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Assessing the South African sardine resource: two stocks rather than one?

CL de Moor and DS Butterworth

Supplementary Material

This supplementary material contains three sections:

- S1) Further methods, results and discussion
- S2) Bayesian assessment for the South African sardine resource
- S3) Calculating the bias in estimates of sardine from the May and November hydro-acoustic surveys

S1) Further methods, results and discussion

The Bayesian integration was implemented numerically using Markov Chain Monte Carlo (Gelman et al. 1995) in AD Model Builder (Fournier et al. 2012). Convergence to the posterior distribution was tested using the Bayesian Output Analysis package (Smith 2005). A chain was accepted as having converged only once the diagnostics from the tests of Geweke (1992), Gelman and Rubin (1992), Raftery and Lewis (1992) and Heidelberger and Welch (1983) were satisfactory. The chains from which resampling was conducted for the results presented in this paper were based on 50 000 000 samples. A burn-in of 2 000 000 for the single stock hypothesis and 10 000 000 for the two stock hypothesis was discarded and the remaining chain was thinned by 2 000 for the single stock hypothesis and 1 000 for the two stock hypothesis only, standard deviation about the mean length of 2+ sardine (Table S2) were found to mix slowly during initial test simulations, and were thus fixed at their values from the joint posterior mode (Table S1) as the posterior distributions for remaining parameters were found to be insensitive to realistically small changes in these fixed values. The standard deviation in the residuals about the stock recruitment curve were also found to mix slowly. Multiple chains of different fixed values were run (see footnote 1 on page 4).

The 90% probability intervals of the Bayesian model are shown in Figures S1 and S2. Although there is a run of positive residuals from the model fit to the south stock November survey estimates of 1+ biomass (Figure 2 of the main text), these residuals are not large, with the model predicted values being within the 95% confidence intervals of the survey estimates. As this has no substantial effect on the productivity of the south stock, with the peak in the south stock biomass being primarily dependent on the influx of recruits from the west stock, no adjustment to the model was merited. The largest residual for the model fit to the recruit survey data is for the south stock in May 2001 (see Figure 3 of the main text). A larger recruitment than observed is predicted by the model as non-negligible 0-year-old catch occurred to the east of Cape Agulhas before November 2001, while the model is limited by the assumption that recruits move in a single pulse in November. In reality, the continuous movement of recruits during the latter half of the year was likely substantial in 2001, resulting in recruits originating from the west stock being caught to the east of Cape Agulhas before November.

The bias associated with the hydro-acoustic survey is estimated to be very similar for the two stockstructure hypotheses (Figure S3a), while the coverage of the recruit survey in comparison to that of the November survey is estimated to be higher under the two stock hypothesis (Figure S3c). The coverage of the south stock recruits relative to those from the west stock during the recruit survey, k_{cov}^{s} , is estimated to be 100% at the joint posterior mode, and is robust to the choice (if any) of stockrecruitment relationship. Convergence of this parameter to the posterior distribution was, however, difficult due to correlation primarily with the south stock maximum recruitment parameter, and thus MCMC chains were run with k_{cov}^{s} fixed at different values (e.g. Figures S3 to S5). The recruit survey is estimated, at the posterior mean, to survey 38% of the true recruitment assuming a single stock hypothesis, 47% (if $k_{cov}^{s} = 1$) or 53% (if $k_{cov}^{s} = 0.4$) of the west stock, and 47% (if $k_{cov}^{s} = 1$) or 21% (if $k_{cov}^{s} = 0.4$) of the south stock (Figure S3b), compared to 68% of the November biomass (Figure S3a). The lower bias for the south stock (and corresponding higher bias for the west stock) corresponds to higher maximum recruitment (in median terms) being estimated for the south stock (Figure S4), so that the model predicted recruitment corresponding to the survey remains the same.

The rapid juvenile growth is reflected in the estimated growth curve (Figure S6). The variability about this growth curve is estimated to be larger for the smallest age groups, reflecting the extended spawning season for sardine (Figure S7).

Table S1 lists the parameter estimates at the joint posterior mode and posterior median.

Table S1: Key model parameter values estimated at the joint posterior mode and posterior median for both the two stock and single stock hypotheses. Numbers are reported in billions and biomass in thousands of tonnes. j=1 denotes the "west" stock and j=2 denotes the "south" stock. **Bold** values denote those fixed for MCMC simulations.

Parameter	Single	stock	Two s	stock	Parameter	Single stock Two sto		stock	
	Mode	Median	Mode	Median		Mode	Median	Mode	Median
$k_{j=1,N}^{S} = k_{ac}^{S}$	0.72	0.68	0.75	0.69	$a_{j=1}^{S}$	58.6	67.1	79.1	101.14
$k_{j=2,N}^{S} = k_{ac}^{S}$			0.75	0.69	$a_{j=2}^{s}$			2.6	2.9
$k_{ m cov}^{S}$	0.54	0.56	0.67	0.68	$b_{j=1}^{S}$	596	749	540	740
$k_{\mathrm{cov}S}^{S}$			1.00	1.00	$b_{j=2}^{s}$			0.4	2.5
$k_{j=1,r}^S$	0.39	0.38	0.50	0.46	$K_{j=1}^S$	2203	2524	2857	3651
$k_{j=2,r}^{S}$			0.50	0.46	$K_{j=2}^S$			105	121
$k_{j=1,r}^{S}/k_{j=1,N}^{S}$	0.54	0.56	0.67	0.68	$\sigma^{\scriptscriptstyle S}_{\scriptscriptstyle j=1,r}$	0.40	0.45 ¹	0.40	0.5 ¹
$k_{j=2,r}^{S} / k_{j=2,N}^{S}$			0.67	0.68	$\sigma^{\scriptscriptstyle S}_{\scriptscriptstyle j=2,r}$ or $\sigma^{\scriptscriptstyle S}_{\scriptscriptstyle r, peak}$	0.89	1.50	0.40	0.5
$\left(\lambda_{j=1,N}^{S} ight)^{2}$	0.00	0.00	0.00	0.00	$L_{j=1,\infty}$	19.9	19.9	19.1	18.44
$\left(\lambda_{j=2,N}^{S}\right)^{2}$			0.00	0.00	$L_{j=2,\infty}$			19.7	19.20
$\left(\lambda_{j=1,r}^{S}\right)^{2}$	0.00	0.00	0.00	0.00	$\kappa_{j=1}$	1.07	1.04	1.22	1.41
$\left(\lambda_{j=2,r}^{S}\right)^{2}$			0.00	0.00	$\kappa_{j=2}$			1.18	1.35
$N^{S}_{j=1,1983,0}$	6.83	8.49	5.56	6.60	t_0	0.09	0.06	0.11	0.16
$N^{S}_{j=1,1983,1}$	4.39	5.50	2.43	3.32	\mathcal{G}_0	3.0	2.6	3.0	3.0
$N_{j=1,1983,2}^{S}$	<0.001	0.03	<0.001	0.02	ϑ_1	2.4	2.5	2.7	2.80
$N_{j=2,1983,0}^{S}$			0.01	0.04	$\mathcal{9}_{2^+}$	1.6	1.6	1.8	2.09
$Finit_{j=1}$	0.57	0.51	0.50	0.50	<i>Finit</i> $_{j=2}$			<0.001	0.07

