Supplementary material for "Multivariate exponential smoothing and dynamic factor model applied to hourly electricity price analysis" published by Technometrics

Jaime Carpio, Jesús Juan, Damián López

Comparative study between exponential smoothing and other two different short term electricity price forecasting techniques

1 Introduction

We have conducted a comparative study of our methodology with two other short term electricity price forecasting techniques. The two main time series approaches are : (A) a complex seasonal ARIMA model to the hourly price series and (B) 24 independent simple ARIMA models, one for each hour of the day. In this document we explain how these two procedures are used for price forecasting in three electricity markets: Powernext (France), Nord Pool (Denmark, Finland, Norway and Sweden), and Omel (Spain).

Looking specifically at univariate time series models, we can distinguish two different approaches in the literature.

A. A single model is not sufficient to collect all the peculiarities of price dynamics. Conejo et al. (2005), one of the most referenced articles in this matter, uses a different model for each season for the PJM Interconnections (Pennsylvania - New Jersey - Maryland).

B. The other approach, which is preferred by several authors, is to use different models for each of the 24 hours in a day (Misiorek et al., 2006). In some cases differentiating between weekdays and weekends is recommended, with the result that 48 models are used (García-Martos et al., 2007).

The structure of these models varies from one hour to another. Therefore, when the literature uses the term *univariate model* it actually refers to *multiple* univariate models. In this comparison, models were estimated with data corresponding to week-days from January 1, 2007 to May 29, 2009 (126 weeks of 5 days). To evaluate model forecasting performance, we have set aside a data subset to serve as a control (from June 1, 2009 to December 25, 2009, 30 weeks of 5 days).

Various programs (SCA, EViews or TRAMO) can deal with ARIMA model automatic identification. In this paper, for the estimation and subsequent forecasting, we have used the "forecast" package from R (Hyndman, 2008). Models are selected using Information Criteria (AIC and SIC). In the appendix we provide the programs developed in this comparative study. The two approaches (A) and (B) are described in the following sections and the results are presented in the third section.

2 Seasonal ARIMA models for hourly series

In this section we consider a single series consisting of hourly prices. There are several articles that follow this approach. Some of them use explanatory variables or regressors. The results and recommendations vary from one work to another. Nogales et al. applied transfer functions and dynamic regression to electricity price forecasting. Contreras et al. (2003) forecasted electricity prices in the Spanish and Californian markets by applying ARIMA models. Troncoso et al. (2002) compared the kWNN (Weighted Nearest Neighbours) technique with dynamic regression. Crespo-Cuaresma et al. (2004) have suggested a group of univariate models to predict electricity prices in the Leipzig market, the most important spot market in Germany. Conejo et al. (2005) compared several methods including wavelet approximation, ARIMA models and neural networks. Nogales et al. (2006) forecasted the prices in the PJM Interconnection through transfer functions, showing that the inclusion of explanatory variables does not significantly reduce prediction errors.

Given the available data in the comparative study, we consider an ARIMA model with seasonality. Some studies include double seasonality, daily with a 24-hour period and weekly with a period of 168 hours. As explained and justified in the article, weekend data have been removed, which leads to a reduction of weekly seasonality. The models selected for the authomatic estimation procedure are:

$$\begin{aligned} \text{Omel:ARIMA} \quad & (6,1,4)\times(4,0,5)_{24} \\ \text{Powernext:ARIMA} \quad & (5,1,3)\times(5,0,4)_{24} \\ \text{Nordpool:ARIMA} \quad & (4,1,3)\times(3,0,3)_{24} \end{aligned}$$

As an example, the model estimated for Nordpool is the following:

```
ARIMA(4,1,3)(3,0,3)[24]

Coefficients:
         ar1      ar2    ar3      ar4      ma1     ma2      ma3    sar1
       0.782  -0.2885  0.352  -0.1355  -0.6241  0.1014  -0.3905  0.8781
s.e.     NaN      NaN    NaN      NaN      NaN     NaN      NaN  0.0037
          sar2    sar3     sma1    sma2     sma3
       -0.5938  0.6777  -0.5832  0.6500  -0.5349
s.e.       NaN     NaN   0.0124  0.0034      NaN

sigma^2 estimated as 0.001181:  log likelihood=29488.7
AIC=-58949.4   AICc=-58949.37   BIC=-58842.66
```

Figure 1: Estimated ARIMA model for Nordpool.

