234(0.000)	0.024(0.000)
	18	-0.05	0.02	-0.03	1.02	1.18	1.02	-0.68	-0.46	0.07	2.53	1.57	3.04	0.189(0.000)	0.233(0.000)	0.021(0.000)
	19	-0.04	-0.14	-0.02	0.45	0.29	0.98	4.25	3.27	-0.04	21.56	14.40	2.96	0.451(0.000)	0.451(0.000)	0.008(0.483)
	20	0.00	-0.12	0.04	0.72	0.64	0.98	2.12	1.75	0.03	6.43	4.87	2.98	0.364(0.000)	0.364(0.000)	0.016(0.014)
3	1	0.03	-0.07	0.01	1.02	1.16	1.01	0.90	0.70	-0.00	2.54	1.76	2.95	0.248(0.000)	0.296(0.000)	0.010(0.310)
	2	-0.03	-0.17	-0.01	0.80	0.69	0.99	2.19	1.75	-0.03	7.01	4.79	2.99	0.355(0.000)	0.355(0.000)	0.010(0.297)
	3	0.03	-0.09	0.04	1.00	1.09	1.01	1.10	0.88	-0.01	2.89	2.08	2.94	0.261(0.000)	0.297(0.000)	0.018(0.002)
	4	-0.03	-0.16	-0.02	0.68	0.48	0.99	3.08	2.42	-0.04	12.24	8.05	2.94	0.409(0.000)	0.409(0.000)	0.011(0.180)
	5	-0.04	-0.17	-0.02	0.64	0.45	0.97	3.25	2.53	-0.01	13.65	8.70	3.00	0.416(0.000)	0.416(0.000)	0.012(0.139)
	6	0.04	-0.05	0.03	1.04	1.20	1.00	0.80	0.60	-0.00	2.37	1.62	3.06	0.229(0.000)	0.280(0.000)	0.017(0.005)
	7	-0.02	-0.15	-0.01	0.65	0.40	0.99	3.79	2.80	-0.02	18.79	10.46	2.93	0.428(0.000)	0.428(0.000)	0.008(0.530)
	8	0.03	-0.07	0.02	1.03	1.18	1.01	0.83	0.64	0.01	2.40	1.68	2.92	0.240(0.000)	0.289(0.000)	0.015(0.019)
	9	-0.02	-0.15	0.00	0.66	0.44	0.96	3.44	2.62	-0.05	15.26	9.24	2.95	0.420(0.000)	0.420(0.000)	0.011(0.153)
	10	-0.07	-0.09	-0.04	0.94	1.14	1.02	0.18	0.19	0.01	2.33	1.51	3.05	0.154(0.000)	0.208(0.000)	0.017(0.006)
	11	-0.04	-0.17	-0.02	0.71	0.56	0.97	2.67	2.12	-0.01	9.57	6.45	2.90	0.389(0.000)	0.389(0.000)	0.010(0.303)
	12	-0.02	-0.16	-0.01	0.76	0.60	0.97	2.56	2.02	-0.01	9.02	5.96	2.98	0.380(0.000)	0.380(0.000)	0.010(0.299)
	13	-0.03	-0.06	-0.03	0.98	1.19	1.00	0.32	0.27	-0.03	2.17	1.46	3.06	0.175(0.000)	0.229(0.000)	0.013(0.084)
	14	0.05	-0.05	0.03	1.02	1.18	1.02	0.81	0.62	-0.04	2.35	1.65	3.04	0.233(0.000)	0.281(0.000)	0.015(0.023)
	15	-0.03	-0.17	-0.01	0.74	0.61	0.99	2.43	1.93	-0.02	8.27	5.59	3.02	0.374(0.000)	0.374(0.000)	0.007(0.647)
	16	-0.03	-0.05	-0.00	0.95	1.17	1.00	0.21	0.19	-0.02	2.12	1.45	2.96	0.154(0.000)	0.209(0.000)	0.007(0.664)
	17	-0.03	-0.16	-0.01	0.64	0.41	0.98	3.61	2.73	0.03	16.50	10.02	2.97	0.425(0.000)	0.425(0.000)	0.009(0.352)
	18	-0.02	-0.14	-0.00	0.64	0.40	0.97	3.76	2.78	-0.04	18.09	10.35	2.98	0.427(0.000)	0.427(0.000)	0.006(0.844)
	19	-0.00	-0.06	0.00	1.00	1.22	1.00	0.46	0.36	0.01	2.03	1.46	2.99	0.191(0.000)	0.245(0.000)	0.006(0.904)
	20	0.04	-0.08	0.01	1.02	1.10	1.05	1.08	0.87	-0.03	2.85	2.04	2.87	0.256(0.000)	0.292(0.000)	0.015(0.029)

Note: The numbers in parentheses represent the p-value of the K-S test for r_{ij}.

Table 3: Comparison of Pearson's residual ($r_{ij}^{(P)}$), deviance residual ($r_{ij}^{(D)}$), normalized surrogate residual ($r_{ij}^{(N)}$), for Scenario III.