¹ Higher standard deviations were selected as a precautionary measure, for use in the Operating Models when simulating testing candidate Management Procedures. Alternative Operating Models, called robustness tests, based on different fixed values were also constructed.



Figure S1: Acoustic survey estimated and associated model predicted posterior median and 90% probability intervals for sardine 1+ biomass from November 1984 to 2011 under both the two stock and single stock hypotheses. The observations are shown together with their 95% confidence intervals. The combined west and south stock biomass is shown by the grey line together with the biomass estimated under the single stock hypothesis.



Figure S2: Acoustic survey estimated and associated model predicted posterior median and 90% probability intervals for sardine recruitment from May 1985 to 2011 under both the two stock and single stock hypotheses. The observations are shown together with their 95% confidence intervals. Note the scale of the vertical axis for the south stock is different from the others. The combined west and south stock recruitment is shown by the grey line together with the recruitment estimated under the single stock hypothesis.



Figure S3: Posterior distributions for the multiplicative bias associated a) with the hydro-acoustic survey, $k_{ac}^{S} = k_{j,N}^{S}$, b) the recruit survey of stock j, $k_{j,r}^{S}$, and c) the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey, k_{cov}^{S} . Distributions for the two stock hypothesis are plotted for two possibly extreme alternative values for k_{covS}^{S} , denoted by "k=1" and "k=0.4" in the legend.



Figure S4: Posterior distributions for the maximum recruitment for each stock, a_j^s . Distributions for the two stock hypothesis are plotted for two possibly extreme alternative values for k_{covs}^s , denoted by "k=1" and "k=0.4" in the legend.



Figure S5: Posterior distributions for the spawner biomass below which the expectation for recruitment is reduced from the maximum for each stock, b_j^s (the "kink" point on that stock-recruitment curve). Distributions for the two stock hypothesis are plotted for two possibly extreme alternative values for k_{covs}^s , denoted by "k=1" and "k=0.4" in the legend.



Figure S6: The von Bertalanffy growth curve estimated at the joint posterior mode.



Figure S7: Proportions-at-length estimated at the joint posterior mode for a) age 0 and b) age 1 corresponding to the middle of each quarter of the year when commercial catches are assumed to be taken and c) ages 2 to 5+, corresponding to the middle of the first quarter.

S2) Bayesian assessment for the South African sardine resource

The assessment is run from November $y_1 = 1984$ to November $y_n = 2011$, with quarters q = 1 denoting November y-1 to January y, q=2 denoting February to April y, q=3 denoting May to July y and q=4 denoting August to October y. All parameters are defined in Table S2.

Population Dynamics

Numbers-at-age at 1 November

$$N_{j,y,a}^{S*} = \left(\left(\left(N_{j,y-1,a-1}^{S} e^{-M_{a-1}^{S}/8} - C_{j,y,1,a-1}^{S} \right) e^{-M_{a-1}^{S}/4} \right) - C_{j,y,2,a-1}^{S} \right) e^{-M_{a-1}^{S}/4} - C_{j,y,3,a-1}^{S} \right) e^{-M_{a-1}^{S}/4} - C_{j,y,4,a-1}^{S} e^{-M_{a-1}^{S}/8} \\ y_{1} \le y \le y_{n}, 1 \le a \le 4$$

$$N_{j,y,5+}^{S*} = \left(\left(\left(N_{j,y-1,4}^{S} e^{-M_{4}^{S}/8} - C_{j,y,1,4}^{S} \right) e^{-M_{4}^{S}/4} \right) - C_{j,y,2,4}^{S} \right) e^{-M_{4}^{S}/4} - C_{j,y,3,4}^{S} \right) e^{-M_{4}^{S}/4} - C_{j,y,4,4}^{S} \right) e^{-M_{4}^{S}/8} \\ + \left(\left(\left(N_{j,y-1,5+}^{S} e^{-M_{5+}^{S}/8} - C_{j,y,1,5+}^{S} \right) e^{-M_{5+}^{S}/4} \right) - C_{j,y,2,5+}^{S} \right) e^{-M_{5+}^{S}/4} - C_{j,y,3,5+}^{S} \right) e^{-M_{5+}^{S}/4} - C_{j,y,4,5+}^{S} \right) e^{-M_{5+}^{S}/8} \\ y_{1} \le y \le y_{n}$$
(S1)

Movement of west stock (j=1) recruits to the south stock (j=2) in the two stock hypothesis; in the single stock hypothesis move $_{y} = 0$

$$N_{1,y,1}^{S} = (1 - move_{y})N_{1,y,1}^{S*}$$

$$N_{2,y,1}^{S} = N_{2,y,1}^{S*} + move_{y}N_{1,y,1}^{S*}$$

$$y_{1} \le y \le y_{n}$$

$$N_{j,y,a}^{S} = N_{j,y,a}^{S*}$$

$$y_{1} \le y \le y_{n}, \ 2 \le a \le 5 +$$
(S2)