Slight modifications are obtained by modifying the search criteria, but from the point of view of accuracy the results do not vary substantially.

3 24 independent models (one for each hour)

This procedure is recommended in García-Martos et al. (2007) and is also mentioned in Weron (2006) and in Misiorek et al. (2006). In this modality two different strategies are proposed. In the first option, models are estimated for each of the 24 hours, without distinguishing weekdays and weekends, and in the second option different models were considered depending on the type of day, which involves 48 models. In our case, as explained above, weekends are eliminated, so we only estimate the 24 models for weekdays. For greater flexibility, the automatic models are allowed to have a different structure for different days. Weekly seasonal pattern is considered with a 5 day periodicity.

Table I presents the models corresponding to some hours in the Nordpool market.

Hour	Model
4	ARIMA $(0,1,2)\times(0,0,1)_5$
8	ARIMA $(1,1,1)$
12	ARIMA $(1,1,2)$
16	ARIMA $(0,1,1)\times(2,0,1)_5$
20	ARIMA $(1,1,2)\times(2,0,3)_5$

Table I. *Model structure for 5 selected hours in Nordpool market.*

As in the models estimated in the previous section, we modified the search criteria, and accuracy differences between models were not significant. The models include intercept and drift parameters.

4 Comparison

We compared the prediction accuracy of the exponential smoothing method with other results obtained with the univariated models used for short-term hourly price forecasting. We have to note that there is no standard measure of forecasting accuracy. Some commonly used measures based on relative errors are misleading when applied to electricity prices. In particular, when electricity prices drop to zero, relative errors become very large regardless of the true absolute error. Alternative normalizations have been proposed in the literature. For instance, let $y_{i,t}$, $\hat{y}_{i,t}$ are the observed and predicted hourly log-price and $\bar{y}$ the mean log-price for a week. The Mean Week Error (MWE_s) is defined as

$$\mathrm{MWE}_s = \frac{1}{120}\frac{\sum_{t=s+1}^{5+s}\sum_{i=1}^{24}|y_{i,t}-\hat{y}_{i,t}|}{\bar{y}}, \qquad s=1,...,30$$

Table 1 and Figure 2 show the MWE values corresponding to the 30 forecasted weeks per market. In this analysis we kept the data as they were recorded without removing the outliers,

Table 1: Mean weekly error MWE corresponding to 30 weeks (01/06 - 25/12 2009) for each market. ES: exponential smoothing model, MA: Model A, MB: Model B

	OMEL			POWERNEXT			NORD POOL		
	ES	MA	MB	ES	MA	MB	ES	MA	MB
1	0.0135	0.1315	0.0098	0.1267	0.1647	0.1065	0.0251	0.0441	0.0184
2	0.0178	0.0537	0.0210	0.0285	0.0804	0.0450	0.0076	0.0983	0.0098
3	0.0102	0.0646	0.0112	0.0157	0.0712	0.0294	0.0144	0.0940	0.0173
4	0.0132	0.0643	0.0130	0.0342	0.0909	0.0324	0.0099	0.0657	0.0102
5	0.0106	0.0883	0.0099	0.0296	0.0500	0.0334	0.0103	0.0952	0.0083
6	0.0131	0.0692	0.0151	0.0311	0.0535	0.0273	0.0062	0.0633	0.0081
7	0.0187	0.0519	0.0147	0.0322	0.0844	0.0335	0.0090	0.0604	0.0070
8	0.0226	0.0445	0.0251	0.0198	0.0833	0.0264	0.0438	0.0494	0.0293
9	0.0209	0.0720	0.0211	0.0251	0.0964	0.0416	0.0753	0.0915	0.0625
10	0.0136	0.0750	0.0144	0.0384	0.0797	0.0263	0.0440	0.0660	0.0256
11	0.0124	0.0461	0.0134	0.0543	0.1146	0.0480	0.0241	0.0969	0.0068
12	0.0162	0.0642	0.0132	0.0356	0.0925	0.0341	0.0085	0.0905	0.0074
13	0.0121	0.0238	0.0132	0.0386	0.1223	0.0373	0.0113	0.1108	0.0118
14	0.0115	0.0360	0.0124	0.0360	0.0870	0.0491	0.0167	0.0931	0.0175
15	0.0138	0.0397	0.0113	0.0275	0.0867	0.0318	0.0112	0.0763	0.0105
16	0.0195	0.0259	0.0186	0.0320	0.0865	0.0230	0.0080	0.0763	0.0100
17	0.0230	0.0277	0.0183	0.0256	0.0944	0.0239	0.0142	0.0593	0.0127
18	0.0216	0.0340	0.0199	0.0226	0.1449	0.0264	0.0132	0.1029	0.0159
19	0.0167	0.0215	0.0129	0.0304	0.1512	0.0320	0.0166	0.0923	0.0178
20	0.0362	0.0377	0.0273	0.0560	0.1132	0.0579	0.0094	0.1315	0.0122
21	0.0322	0.0421	0.0352	0.1059	0.1692	0.0984	0.0069	0.1484	0.0076
22	0.0193	0.0396	0.0183	0.0434	0.1455	0.0473	0.0083	0.1413	0.0064
23	0.0646	0.0955	0.0594	0.0584	0.1397	0.0631	0.0085	0.1607	0.0077
24	0.0601	0.1121	0.0551	0.0450	0.1344	0.0552	0.0103	0.1320	0.0121
25	0.0343	0.0834	0.0360	0.0393	0.1016	0.0433	0.0085	0.0933	0.0105
26	0.0389	0.0692	0.0408	0.0306	0.0926	0.0279	0.0107	0.0707	0.0102
27	0.0318	0.0937	0.0303	0.0363	0.0999	0.0338	0.0155	0.1158	0.0178
28	0.0453	0.0810	0.0443	0.0408	0.0758	0.0325	0.0083	0.1492	0.0059
29	0.0668	0.1196	0.0664	0.0432	0.1311	0.0526	0.0410	0.2692	0.0341
30	0.0959	0.1581	0.0928	0.0800	0.1426	0.0919	0.0297	0.2999	0.0229