j	i	mean			variance			skewness			kurtosis			K-S test		
		$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$	$r_{ij}^{(P)}$	$r_{ij}^{(D)}$	$r_{ij}^{(N)}$
1	1	-0.03	0.06	-0.01	1.02	1.20	0.99	-0.74	-0.59	-0.01	2.20	1.60	2.98	0.244(0.000)	0.288(0.000)	0.007(0.694)
	2	-0.01	0.08	0.00	1.01	1.19	1.00	-0.75	-0.60	-0.04	2.27	1.63	2.97	0.247(0.000)	0.292(0.000)	0.007(0.746)
	3	-0.04	0.10	-0.02	1.03	1.04	1.00	-1.33	-1.08	0.00	3.49	2.45	2.96	0.289(0.000)	0.325(0.000)	0.013(0.071)
	4	-0.04	0.07	-0.00	1.02	1.17	0.99	-0.87	-0.70	0.01	2.36	1.73	2.98	0.259(0.000)	0.300(0.000)	0.006(0.924)
	5	-0.07	0.07	-0.00	1.07	1.07	1.01	-1.29	-1.05	0.04	3.30	2.34	2.99	0.288(0.000)	0.320(0.000)	0.010(0.291)
	6	-0.04	0.10	-0.03	1.02	1.04	1.00	-1.29	-1.03	0.04	3.39	2.37	2.99	0.282(0.000)	0.317(0.000)	0.016(0.020)
	7	0.19	0.23	0.10	0.82	1.02	0.97	-0.42	-0.42	-0.03	2.55	1.73	2.97	0.204(0.000)	0.257(0.000)	0.049(0.000)
	8	0.11	0.19	0.03	0.78	0.92	0.98	-1.01	-0.84	-0.04	3.47	2.31	2.98	0.252(0.000)	0.285(0.000)	0.020(0.002)
	9	0.03	0.14	0.01	0.90	0.99	0.98	-1.17	-0.96	-0.01	3.31	2.34	3.04	0.272(0.000)	0.306(0.000)	0.010(0.328)
	10	-0.04	0.09	-0.00	1.02	1.06	1.00	-1.23	-0.97	-0.00	3.29	2.26	2.98	0.272(0.000)	0.307(0.000)	0.007(0.753)
	11	-0.02	0.07	0.00	1.00	1.17	1.00	-0.79	-0.64	-0.01	2.24	1.67	2.98	0.254(0.000)	0.297(0.000)	0.004(0.996)
	12	-0.06	0.07	-0.01	1.06	1.14	1.01	-1.04	-0.83	0.06	2.72	1.92	3.03	0.270(0.000)	0.311(0.000)	0.013(0.103)
	13	-0.06	0.08	-0.01	1.07	1.05	1.00	-1.33	-1.07	-0.01	3.47	2.41	3.00	0.287(0.000)	0.321(0.000)	0.010(0.361)
	14	-0.07	0.07	-0.01	1.07	1.07	1.00	-1.27	-1.05	0.07	3.23	2.33	3.05	0.288(0.000)	0.319(0.000)	0.014(0.051)
	15	0.02	0.10	0.01	0.97	1.17	1.03	-0.67	-0.55	-0.01	2.22	1.61	3.06	0.240(0.000)	0.287(0.000)	0.008(0.557)
	16	-0.07	0.08	-0.03	1.09	1.07	1.03	-1.30	-1.06	0.01	3.33	2.36	2.98	0.288(0.000)	0.320(0.000)	0.017(0.008)
	17	-0.05	0.07	-0.04	1.04	1.16	1.01	-0.95	-0.76	0.00	2.52	1.82	2.90	0.265(0.000)	0.306(0.000)	0.021(0.001)
	18	0.05	0.15	0.02	0.86	0.98	0.98	-1.08	-0.89	0.00	3.17	2.28	3.02	0.256(0.000)	0.290(0.000)	0.015(0.035)
	19	0.09	0.14	0.05	0.92	1.14	0.97	-0.51	-0.46	0.00	2.15	1.59	2.96	0.218(0.000)	0.268(0.000)	0.023(0.000)
	20	0.13	0.22	0.04	0.78	0.91	0.97	-1.06	-0.88	-0.00	3.48	2.41	3.07	0.258(0.000)	0.291(0.000)	0.022(0.000)
2	1	0.04	-0.09	0.03	1.02	1.02	1.01	1.34	1.05	-0.01	3.63	2.48	3.00	0.280(0.000)	0.315(0.000)	0.014(0.053)
	2	0.02	-0.12	0.00	0.98	1.00	1.02	1.36	1.07	0.03	3.75	2.55	2.94	0.287(0.000)	0.326(0.000)	0.008(0.652)
	3	-0.04	-0.15	-0.01	0.51	0.34	0.98	3.71	3.00	-0.04	16.37	11.92	2.98	0.440(0.000)	0.440(0.000)	0.009(0.450)
	4	0.04	-0.11	0.02	0.99	0.93	0.99	1.59	1.26	0.02	4.40	2.99	3.02	0.304(0.000)	0.332(0.000)	0.009(0.440)
	5	0.03	-0.12	0.02	0.83	0.57	1.01	2.70	2.22	-0.04	9.34	6.72	2.98	0.392(0.000)	0.392(0.000)	0.014(0.058)
	6	-0.03	-0.14	-0.01	0.48	0.32	1.00	3.79	3.05	-0.07	17.09	12.38	2.96	0.442(0.000)	0.442(0.000)	0.007(0.783)
	7	-0.17	-0.24	-0.09	0.75	0.91	0.95	0.89	0.71	0.04	3.50	2.21	2.99	0.224(0.000)	0.272(0.000)	0.043(0.000)
	8	-0.07	-0.14	-0.02	0.25	0.18	0.95	5.33	3.66	-0.08	35.26	20.37	2.95	0.469(0.000)	0.469(0.000)	0.012(0.134)
	9	-0.06	-0.14	-0.03	0.34	0.24	0.98	4.50	3.43	-0.07	24.22	16.29	2.87	0.459(0.000)	0.459(0.000)	0.011(0.177)
	10	-0.04	-0.14	-0.01	0.42	0.29	0.99	4.08	3.22	-0.03	19.67	13.93	2.95	0.449(0.000)	0.449(0.000)	0.009(0.393)
	11	0.03	-0.11	0.03	0.99	0.97	1.02	1.45	1.14	0.01	3.99	2.68	3.03	0.295(0.000)	0.328(0.000)	0.012(0.137)
	12	0.07	-0.09	0.03	1.04	0.88	1.06	1.80	1.42	0.04	5.14	3.45	3.00	0.308(0.000)	0.316(0.000)	0.015(0.033)
	13	-0.00	-0.13	0.00	0.65	0.43	1.01	3.23	2.66	-0.03	12.64	9.32	2.94	0.421(0.000)	0.421(0.000)	0.013(0.094)
	14	0.03	-0.12	0.02	0.82	0.56	1.02	2.72	2.25	-0.04	9.47	6.89	2.98	0.395(0.000)	0.395(0.000)	0.016(0.023)
	15	-0.01	-0.13	-0.01	0.95	1.02	1.01	1.21	0.95	0.01	3.40	2.33	2.91	0.270(0.000)	0.314(0.000)	0.011(0.241)
	16	0.02	-0.12	0.03	0.79	0.55	1.01	2.77	2.28	-0.03	9.78	7.05	2.87	0.397(0.000)	0.397(0.000)	0.017(0.007)
	17	0.08	-0.08	0.03	1.09	0.95	1.05	1.64	1.29	-0.03	4.55	3.05	2.89	0.289(0.000)	0.314(0.000)	0.020(0.001)
	18	-0.06	-0.14	-0.02	0.29	0.21	0.98	4.86	3.52	-0.04	29.09	17.78	2.96	0.463(0.000)	0.463(0.000)	0.010(0.361)
	19	-0.07	-0.16	-0.05	0.88	1.01	0.97	0.95	0.78	-0.00	3.01	2.13	2.97	0.245(0.000)	0.290(0.000)	0.021(0.001)
	20	-0.07	-0.15	-0.01	0.23	0.17	0.94	5.53	3.71	-0.11	38.38	21.69	3.02	0.473(0.000)	0.473(0.000)	0.014(0.067)
3	1	-0.03	-0.17	-0.01	0.71	0.53	1.00	2.85	2.29	-0.03	10.49	7.21	3.09	0.398(0.000)	0.398(0.000)	0.009(0.453)
	2	-0.02	-0.16	0.01	0.73	0.54	0.99	2.79	2.26	-0.03	10.11	7.02	2.87	0.396(0.000)	0.396(0.000)	0.010(0.317)
	3	0.06	-0.11	0.03	1.04	0.89	1.03	1.78	1.43	0.01	5.05	3.44	2.96	0.307(0.000)	0.323(0.000)	0.018(0.006)
	4	-0.01	-0.16	-0.01	0.78	0.58	0.98	2.67	2.18	-0.00	9.29	6.55	2.98	0.389(0.000)	0.389(0.000)	0.010(0.293)
	5	0.04	-0.13	0.02	1.01	0.77	1.01	2.19	1.78	-0.02	6.77	4.62	3.01	0.348(0.000)	0.348(0.000)	0.018(0.005)
	6	0.05	-0.11	0.02	1.03	0.91	1.03	1.70	1.35	0.01	4.76	3.24	2.93	0.297(0.000)	0.320(0.000)	0.017(0.007)
	7	-0.06	-0.18	-0.02	0.51	0.37	0.96	3.65	2.61	-0.05	17.63	10.17	2.99	0.430(0.000)	0.429(0.000)	0.013(0.073)
	8	-0.08	-0.18	-0.05	0.75	0.85	0.97	1.22	0.96	-0.03	4.06	2.65	3.03	0.257(0.000)	0.274(0.000)	0.024(0.000)
	9	-0.01	-0.14	-0.01	0.88	0.90	1.00	1.45	1.16	-0.04	4.12	2.85	2.96	0.277(0.000)	0.305(0.000)	0.006(0.927)
	10	0.06	-0.09	0.04	1.04	0.95	1.02	1.56	1.22	-0.06	4.32	2.91	2.90	0.279(0.000)	0.311(0.000)	0.024(0.000)
	11	-0.02	-0.16	-0.01	0.75	0.55	1.01	2.78	2.26	-0.02	10.03	7.00	2.93	0.395(0.000)	0.395(0.000)	0.007(0.731)
	12	-0.02	-0.18	0.00	0.79	0.58	1.00	2.70	2.21	-0.00	9.43	6.68	3.02	0.390(0.000)	0.390(0.000)	0.005(0.985)
	13	0.06	-0.11	0.03	1.06	0.85	1.05	1.95	1.56	-0.00	5.71	3.85	2.93	0.324(0.000)	0.325(0.000)	0.020(0.001)
	14	0.05	-0.12	0.04	1.03	0.78	1.02	2.15	1.76	0.03	6.54	4.54	2.95	0.346(0.000)	0.346(0.000)	0.016(0.013)
	15	-0.04	-0.18	-0.03	0.66	0.49	0.99	3.00	2.38	0.01	11.63	7.83	3.00	0.407(0.000)	0.407(0.000)	0.014(0.065)
	16	0.05	-0.12	0.02	1.04	0.79	1.04	2.14	1.75	0.04	6.49	4.50	2.96	0.345(0.000)	0.345(0.000)	0.015(0.034)
	17	-0.04	-0.19	-0.02	0.73	0.54	0.98	2.82	2.30	-0.04	10.25	7.19	2.99	0.398(0.000)	0.398(0.000)	0.012(0.158)
	18	-0.04	-0.15	-0.02	0.83	0.90	0.99	1.32	1.04	-0.05	3.86	2.68	2.94	0.263(0.000)	0.282(0.000)	0.007(0.722)
	19	-0.05	-0.18	-0.02	0.60	0.44	0.97	3.32	2.48	-0.04	14.74	8.65	3.03	0.415(0.000)	0.415(0.000)	0.014(0.056)
	20	-0.10	-0.20	-0.06	0.76	0.85	0.97	1.28	1.00	-0.03	4.17	2.75	3.03	0.265(0.000)	0.282(0.000)	0.028(0.000)