Spawning biomass and biomass associated with the November survey

 Recruitment

Carrying Capacity

$$K_{j}^{S} = a_{j}^{S} e^{-0.5(\sigma_{j,r}^{S})^{2}} \sum_{a=1}^{4} \overline{w}_{j,a}^{S} e^{-M_{j}^{S} - (a-1)\overline{M}_{ad}^{S}} + \overline{w}_{j,5+} e^{-M_{j}^{S} - 3\overline{M}_{ad}^{S}} \frac{1}{1 - e^{-\overline{M}_{ad}^{S}}}$$
$$K_{peak}^{S} = a_{j}^{S} e^{-0.5(\sigma_{r,peak}^{S})^{2}} \sum_{a=1}^{4} \overline{w}_{j,a}^{S} e^{-M_{j}^{S} - (a-1)\overline{M}_{ad}^{S}} + \overline{w}_{j,5+} e^{-M_{j}^{S} - 3\overline{M}_{ad}^{S}} \frac{1}{1 - e^{-\overline{M}_{ad}^{S}}}$$
(S7)

Number of recruits associated with the recruit survey

$$N_{j,y,r}^{S} = k_{j,r}^{S} \left(\left((N_{j,y-1,0}^{S} e^{-M_{0}^{S}/8} - C_{j,y,1,0}^{S}) e^{-M_{0}^{S}/4} - C_{j,y,2,0}^{S} \right) e^{-0.5t_{y}^{S} \times M_{0}^{S}/12} - \widetilde{C}_{j,y,0bs}^{S} \right) e^{-0.5t_{y}^{S} \times M_{0}^{S}/12}$$

$$y_{1} \leq y \leq y_{n}$$
(58)

Multiplicative survey bias

$$k_{j,N}^{S} = k_{ac}^{S}$$
(S9)

$$k_{1,r}^S = k_{\rm cov}^S \times k_{ac}^S \tag{S10}$$

$$k_{2,r}^{S} = k_{\text{cov}S}^{S} \times k_{\text{cov}}^{S} \times k_{ac}^{S}$$
 (for the two stock hypothesis only) (S11)

Numbers-at-length

where
$$A_{j,a,l}^{sur} \sim N\left(L_{j,\infty}\left(1 - e^{-\kappa_{j}(a-t_{0})}\right), \mathcal{G}_{j,a}^{2}\right)$$
 $0 \le a \le 5 + , 3.5cm \le l \le 23cm$ (S13)

Proportion-at-length associated with the November survey

 $^{2} \sigma_{j,r}^{s}$ is replaced with $\sigma_{r,peak}^{s}$ during the peak years of 2000-2004 in the single stock hypothesis (see Table S2).

Commercial selectivity

$$S_{j,y,l} = \begin{cases} 0 & l \le 5.5cm \\ \chi_{j} \exp\left\{-\frac{\left(l+0.25-\bar{l}_{1,j}\right)^{2}}{\left(\sigma_{1}^{sel}\right)^{2}}\right\} + \exp\left\{-\frac{\left[\ln\left(\left(l+0.25-23.5\right)/\left(\bar{l}_{2,j}-23.5\right)\right)\right]^{2}}{\left(\sigma_{2}^{sel}\right)^{2}}\right\} & 6cm \le l \le 23cm \\ y_{1} \le y \le y_{n} & (S17) \end{cases}$$

where the 23.5cm is one length class above the maximum for which observations can be predicted.

$$S_{j,y,q,a} = \sum_{l} A_{j,q,a,l}^{com} S_{j,y,l} \qquad y_1 \le y \le y_n, \ 1 \le q \le 4, \ 0 \le a \le 5 +$$
(S18)

Bycatch in the anchovy directed fishery

$$C_{j,y,1,0}^{bycatch} = \sum_{m=1}^{12} \sum_{l < lcut_{y,m}} C_{j,y-1,m,l}^{RLF, fleet=3} + \sum_{l < lcut_{y,m}} C_{j,y,1,l}^{RLF, fleet=3} C_{j,y,1,l}^{bycatch} = \sum_{m=11}^{12} \sum_{l > lcut_{y,m}} C_{j,y-1,m,l}^{RLF, fleet=3} + \sum_{l > lcut_{y,m}} C_{j,y,1,l}^{RLF, fleet=3} + \sum_{l < lcut_{y,m}} C_{j,y,1,l}^{RLF, fleet=3} +$$

Catch in the directed sardine and round herring bycatch fisheries

$$C_{j,y,1,a}^{dir} = \left(N_{j,y-1,a}^{S}e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{bycatch}\right)S_{j,y,1,a}F_{j,y,1}$$

$$C_{j,y,2,a}^{dir} = \left(\left(N_{j,y-1,a}^{S}e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S}\right)e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{bycatch}\right)S_{j,y,2,a}F_{j,y,2}$$

$$C_{j,y,3,a}^{dir} = \left(\left(\left(N_{j,y-1,a}^{S}e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S}\right)e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{S}\right)e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{bycatch}\right)S_{j,y,3,a}F_{j,y,3}$$

$$C_{j,y,4,a}^{dir} = \left(\left(\left(N_{j,y-1,a}^{S}e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S}\right)e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{S}\right)e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{S}\right)e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{bycatch}\right)S_{j,y,4,a}F_{j,y,4}$$
(S20)

Total catch

$$C_{j,y,q,a}^{S} = C_{j,y,q,a}^{bycatch} + C_{j,y,q,a}^{dir} \qquad y_{1} \le y \le y_{n}, 1 \le q \le 4, 0 \le a \le 5 +$$
(S21)

