Supplementary Material (Section A): Parameter Transformations in the mlVAR Model in Mplus

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Parameter Transformations in the mlVAR Model in Mplus

This section shows how to perform parameter transformations to obtain random innovation variances and covariances/correlations in the original mlVAR model specified in Equations 5 and 6 in the main manuscript. For simplicity's sake, person- and time indices were omitted from all variables. Note that the following formulation was needed only in Mplus to allow for polarity of the person-specific innovation covariances/correlations.

As presented in the within-level part of the Mplus code, we introduced two "phantom" latent variables that were essentially equated to the observed variables (see, `f1 by y1@1; f2 by y2@1;`), and allowed the random innovation covariances to be modeled by regressing `f1` on `f2` (see, `s|f1 on f2;`). Following this parameterization, the log residual variance of `f1` and variance of `f2`, labeled respectively as `v1` and `v2` in our code (see, `v1|f1; v2|f2;`), could be linked to random innovation variances in the original model via the parameter transformation procedures described below.

First, the regression model could be summarized as:

$$\begin{aligned} f_1 &= sf_2 + \zeta_{f_1},\ \zeta_{f_1} \sim N(0, e^{v_1}) \\ f_2 &= \zeta_{f_2},\ \zeta_{f_2} \sim N(0, e^{v_2}) \end{aligned} \tag{1}$$

where f_1 and f_2 were the two latent variables; f_1 was regressed on f_2 with a regression coefficient s; ζ_{f_1} and ζ_{f_2} were residuals, the variances of which were e^{v_1} and e^{v_2}, respectively (here v_1 and v_2 represented `v1` and `v2` in the code). Then, the variances of f_1 and f_2, and the covariance between f_1 and f_2 could be obtained as:

$$\begin{aligned} Var(f_2) &= Var(\zeta_{f_2}) = e^{v_2} \\ Var(f_1) &= Var(s\zeta_{f_2} + \zeta_{f_1}) = s^2 Var(\zeta_{f_2}) + Var(\zeta_{f_1}) = s^2 e^{v_2} + e^{v_1} \\ cov(f_1, f_2) &= cov[(s\zeta_{f_2} + \zeta_{f_1}), \zeta_{f_2}] = E[(s\zeta_{f_2} + \zeta_{f_1})\zeta_{f_2}] = E(s\zeta_{f_2}^2) = sVar(\zeta_{f_2}) = se^{v_2} \end{aligned} \tag{2}$$

In addition to the above regression model linking two latent variables, f_1 and f_2, there were also measurement models linking f_1 and f_2 to observed variable, y_1 and y_2, respectively, which could be summarized as:

$$\begin{aligned} y_1 &= f_1 + e_1,\ e_1 \sim N(0, 0.2) \\ y_2 &= f_2 + e_2,\ e_2 \sim N(0, 0.2) \end{aligned} \tag{3}$$

where e_1 and e_2 were measurement errors related to observed variables, following a normal distribution with zero mean and variance of 0.2 (see, `y1@0.2; y2@0.2;` in the code example). Therefore, the random innovation variances (i.e., $\sigma^2_{1,i}$ and $\sigma^2_{2,i}$ in Equation 5 in the main manuscript) were equal to $Var(f_1) + 0.2$ and $Var(f_2) + 0.2$, respectively, and the random innovation covariance was equal to $cov(f_1, f_2)$ (see Equation 2).

Since we have obtained the random innovation variances and the covariance, we can then obtain the random innovation correlation (i.e., r_i in Equation 5 in the main manuscript), transform them to $\log(\sigma_{1,i})$, $\log(\sigma_{2,i})$, and Fisher-z(r_i), and regress the transformed values on the two predictors as presented in the between-level model. The R code example of these transformations and regression procedures is available in our GitHub repository (see the "MLVAR_Mplus_Code.R" file).

As $Var(e_1)$ and $Var(e_2)$ approached zero, the formulation in Equation 1 could be regarded as an alternative (albeit somewhat redundant) parameterization of random innovation variances and covariances/correlations in the original mlVAR model. In MCMC sampling, however, it is not possible to force $Var(e_1)$ and $Var(e_2)$ to be exactly zero, so we resorted to constraining these variances to a small, positive constant value, 0.2.