Note: The numbers in parentheses represent the p-value of the K-S test for r_{ij}.

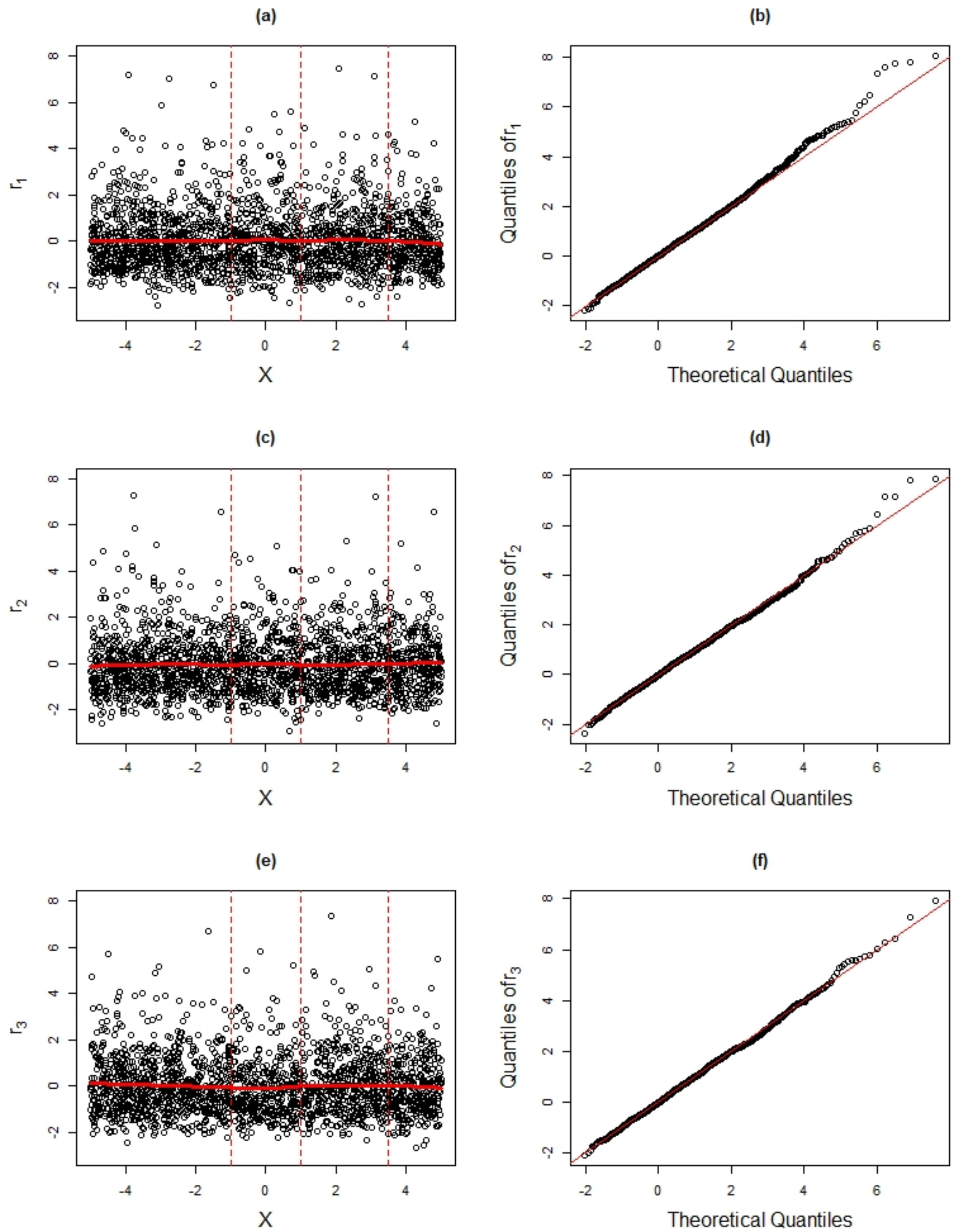

Figure S1: Model diagnostics for the improved model in Example 2 with the surrogate residual. Figures (a), (c) and (e) show the residuals-by-covariate plots of r_1, r_2 and r_3 respectively; vertical dashed lines $X = -1$, 1, and 3.5, and loess curves (red solid) are added as references. Figures (b), (d) and (f) are the corresponding QQ-plots (quantile-quantile plots) of above mentioned residuals.