Fished proportion of the available biomass from the directed catch and round herring bycatch fisheries

$$F_{j,y,1} = \frac{\sum_{l=0}^{2} \sum_{m=1}^{12} \sum_{l\geq 6cm} C_{j,y-1,m,l}^{RFL,fleet} + \sum_{fleet=1}^{2} \sum_{l\geq 6cm} C_{j,y,1,l}^{RFL,fleet}}{\sum_{a=0}^{5+} \left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{bycatch} \right)} S_{j,y,1,a}$$

$$F_{j,y,2} = \frac{\sum_{fleet=1}^{2} \sum_{m=2}^{4} \sum_{l\geq 6cm} C_{j,y,m,l}^{RFL,fleet}}{\sum_{a=0}^{5+} \left(\left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{bycatch} \right) s_{j,y,2,a}}$$

$$F_{j,y,3} = \frac{\sum_{fleet=1}^{2} \sum_{m=5}^{7} \sum_{l\geq 6cm} C_{j,y,m,l}^{RFL,fleet}}{\sum_{a=0}^{5+} \left(\left(\left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{bycatch} \right) s_{j,y,3,a}}$$

$$F_{j,y,4} = \frac{\sum_{fleet=1}^{2} \sum_{m=8}^{1} \sum_{l\geq 6cm} C_{j,y,n,l}^{RFL,fleet}}{\sum_{a=0}^{2} \left(\left(\left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,$$

A penalty is imposed within the model to ensure that $S_{j,y,q,a}F_{j,y,q} < 0.95$. Fish <6cm were caught in less than 10% of the quarters and were thus not used in fitting this model. Commercial selectivity-at-length is fixed to zero for length classes < 6cm (equation S17)

Catch-at-length from the directed and round herring bycatch fisheries

$$C_{j,y,1,l}^{dir} = \sum_{a=0}^{5+} \left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{bycatch} \right) A_{j,1,a,l}^{com} S_{j,y,l} F_{j,y,1}$$

$$C_{j,y,2,l}^{dir} = \sum_{a=0}^{5+} \left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{bycatch} \right) A_{j,2,a,l}^{com} S_{j,y,l} F_{j,y,2}$$

$$C_{j,y,3,l}^{dir} = \sum_{a=0}^{5+} \left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{bycatch} \right) A_{j,3,a,l}^{com} S_{j,y,l} F_{j,y,3}$$

$$C_{j,y,4,l}^{dir} = \sum_{a=0}^{5+} \left(\left(N_{j,y-1,a}^{S} e^{-M_{a}^{S}/8} - C_{j,y,1,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,2,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,3,a}^{S} \right) e^{-M_{a}^{S}/4} - C_{j,y,4,a}^{S} \right) A_{j,4,a,l}^{com} S_{j,y,l} F_{j,y,4}$$

$$Y_{1} \leq y \leq y_{n}, 3.5cm \leq l \leq 23cm \qquad (S23)$$
where $A_{j,q,a,l}^{com} \sim N \left(L_{j,\infty} \left(1 - e^{-\kappa_{j}(a + (2q-1)/8 - t_{0})} \right) \beta_{j,a}^{2} \right) \qquad 0 \leq a \leq 5 + , 3.5cm \leq l \leq 23cm \qquad (S24)$

Proportion-at-length associated with the directed catch and round herring bycatch

Initial numbers-at-age

$$N_{1,1983,a}^{S} = N_{1,1983,a-1}^{S} e^{-Finit_1 - M_a^{S}} \qquad 3 \le a \le 4$$

$$N_{2,1983,a}^{S} = N_{2,1983,a-1}^{S} e^{-Finit_2 - M_a^{S}} \qquad 1 \le a \le 4$$

$$N_{j,1983,5+}^{S} = N_{j,1983,4}^{S} \frac{e^{-Finit_{j} - M_{5+}^{S}}}{1 - e^{-Finit_{j} - M_{5+}^{S}}}$$
(S26)

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Fitting the Model to Observed Data (Likelihood)

$$-\ln L = -\ln L^{Nov} - \ln L^{rec} - \ln L^{sur \ propl \ min} - \ln L^{sur \ propl} - \ln L^{com \ propl}$$
(S27)
where

 $-\ln L^{Nov} = \frac{1}{2} \sum_{j} \sum_{y=yl}^{yn} \left\{ \begin{cases} \frac{\left| \ln(\hat{B}_{j,y}^{s}) - \ln(B_{j,y}^{s}) \right|}{\sqrt{(\sigma_{j,y,Nov}^{s})^{2} + (\phi_{ac}^{s})^{2} + (\lambda_{j,N}^{s})^{2}}} \right)^{5} \\ \frac{1}{5^{5} + \left(\frac{\left| \ln(\hat{B}_{j,y}^{s}) - \ln(B_{j,y}^{s}) \right|}{\sqrt{(\sigma_{j,y,Nov}^{s})^{2} + (\phi_{ac}^{s})^{2} + (\lambda_{j,N}^{s})^{2}}} \right)^{5}} \right\}^{2} + \ln \left[2\pi \left((\sigma_{j,y,Nov}^{s})^{2} + (\phi_{ac}^{s})^{2} + (\lambda_{j,N}^{s})^{2}} \right) \right] \right\}$