to illustrate the large variations observed among weeks. We preferred to retain the comparison in logarithms to simplify the graphs.

As shown in the Figure 2, the model that shows the worst performance in all three markets is the Model A that analyzes in a single equation the entire process. Model B, which studies each hour separately, and the exponential smoothing technique give very similar results.

The behavior of many models depends on its particular implementation that includes many small details to consider (for example the treatment of outliers, the numerical algorithms used, the length of the series,...). Model behaviour may depend on these implementation details. One aspect that may be secondary to the methodology but which is transcendental for the comparison is the treatment of outliers. Any market throughout the year has several days or weeks with prices that completely distort the model. In the estimation stage it is essential to identify and filter these observations. Once the model is established, if we want to measure its prediction accuracy with “new observations”, then the problem of what to do with outliers in

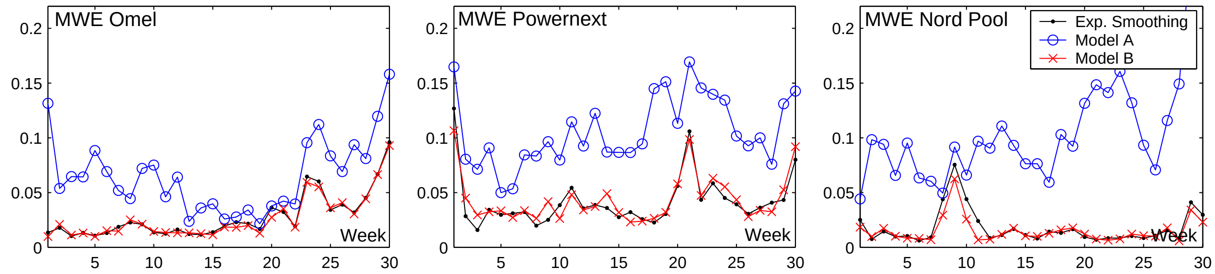

Figure 2: Mean weekly error MWE corresponding to 30 weeks (01/06 - 25/12 2009) for each market calculated with exponential smoothing model (order $r = 10$), Model A, and Model B.

the "new observations" resurfaces. In our analysis we have kept the outliers in order to illustrate the differences in prediction errors for different weeks.

5 Conclusions

Electricity price analysis and prediction is an interesting problem that is being extensively studied in statistical and economic literature. Many different models and methods have been proposed in these studies to make predictions, but it is not possible to conclude that one method is superior to another. In the context of time series models, for the markets analyzed with the information available, the prediction errors of our model are similar to the prediction errors of other forecasting models.

Besides good predictive accuracy, the advantage of our proposed model is its conceptual simplicity. Our approach permits the dynamic factor analysis of the process which is useful for market behaviour description.