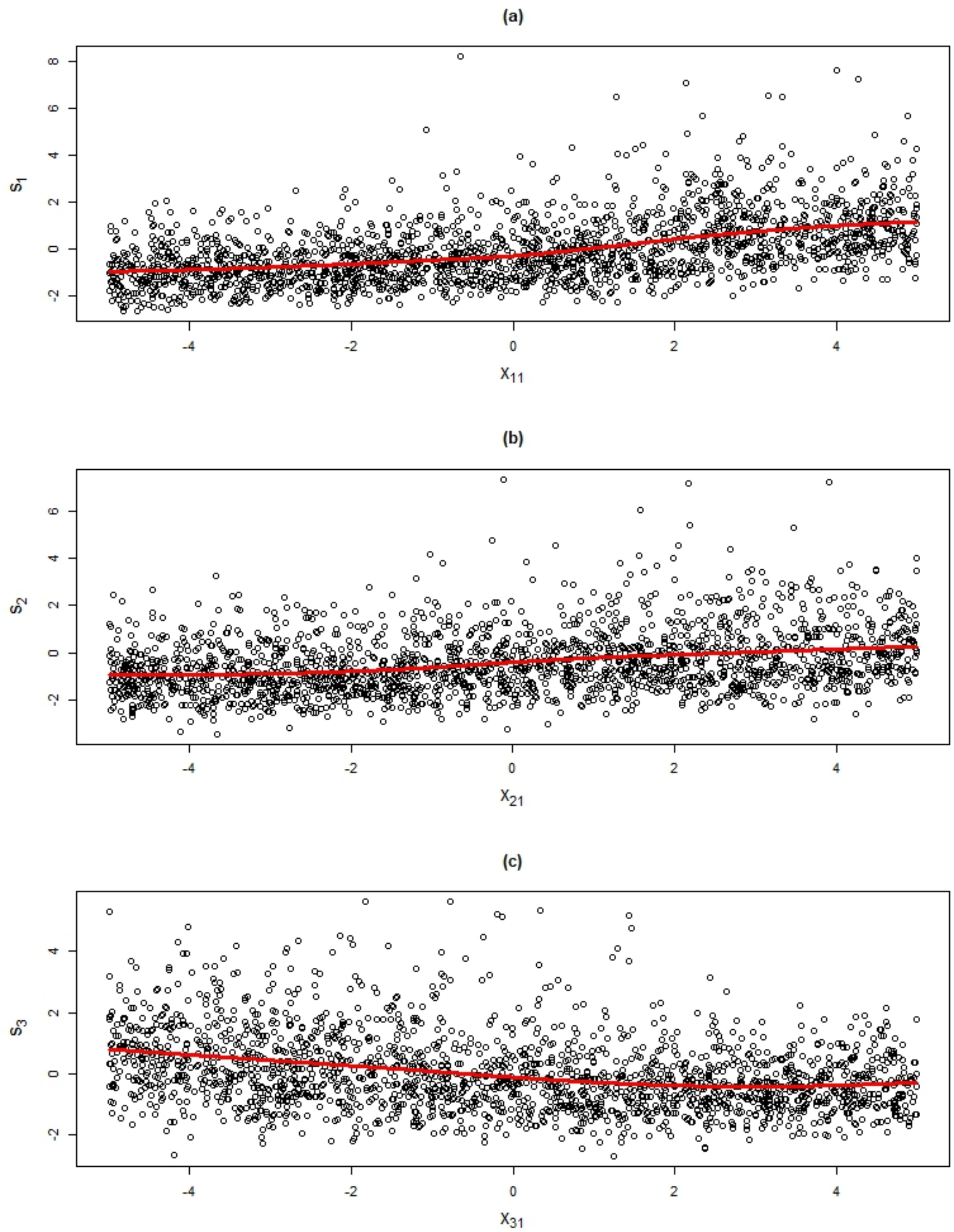

Figure S2: Scatterplots of the surrogate response $\boldsymbol{s}$ versus x_1 with a nonparametric loess smooth (red line). (a): s_1-by-x_{11}; (b): s_2-by-x_{21}; (c): s_3-by-x_{31}.

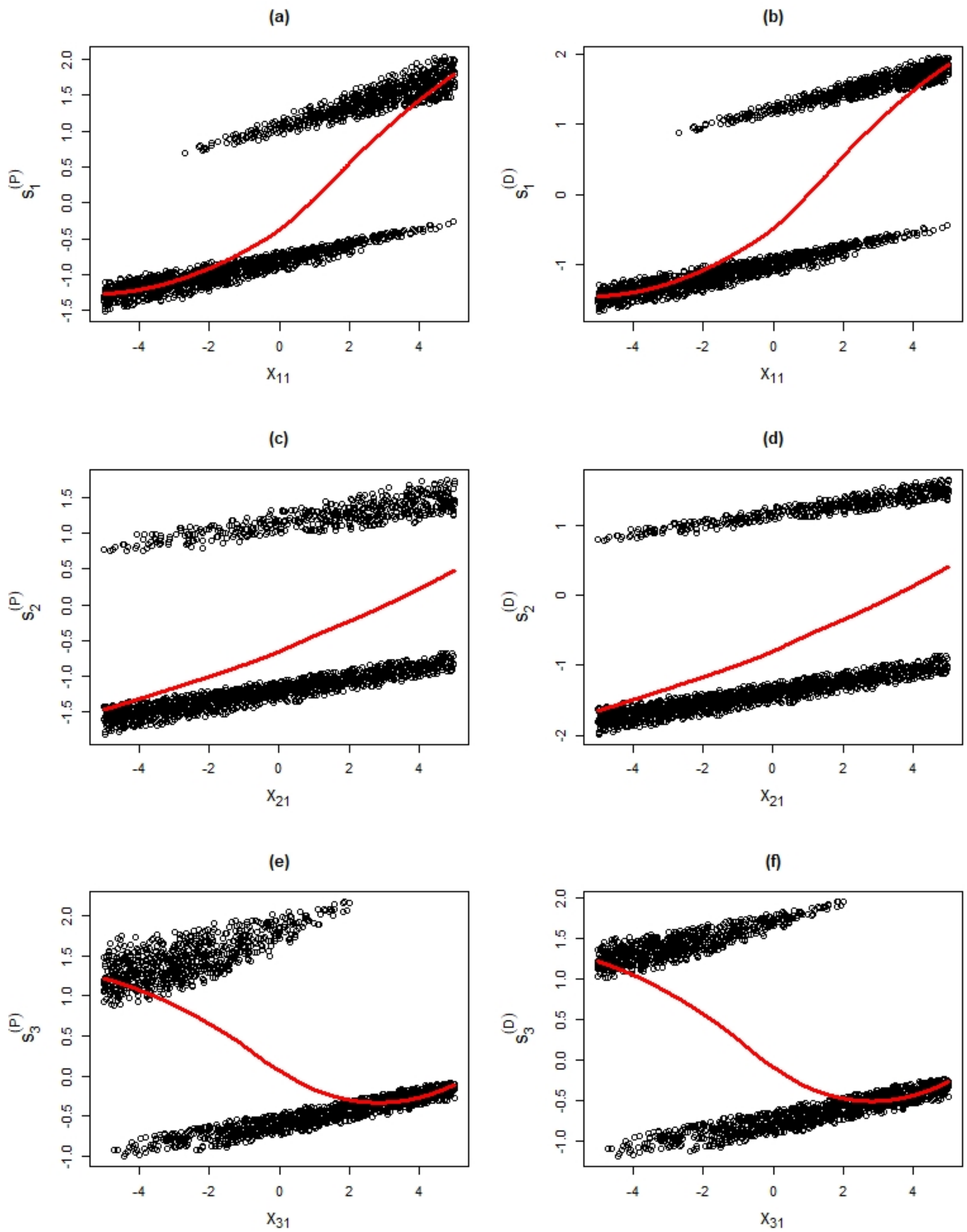

Figure S3: Scatterplots of the surrogate response by Pearson's residual ($\boldsymbol{s}^{(P)}$) and deviance residual ($\boldsymbol{s}^{(D)}$) versus x_1 with a nonparametric loess smooth (red line). Left column: $\boldsymbol{s}^{(P)}$ by x_1; Right column: $\boldsymbol{s}^{(D)}$ by x_1.