$$-\ln L^{rec} = \frac{1}{2} \sum_{j} \sum_{y=y_{l+1}}^{y_n} \left\{ \begin{cases} 5^{5} \left(\frac{\left| \ln(\hat{N}_{j,y,r}^{S}) - \ln(N_{j,y,r}^{S}) \right|}{\sqrt{(\sigma_{j,y,rec}^{S})^2 + (\phi_{ac}^{S})^2 + (\lambda_{j,r}^{S})^2}} \right)^5 \\ + \ln\left[2\pi \left((\sigma_{j,y,rec}^{S})^2 + (\phi_{ac}^{S})^2 + (\lambda_{j,r}^{S})^2 \right) \right] \\ - \ln L^{sur \ propl \min} = w_{propl \min}^{sur} \sum_{j} \sum_{y=y_{l}}^{y_n} \left\{ \frac{\hat{p}_{j,y,l}^{S} \min\left(\ln(\hat{p}_{j,y,l}^{S}) - \ln(N_{j,y,r}^{S})\right)}{2(\sigma_{j}^{S,surl \min})^2} + \ln\left(\frac{\sigma_{j}^{S,surl \min}}{\sqrt{\hat{p}_{j,y,l \min}^{S}}}\right) \right\} \\ - \ln L^{sur \ propl \min} = w_{propl \min}^{sur} \sum_{j} \sum_{y=y_{l}}^{y_n} \left\{ \frac{\hat{p}_{j,y,l}^{S} \left(\ln(\hat{p}_{j,y,l}^{S}) - \ln(p_{j,y,l}^{S})\right)}{2(\sigma_{j}^{S,surl \min})^2} + \ln\left(\frac{\sigma_{j}^{S,surl \min}}{\sqrt{\hat{p}_{j,y,l \min}^{S}}}\right) \right\} \\ - \ln L^{sur \ propl} = w_{propl}^{sur} \sum_{j} \sum_{y=y_{l}}^{y_n} \sum_{l=1}^{l \max_{min+1}} \left\{ \frac{\hat{p}_{j,y,l}^{S} \left(\ln(\hat{p}_{j,y,l}^{S,conl}) - \ln(p_{j,y,l}^{S})\right)^2}{2(\sigma_{j}^{S,surl})^2} + \ln\left(\frac{\sigma_{j}^{S,surl}}{\sqrt{\hat{p}_{j,y,q,l}^{S}}}\right) \right\} \\ - \ln L^{com \ propl} = w_{propl}^{com} \sum_{j} \sum_{y=y_{l}}^{y_n} \sum_{l=1}^{l} \sum_{l=6cm} \left\{ \frac{\hat{p}_{j,y,q,l}^{S,coml} \left(\ln(\hat{p}_{j,y,q,l}^{S,coml}) - \ln(p_{j,y,q,l}^{S,coml})\right)^2}{2(\sigma_{j}^{S,coml})^2} + \ln\left(\frac{\sigma_{j}^{S,coml}}{\sqrt{\hat{p}_{j,y,q,l}^{S,coml}}}\right) \right\}$$
(S28)

A "robustified likelihood" is used for the contributions from the hydro-acoustic surveys to ensure no undue influence from any extreme (outlying) values for residuals (the functional form chosen to robustify makes negligible difference for standardised residuals of magnitude three or less, but essentially treats large standardised residuals as if they do not exceed five in magnitude).

The prior on the variance-related parameters for the log-transformed survey proportion-at-length data for the minus length class of stock j, $\sigma_j^{S,surl \min}$, is taken to be proportional to $(\sigma_j^{S,surl \min})^{-3}$. This has the convenience that integration over $\sigma_j^{S,surl \min}$ can be performed analytically and leads to the result that this parameter can be substituted by its maximum likelihood estimate following this integration (Walters and Ludwig 1994). This holds similarly for all the variance-related parameters in equation (S28):

$$\sigma_{j}^{S,surl\min} = \sqrt{\frac{\sum_{y=y1}^{yn} \hat{p}_{j,y,l\min}^{S} (\ln \hat{p}_{j,y,l\min}^{S} - \ln p_{j,y,l\min}^{S})^{2}}{\sum_{y=y1}^{yn} 1}}$$
(S29)
$$\sigma_{j}^{S,surl} = \sqrt{\frac{\sum_{y=y1}^{yn} \sum_{l=1\min+1}^{l\max} \hat{p}_{j,y,l}^{S} (\ln \hat{p}_{j,y,l}^{S} - \ln p_{j,y,l}^{S})^{2}}{\sum_{y=y1}^{yn} \sum_{l=1\min+1}^{l\max} 1}}$$
(S30)
$$\sigma_{j}^{S,conl} = \sqrt{\frac{\sum_{y=y1}^{yn} \sum_{q=1}^{4} \sum_{l>=6cm} \hat{p}_{j,y,q,l}^{S,conl} (\ln \hat{p}_{j,y,q,l}^{S,conl} - \ln p_{j,y,q,l}^{conl})^{2}}{\sum_{y=y1}^{yn} \sum_{q=1}^{4} \sum_{l>=6cm} 1} \sum_{l>=6cm} (S31)$$

Table S2: Assessment model parameters and variables. As the majority of prior distributions are uninformative, notes are provided only for informative priors and/or bounds.

Ρ	arameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
	$N^{S}_{j,y,a}$	Model predicted numbers-at-age a at the beginning of November in year y of stock j	Billions		S1, S2 and S6	
nd biomass	$M_a^{\ S}$	Rate of natural mortality of age <i>a</i>	Year ⁻¹	=1.0, $a = 0$ = 0.8, $1 \le a \le 5 +$		Selected based on maximized joint posterior, but subject to $0.5 \le k_{j,r}^S / k_{j,N}^S \le 1$ and a compelling reason to modify from previous assessment
ibers an	move _y	Proportion of west stock recruits which move to the south stock at the beginning of November of year y (two stock hypothesis only)	-	$= 0$, $y_1 \le y \le 1993$ ~ $U(0,1)$, $1994 \le y \le y_n$		
ual num	$SSB_{j,y}^{S}$	Model predicted spawning biomass of stock j at the beginning of November in year y	Thousand tonnes		S 3	
Ann	$B_{j,y}^S$	Model predicted 1+ biomass of stock $ j$ at the beginning of November in year $ y$, associated with the November survey	Thousand tonnes		S4	
	$w_{j,y,a}^S$	Mean mass of age a from stock j sampled during the November survey of year y	Grams		S5	
	$\overline{w}_{j,a}^{S}$	Mean mass of age a from stock j sampled during each November survey, averaged over all years	Grams		$\frac{\sum_{y=y_1}^{y_n} w_{j,y,a}^S}{y_n - y_1 + 1}$	