References

Conejo A.J., Contreras J., Espínola R., and Plazas M.A. (2005). Forecasting electricity prices for a day-ahead pool-based electric energy market. International Journal of Forecasting, 21(3), 435–462.

Conejo A.J., Plazas M.A., Espínola R., and Molina A.B. (2002). Day-ahead electricity price forecasting using wavelet transform and ARIMA models. IEEE Trans. Power Syst., vol. 20, pp. 1035–1042.

Contreras J., Espínola R., Nogales F.J., and Conejo A.J. (2003). ARIMA models to predict next-day electricity prices. IEEE Trans. Power Syst., 18, 1014–1020.

Crespo-Cuaresma J., Hlouskova J., Kossmeier S., and Obersteiner M. (2004). Forecasting electricity spot-prices using linear univariate time-series models. Appl. Energy, vol. 77.

García-Martos, C., Rodríguez, J. and Sánchez, M.J. (2007). Mixed models for short-run forecasting of electricity prices: application for the Spanish market. IEEE Transactions on Power Systems.

Hyndman R.J. (2008). Forecasting Functions for Time Series: R package version 2.13. URL http://CRAN.R-project.org/package=forecasting.

Misiorek, A., Trück, S., Weron, R. (2006). Point and Interval Forecasting of Spot Electricity Prices: Linear vs. Non-Linear Time Series Models. Studies in Nonlinear Dynamics and Econometrics, 10(3), Article 2.

Nogales F.J. and Conejo A.J. (2006). Electricity price forecasting through transfer function models. J. Oper. Res. Soc.

Troncoso A., Riquelme J., Gómez A., and Martínez J.L. (2002) A comparison of two techniques for next-day electricity price forecasting. In Proc. 13th Int. Conf. Database and Expert Systems Applications, vol. 2453, Lecture Notes In Computer Science.

Weron R. (2006). Modeling and Forecasting Electricity Load and Prices: A Statistical Approach. The Wiley Finance Series.

Appendix: R Instructions used for the comparative study

The data are in plain text format in the files **nordpool.dat**, **powernext.dat** and **omel.dat**.

Prediction with Model A

Here you can find the instructions about the estimation and prediction of model A described in Section 2 of this document:

```
yn<-read.table('nordpool.dat')
library(forecast)
yn<-ts(t(yn))
zn=yn[1:630,]
zn=as.vector(t(zn))
zn = ts(zn,frequency=24)
mn = auto.arima(zn, d=NA, D=NA, max.p=30, max.q=30,
           max.P=3, max.Q=3, max.order=50, start.p=5, start.q=3,
           start.P=3, start.Q=3, stationary=FALSE,
           ic=c("aic","aicc", "bic"), stepwise=TRUE, trace=TRUE,
           test=c("kpss","adf","pp"), seasonal.test="ocsb",
           allowdrift=TRUE, lambda=NULL)
```

The forecasted values are obtained using

```
nord.fcastA = fcast(yn,mn,631,780)
```

with the function:

```
fcast <- function(yn,m,ini,fin)
{
  fcast=yn
  for (k in (ini:fin))
  {
    print(k)
    fit = Arima(window(y,end=c(k-1,24)),model=m)
    fc = forecast(fit,h=24)
    fcast = c(fcast,fc$mean)
  }
  fcast
}
```

Prediction with Model B

Here you can find the instructions about the estimation and prediction of the Model B described in Section 3 of this document:

```
yn = read.table("nordpool.dat")
library(forecast)
nord.fcastB = pred24(t(yn))
```

fpred24 function

```
pred24 <- function(yn) {
  pred = matrix(0,150,24)
  for (k in (1:24))
  {
    print(k)
    z = ts(yn[1:630,k],frequency=5)
    m = auto.arima(z, d=NA, D=NA, max.p=6, max.q=6,
                   max.P=6, max.Q=6, max.order=25, start.p=2, start.q=2,
                   start.P=1, start.Q=1, stationary=FALSE,
                   ic=c("aic","aicc", "bic"), stepwise=TRUE, trace=FALSE,
                   test=c("kpss","adf","pp"), seasonal.test="ocsb",
                   allowdrift=TRUE, lambda=NULL)
    print(m)
    m2=Arima(yn[631:780,k],model=m)
    fcast = fitted(m2)
    pred[,k]=fcast

  }
  pred = rbind(yn[1:630,],pred)
  pred
}
```