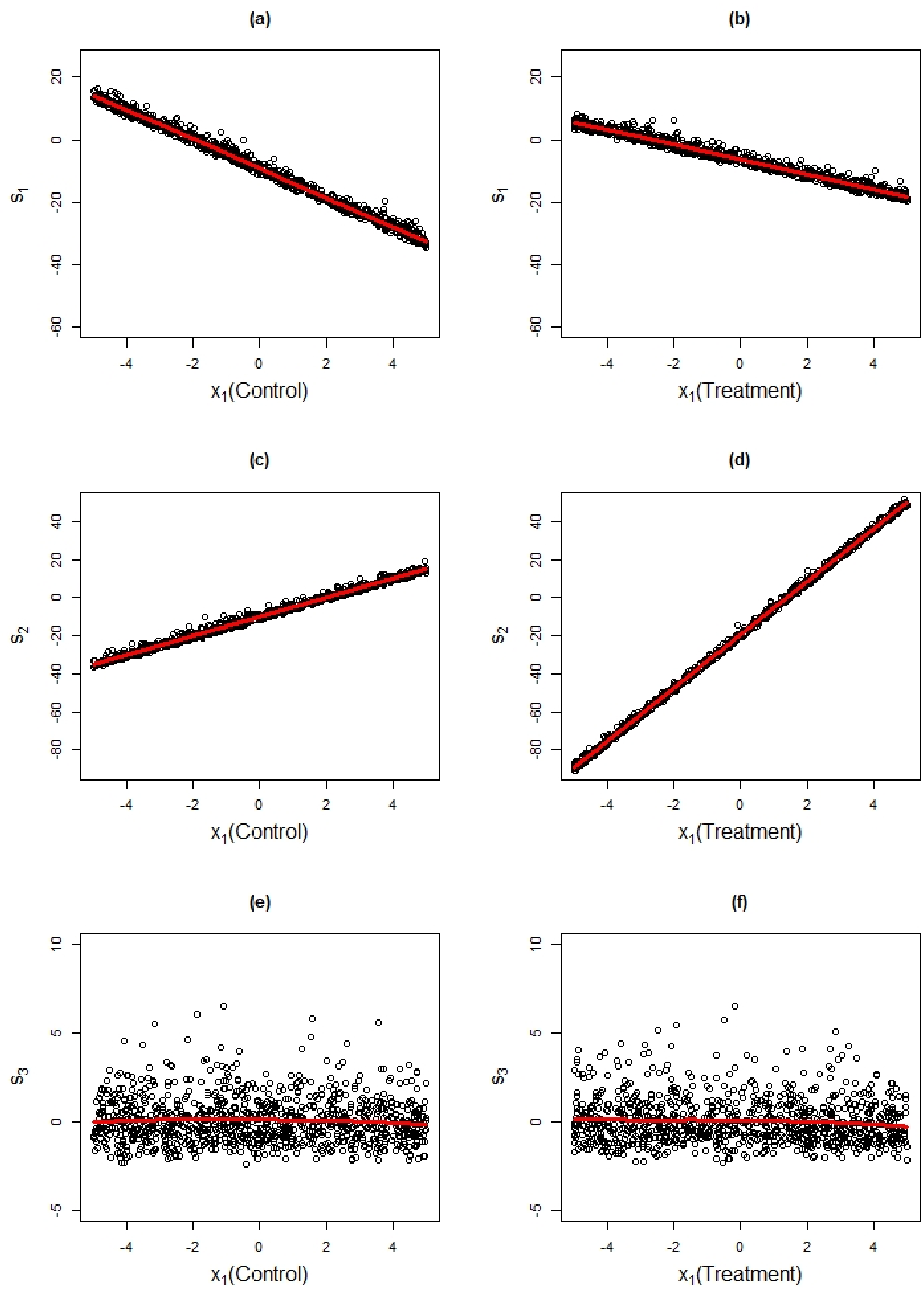

Figure S4: Scatterplots of the surrogate response $\boldsymbol{s}$ versus x_1 with a nonparametric loess smooth (red line). Left column: control group; Right column: treatment group.

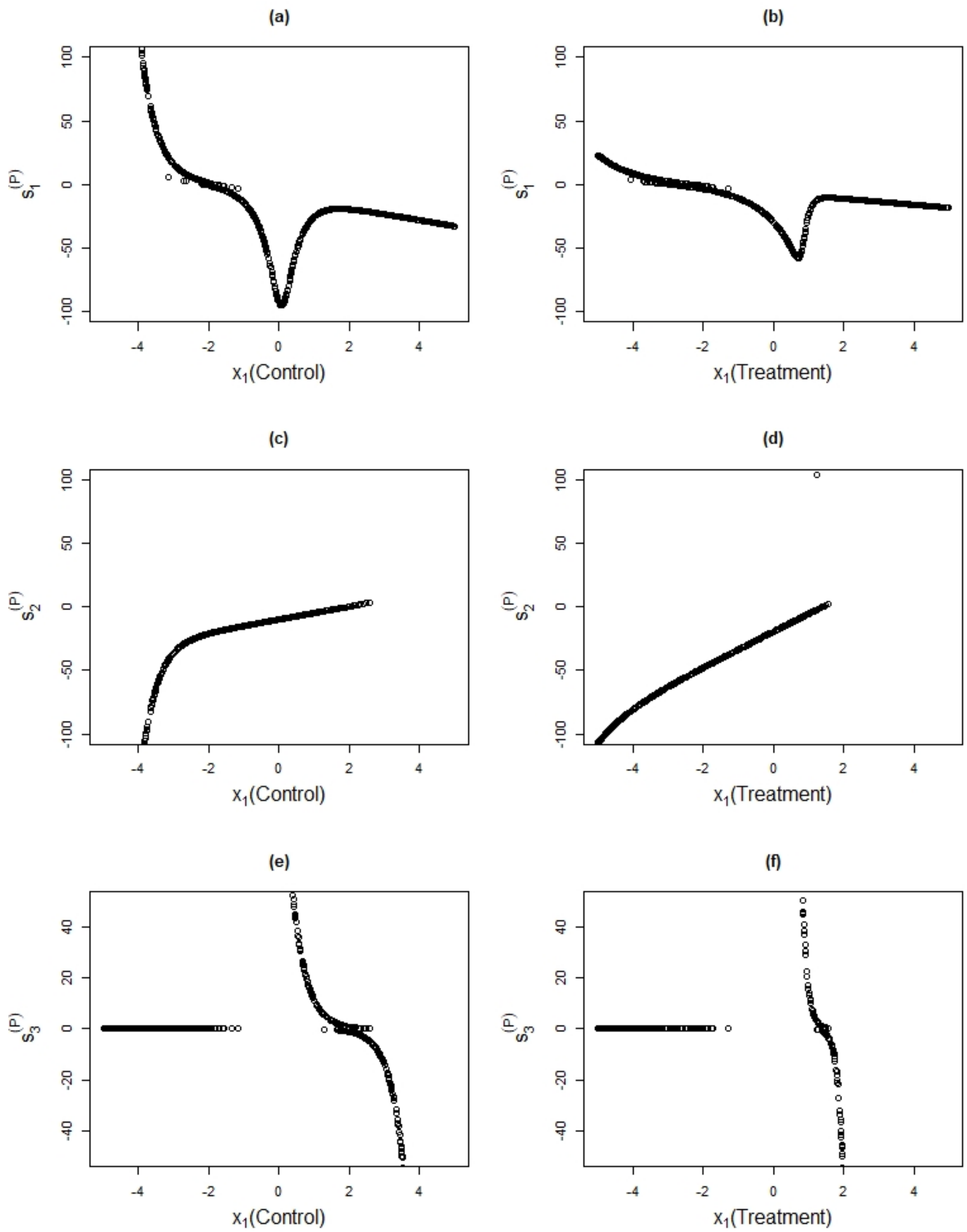

Figure S5: Scatterplots of the surrogate response of Pearson's residual ($\boldsymbol{s}^{(P)}$) versus x_1. Left column: control group; Right column: treatment group. The loess curve was removed, as some points tend to infinity and a loess curve is hard to compute.