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	Parameter/ Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
Annual numbers and biomass	$\frac{\overline{w_{j,a}^S}}{w_{j,1}^S}$	Average ratio of mean mass of stock j aged a to age 1	-	Single stock hypothesis: $\frac{\overline{w}_{j,2}^{S}}{w_{j,1}^{S}} = 1.40, \frac{\overline{w}_{j,3}^{S}}{w_{j,1}^{S}} = 1.69$ $\frac{\overline{w}_{j,4}^{S}}{w_{j,1}^{S}} = 1.88, \frac{\overline{w}_{j,5+}^{S}}{w_{j,1}^{S}} = 2.00$ Two stock hypothesis: $\frac{\overline{w}_{1,2}^{S}}{w_{1,1}^{S}} = 1.39, \frac{\overline{w}_{1,3}^{S}}{w_{1,1}^{S}} = 1.65,$ $\frac{\overline{w}_{1,4}^{S}}{w_{1,1}^{S}} = 1.80, \frac{\overline{w}_{1,5+}^{S}}{w_{1,1}^{S}} = 1.89$ $\frac{\overline{w}_{2,2}^{S}}{w_{2,1}^{S}} = 1.39, \frac{\overline{w}_{2,5+}^{S}}{w_{2,1}^{S}} = 1.68$ $\frac{\overline{w}_{2,4}^{S}}{w_{2,1}^{S}} = 1.89, \frac{\overline{w}_{2,5+}^{S}}{w_{2,1}^{S}} = 2.02$		From the von Bertalanffy growth curve (D Durholtz and C Mtengwane pers comm.)
uitment	a_j^S	Maximum recruitment of stock j in the hockey stick model	Billions	$\ln\!\left(a_j^s\right) \sim U\!\left(0, 5.6\right)$		Uninformative on log-scale as scale is not known <i>a priori</i> , with the maximum corresponding to about 10 million tonnes for K_j^s
Recr	b_j^S	Spawner biomass below which the expectation for recruitment is reduced below the maximum for stock $ j$	Thousand tonnes	$b_j^S / K_j^S \sim U(0,1)$		
	K_{j}^{S}	Carrying capacity for stock j	Thousand tonnes		S7	
	K^{S}_{peak}	Carrying capacity during "peak" years (single stock hypothesis only)	Thousand tonnes		S7	

Ρ	Parameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
lent	$\mathcal{E}_{j,y}^{S}$	Annual lognormal deviation of recruitment of stock <i>j</i>	-	$\begin{split} \mathcal{E}_{j,y}^{S} \sim N\!\!\left(0, \left(\sigma_{j,r}^{S}\right)^{2}\right) \\ \text{Except for} \\ \mathcal{E}_{1,y}^{S} \sim N\!\!\left(0, \left(\sigma_{r,peak}^{S}\right)^{2}\right) \\ 2000 \leq y \leq 2004 \text{ for} \\ \text{cincle steak hyperbasis} \end{split}$		Reflects the assumption of a different distribution applying over the peak period
Recruitm	$\left(\sigma_{j,r}^{s}\right)^{2}$	Variance in the residuals (lognormal deviation) about the stock recruitment curve of stock $ j$	-	$\sim U(0.16, 10)$		Lower bound chosen to restrict the
	$\left(\sigma_{r,peak}^{s}\right)^{2}$	Variance in the residuals (lognormal deviation) about the stock recruitment curve during "peak" years (single stock hypothesis only)	-	$\sim U(0.16, 10)$		stock recruitment curve on the assessment results
	$N^{S}_{j,y,r}$	Model predicted number of juveniles of stock j at the time of the recruit survey in year y	Billions		S8	
	$k_{j,N}^S$	Multiplicative bias associated with the November survey of stock j	-		S9	
	$k_{j,r}^S$	Multiplicative bias associated with the recruit survey of stock j	-		S10 and S11	
ive bias	k_{ac}^{S}	Multiplicative bias associated with the hydro-acoustic survey	-	$\sim N(0.714, 0.077^2)$		Supplementary Material Section S3 Lower bound
Multiplicati	$k_{\rm cov}^{S}$	Multiplicative bias associated with the coverage of the recruits by the recruit survey in comparison to the 1+ biomass by the November survey	-	~ U(0.3,1)		selected in discussions with scientists on these surveys and their field experience
	$k^S_{{ m cov}S}$	Multiplicative bias associated with the coverage of the south stock recruits by the recruit survey in comparison to the west stock recruits during the same survey	-	$\sim U(0,1)$		·

Parameter / Variable		Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
	$N^{S}_{j,y,l}$	Model predicted numbers-at-length l at the beginning of November in year y of stock j	Billions		S12	
	$p_{j,y,l}^S$	Model predicted proportion-at-length l of stock j associated with the November survey in year y	-		S14-S16	
th curve	<i>l</i> min	Minus length class used when fitting the model to survey proportion-at-length data	Cm	= 9		Selected to ensure reasonably positive proportions for all years for simplicity of programming and to avoid undue influence of small samples
grow	<i>l</i> max	Plus length class used when fitting the model to survey proportion-at-length data	Cm	=20		
and g	$A_{j,a,l}^{sur}$	Proportion of age a of stock j that falls in the length group l in November	-		S13	
length a	$p_{j,y,q,l}^{comlS}$	Model predicted proportion-at-length l of stock j in the directed catch and round herring bycatch of quarter q of year y	-		S25	
ons-at	$A^{com}_{j,q,a,l}$	Proportion of age a of stock j that falls in the length group l in quarter q	-		S24	
ortic	$L_{j,\infty}$	Maximum length (in expectation) of stock j	Cm	$\sim U(10,30)$		$\kappa_{j} imes L_{j,\infty}$ assumed
Prop	K _j	Somatic growth rate parameter for stock j	Year ⁻¹	$\kappa_{j} \times L_{j,\infty} \sim U(0,10)$		same for both stocks. Bounds informed by data
	t_0	Age at which the length (in expectation) is zero	Year	$\sim U(-4,4)$		
	$\boldsymbol{\mathcal{G}}_{j,a}$	Standard deviation of the distribution about the mean length for age a of stock j	-	$\sim U(0.01,3), a = 0,1,2 +$		Assumed same for both stocks. Upper bound chosen to preclude unrealistically large lengths for very young fish