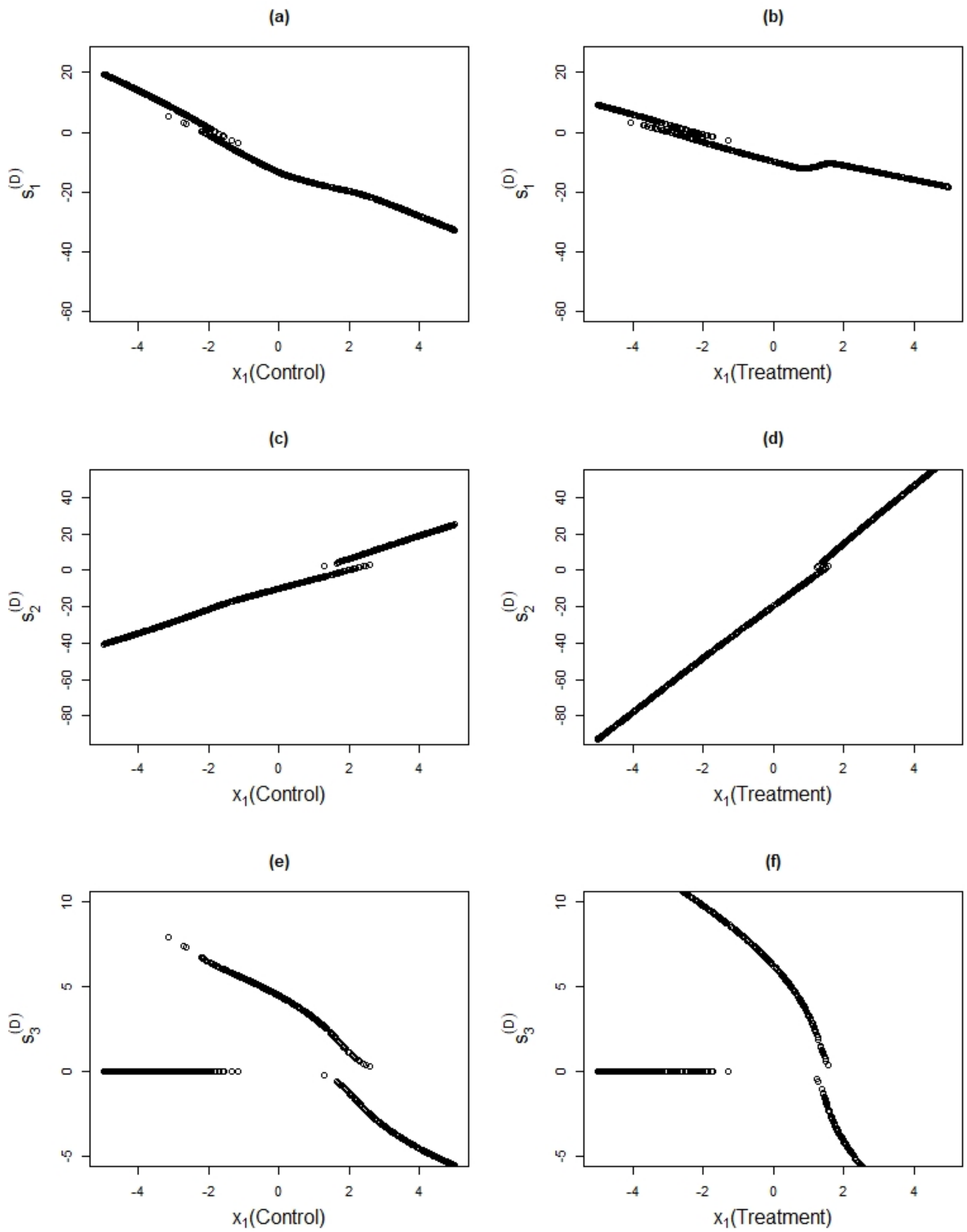

Figure S6: Scatterplots of the surrogate response of deviance residual ($\boldsymbol{s}^{(P)}$) versus x_1. Left column: control group; Right column: treatment group. The loess curve was removed, as some points tend to infinity and a loess curve is hard to compute.

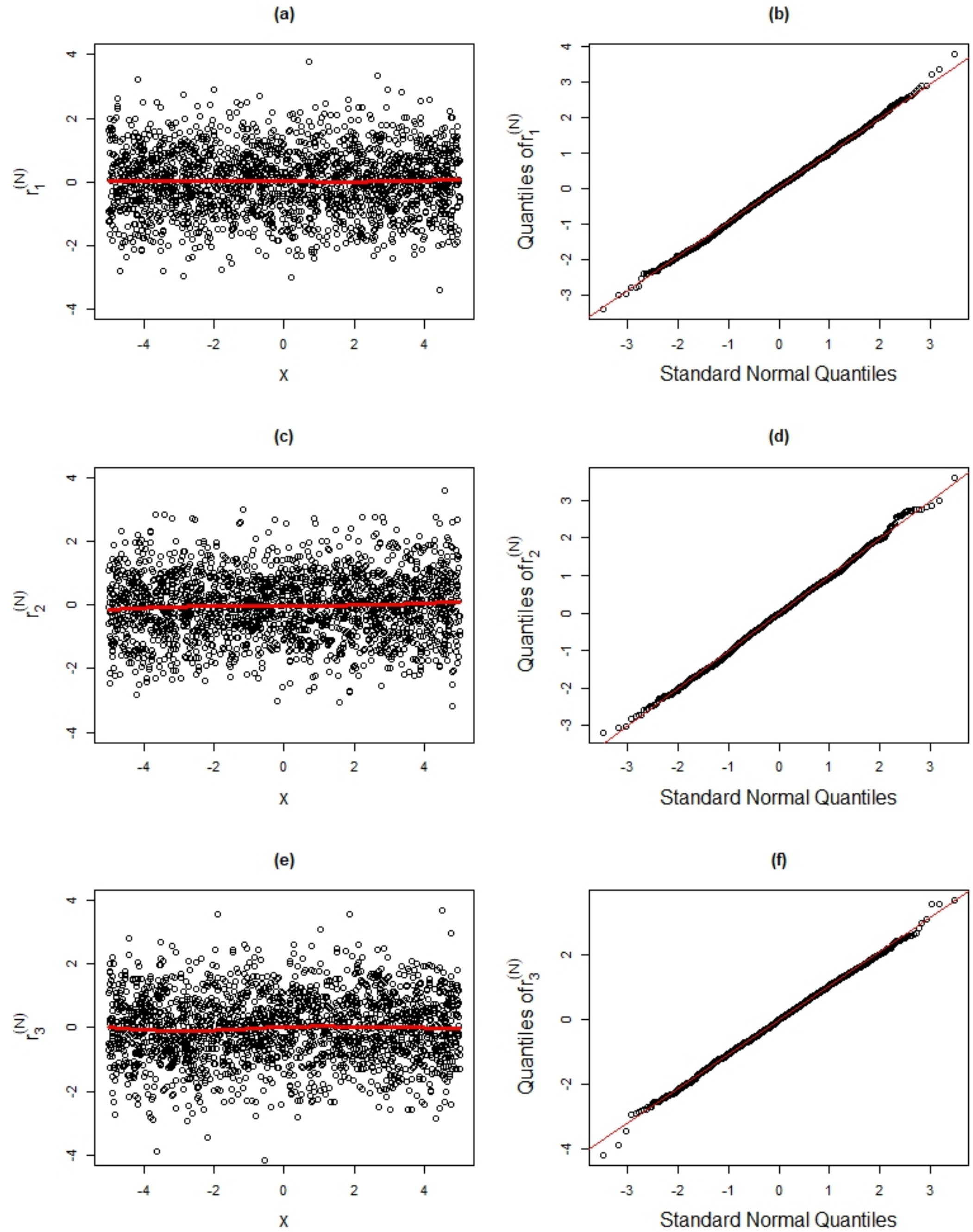

Figure S7: Model diagnostics for the improved model in Example 2 with the normalized surrogate residual defined in Section 3.4.1. Figures (a), (b) and (c) show the residuals-by-covariate plots of $r_1^{(N)}$, $r_2^{(N)}$ and $r_3^{(N)}$ respectively; loess curves (red solid) are added as references. Figures (b), (d) and (f) are the corresponding QQ-plots (quantile-quantile plots) of above mentioned residuals.