P	arameter / Variable	Description		Fixed Value / Prior Distribution	Equation	Notes
electivity	$S_{j,l}^{survey}$	Survey selectivity-at-length l in the November survey for stock $ j$	-	$S_{j,l}^{survey} \sim U(0.6,1.1),$ $3.5 \le l \le 9$ =1, 9.5 \le l \le 19.5 $S_{j,l}^{survey} \sim U(0.9,1.1),$ $20 \le l \le 23$		Only allowed to differ from 1 for the minus and plus group. The smaller lower bound for the minus group reflected a trade-off between the requirement to be able to reflect the data and also not to depart too far from the flat selectivity one hopes will be achieved by the survey's design
Š	$S_{j,y,l}$	Commercial selectivity-at-length l during year y of stock j	-		S17	
	$S_{j,y,q,a}$	Commercial selectivity-at-age a during quarter q of year y of stock j	-		S18	
	χ_{j}	Height of the near-normal curve component for stock j relative to the height of the near-lognormal component	-	$\sim U(0,1)$		
	$ar{l}_{1,j}$	Mean of the near-normal distribution for stock $ j$	Cm	$\sim U(5,15)$		Bounds reflect a
	$\bar{l}_{2,j}$	Median of the near-lognormal distribution for stock j	Cm	$\bar{l}_{2,j} - \bar{l}_{1,j} \sim U(0,15)$		not wanting to
	$(\sigma_1^{sel})^2$	Variance parameter of the near-normal distribution	-	$\sim U(2,7)$		and ensuring that
	$(\sigma_2^{sel})^2$	Variance parameter of the near-lognormal distribution	-	$\sim U(0,2)$		results remained realistic
۲.	$C^{S}_{j,y,q,a}$	Model predicted number of age a fish of stock j caught during quarter q of year y	Billions		S21	
Catc	$lcut_{y,m}$	Cut off length for recruits in month m of year y	Cm	de Moor et al. 2012		Differ by month and year as informed by the recruit surveys

F	arameter / Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
	$C^{bycatch}_{j,y,q,a}$	Number of age a fish of stock j bycaught in the anchovy-directed fishery in quarter q of year y	Billions		S19	
Catch	$C^{dir}_{j,y,q,a}$	Number of age a fish of stock j caught in the sardine-directed and round herring bycatch fisheries in quarter q of year y	Billions		S20	
	$C^{dir}_{j,y,q,l}$	Number of length l fish of stock j caught in the sardine-directed and round herring bycatch fisheries in quarter q of year y	Billions		S23	
	$F_{j,y,q}$	Fished proportion in quarter q of year y for a fully selected age class a of stock j , by the directed and round herring bycatch fisheries	-		S22	
	$-\ln L^{Nov}$	Contribution to the negative log likelihood from the model fit to the November 1+ survey biomass data	-		S28	
	$-\ln L^{rec}$	Contribution to the negative log likelihood from the model fit to the recruit survey data	-		S28	
	$-\ln L^{surpropln}$	Contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data for the minus length class only	-		S28	
	$-\ln L^{sur\ propl}$	Contribution to the negative log likelihood from the model fit to the November survey proportion-at-length data	-		S28	
hood	$-\ln L^{com\ propl}$	Contribution to the negative log likelihood from the model fit to the quarterly commercial proportion-at-length data	-		S28	
Likeli	ϕ^{S}_{ac}	CV associated with factors which cause bias in the acoustic survey estimates and which vary inter-annually rather than remain fixed over time	-	=0.222		Supplementary Material Section S3
	$\left(\lambda_{j,N/r}^{S}\right)^{2}$	Additional variance (over and above $(\sigma^s_{j,y,Nov/rec})^2$ and $(\phi^s_{ac})^2$) associated with the November/recruit surveys of stock j	-	$\sim U(0,10)$		
	W ^{sur} propl min	Weighting applied to the survey proportion-at-length data for the minus length class	-	=0.167		To allow for
	w_{propl}^{sur}	Weighting applied to the remaining survey proportion-at-length data	-	=0.167		autocorrelation

 $^{^{3}}$ Based upon data being available ~6 times more frequently than annual age data which contain maximum information content on this

Table S2	(Continued)	
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	Parameter /Variable	Description	Units / Scale	Fixed Value / Prior Distribution	Equation	Notes
	$\sigma^{S, surl { m min}}_{j}$	Variance-related parameter for the log-transformed survey proportion-at-length data for the minus length class of stock j			S29	
pooq	$\sigma^{S,surl}_{j}$	Variance-related parameter for the log-transformed survey proportion-at-length data of stock j			S30	
Likeli	w_{propl}^{com}	Weighting applied to the commercial proportion-at-length data		=0.04		To allow for autocorrelation ⁴
	$\sigma^{\scriptscriptstyle S,com}_{j}$	Variance-related parameter for the log-transformed commercial proportion-atlength data of stock $ j$			S31	
Initial Values	$N_{j,1983,a}^{S}$	Initial numbers-at-age <i>a</i> in stock <i>j</i>	Billion	$N^{S}_{j,1983,a}$ ~ $U(0,50)$ for $j=1,\ 0\leq a\leq 2$ and $j=2$, $a=0$	S26	
	Finit $_j$	Rate of fishing mortality assumed in the initial year for stock j		$\sim U(0,1)$		

 $^{^4}$ Based upon data being available ~4x6 times more frequently than annual age data which contain maximum information content on this

Quantity	Description	Units / Scale	Shown in Figure
$W^{Sl+}_{j,y}$	Total (1+) mean mass of stock $ j $ sampled during the November survey of year $ y $	Grams	
t_y^s	Time lapsed between 1 May and the start of the recruit survey in year $\ y$	Months	
$\widetilde{C}^{S}_{j,y,0bs}$	Number of juveniles of stock j caught between 1 May and the day before the start of the recruit survey in year y	Billions	
$C^{\!RFL,fleet}_{j,y,m,l}$	Number of fish in length class l landed by $fleet$ in month m of year y of stock j . fleet = 1 denotes the sardine directed fishery, $fleet = 2$ denotes the sardine bycatch with round herring and $fleet = 3$ denotes the juvenile sardine bycatch with anchovy	Billions	
$\hat{B}^{S}_{j,y}$	Acoustic survey estimate of biomass of stock j from the November survey in year y	Thousand tonnes	Figure 2
$\sigma^{\scriptscriptstyle S}_{\scriptscriptstyle j,y,\scriptscriptstyle Nov}$	Survey sampling CV associated with $\hat{B}^{S}_{j,y}$ that reflects survey inter-transect variance	-	Figure 2
$\hat{N}^{S}_{j,y,r}$	Acoustic survey estimate of recruitment of stock j from the recruit survey in year y	Billions	Figure 3
$\sigma^{\scriptscriptstyle S}_{\scriptscriptstyle j,y,\mathit{rec}}$	Survey sampling CV associated with $\hat{N}^{S}_{j,y,r}$ that reflects survey inter-transect variance	-	Figure 3
$\hat{p}^{s}_{j,y,l}$	Observed proportion (by number) of stock j in length group l in the November survey of year y	-	
$\hat{p}^{S,coml}_{j,y,q,l}$	Observed proportion (by number) of the directed catch and round herring bycatch of fish of stock j and length group l during quarter q of year y		

Table S3: Assessment model data, detailed in de Moor et al. (2012).

S3) Calculating the bias in estimates of sardine from the May and November hydro-acoustic surveys

A probability density function (pdf) for the bias in the May and November survey that relate directly to the acoustic survey, rather than, for example the coverage of the stock, k_{ac}^{S} , was calculated as follows. Ten thousand samples were drawn from the individual pdfs for each source of constant error, together with the median values of the individual pdfs of each source of variable error (see Table S4, Anon. 2000). Constant error relates to a factor whose value is not known exactly, but whatever it is, it is the same for each year. In contrast variable errors relate to a factor whose true value will change from one year to the next. A second pdf of the factors causing bias in the acoustic survey estimated which vary inter-annually, ϕ^{S}_{ac} , was then calculated by drawing ten thousand samples from the individual pdfs for each source of variable error. The resultant pdfs on the model predicted biomass (i.e. the inverse of the pdf calculated using the errors provided), together with normal distributions fitted to these pdfs are given in Figures S8 and S9. A prior distribution for the multiplicative bias associated with the acoustic survey, k_{ac}^{S} , is then the normal distribution obtained in Figure S8, with the mean multiplied by the mean of the normal distribution obtained in Figure S9, i.e. $k_{ac}^{S} \sim N(0.969 \times 0.737, 0.077^{2})$. The reason to include the 0.969 mean from Figure S9 here is that the distribution of the annually varying bias factors in combination is not centred on 1; this then takes account of the formulation of equation S28 treating the impact of these factors as a symmetric variance. There may, however, still be systematic errors relating to the target strength that are unaccounted for in these pdfs. These could be taken into account through sensitivity tests using alternative k_{ac}^{S} values.

Table S4: Individual error factors for hydro-acoustic surveys of sardine biomass, where the values define
trapezium form pdfs (Anon. 2002). Note that these error factors apply to the observed biomass, i.e. they
reflect the inverse of the multiplicative bias in this model.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration						
(On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Variable ⁶
(Beam factor)	0.75 ⁵	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Variable ⁶
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable

⁵ This was originally reported as 0.8 in Anon 2000, but subsequently corrected (I. Hampton pers. Comm.).

⁶ This was recorded in Anon. (2000) as random error denoting that it would be positive or negative rather than purely positive or negative.



Figure S8: The probability density function for the overall bias in the estimate of sardine abundance from the acoustic survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of constant error, together with the median values of the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $N(0.737, 0.077^2)$.



Figure S9: The probability density function for the factors which cause bias in the sardine acoustic survey estimates and which vary inter-annually, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of variable and random error. The normal distribution fitted to this pdf is $N(0.969, 0.215^2)$. The CV of this distribution is thus $\phi_{ac}^S = 0.215/0.969 = 0.222$.

References

- Anon. 2000. Survey Errors Workshop. Benguela Environment and Fisheries Interaction and Training programme report. 4-7 December, Breakwater Lodge, Cape Town.
- de Moor CL, Coetzee J, Durholtz D, Merkle D, van der Westhuizen JJ, Butterworth DS. 2012. A record of the generation of data used in the 2012 sardine and anchovy assessments. Report No. FISHEREIS/2012/AUG/SWG-PEL/41. Cape Town: Department of Agriculture, Forestry and Fisheries. Available at <u>http://www.mth.uct.ac.za/maram/pub/2012/FISHERIES_2012_SEP_SWG-PEL_41.pdf</u>.

Gelman A, Carlin JB, Stern HS, Rubin DB (eds). 1995. Bayesian data analysis. New York: Chapman & Hall.

- Gelman A, Rubin DB. 1992. Inference from iterative simulation using multiple sequences. *Statistical Science* 7:457-511.
- Geweke J. 1992. Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments. In: Bernardo JM, Berger JO, Dawid AP, Smith AFM (eds), *Bayesian statistics* 4. Oxford: Oxford University Press. pp 169-193.
- Heidelberger P, Welch PD. 1983. Simulation run length control in the presence of an initial transient. *Operations Research* 31:1109-1144.
- Raftery AE, Lewis SM 1992. How many iterations in the Gibbs Sampler? In: Bernardo JM, Berger JO, Dawid AP, Smith AFM (eds), *Bayesian statistics* 4. Oxford: Oxford University Press. pp 763-773.
- Smith BJ. 2005. Bayesian Output Analysis Program (BOA) Version 1.1.5. The University of Iowa. Available at http://www.public-health.uiowa.edu/boa.
- Walters C, Ludwig D. 1994. Calculation of Bayes Posterior Probability Distributions for Key Population Parameters. *Canadian Journal of Fisheries and Aquatic Sciences* 51:713-722.