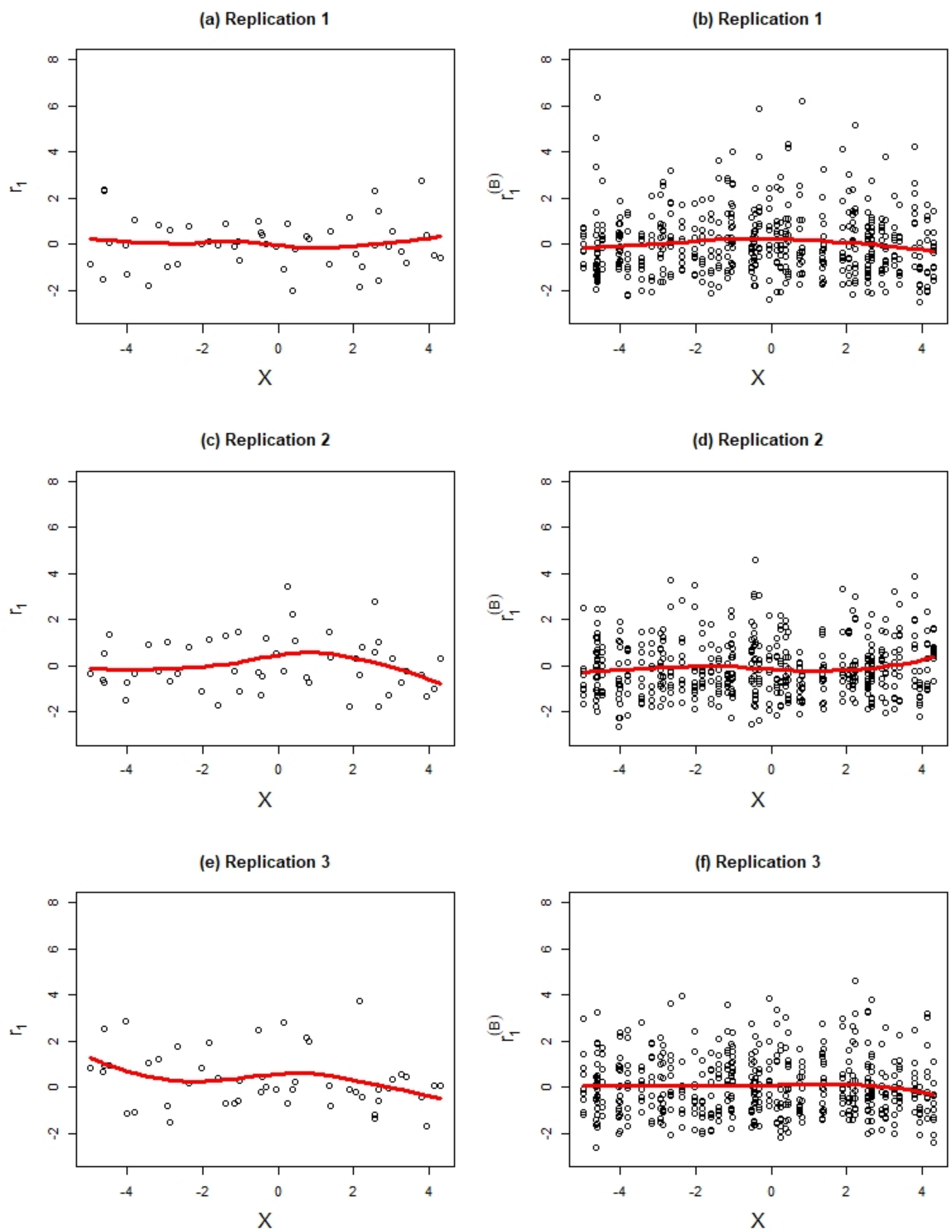

Figure S8: Three replications of r_1-by-x plots by the surrogate residual (r_1) and bootstrapping residual ($r_1^{(B)}$) with 10 copies. Upper row: the first replication; Middle row: the second replication; Bottom row: the third replication.

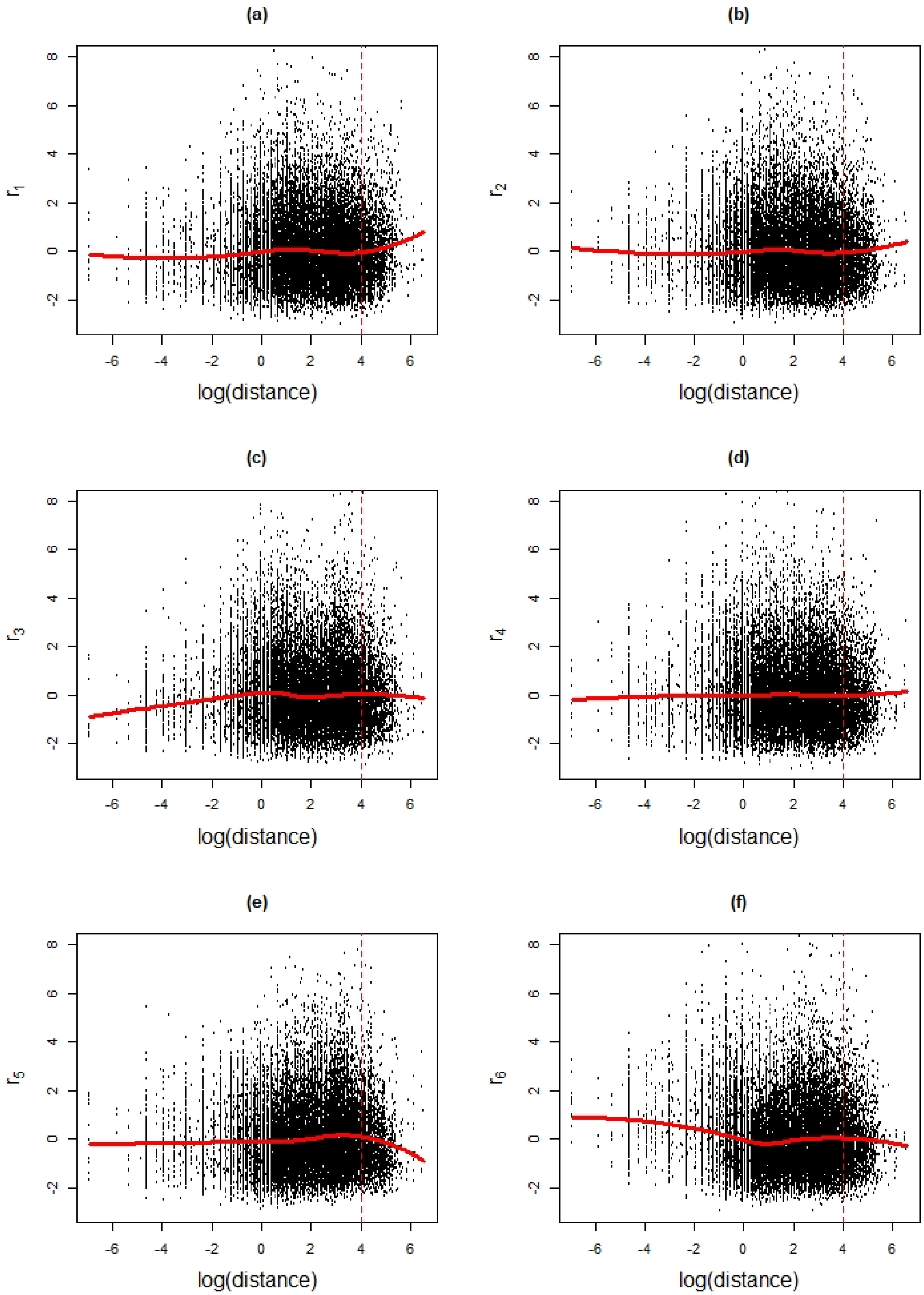

Figure S9: Residual-by-log(distance) plots for our initial multinomial logistic regression fitted to the MPN data. r_1 to r_6 represent the surrogate residuals for the utilities of car as driver, car as passenger, bicycle, ebike, public transport, and walking, respectively. A vertical dashed line at $X = 4$ and a loess curve (solid) are added for reference.

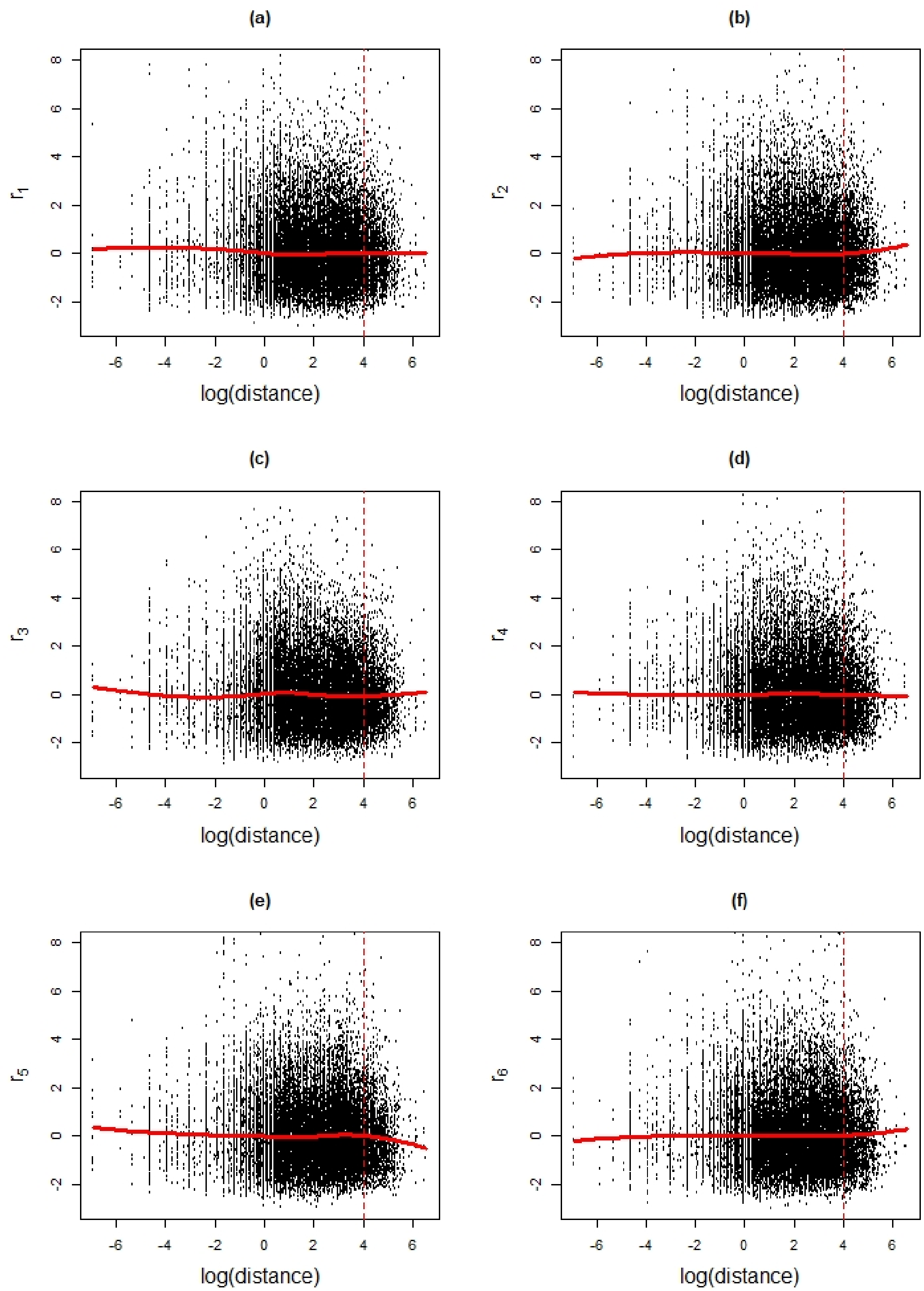

Figure S10: Residual-by-log(distance) plots when natural logarithm term of distance are applied in the multinomial logistic regression fitted to the MPN data. r_1 to r_6 represent the surrogate residuals for the utilities of car as driver, car as passenger, bicycle, ebike, public transport, and walking, respectively. A vertical dashed line at $X = 4$ and a loess curve (solid) are added for reference.

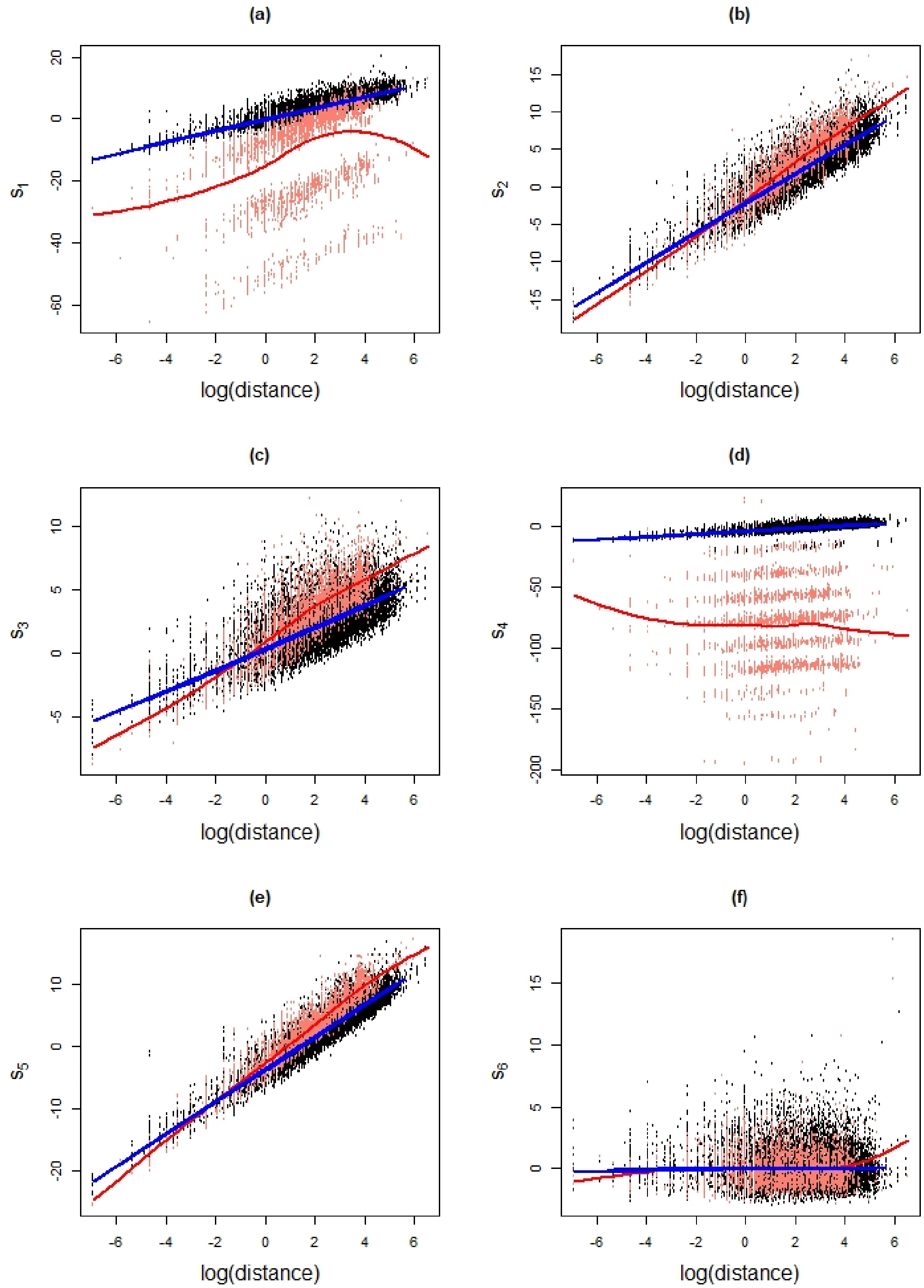

Figure S11: Plots of surrogate response versus log(distance) for the subjects who have a driving license and who do not have a driving license. s_1 to s_6 represent the surrogate responses corresponding to the utilities of car as driver, car as passenger, bicycle, ebike, public transport, and walking, respectively. The black points with a blue loess curve correspond to data from the subjects who have a driving license, while the orange points with a red loess curve correspond to data from the subjects who do not have a driving license.