Reconciled Estimates of Monthly GDP in the US*

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SUPPLEMENTARY ONLINE MATERIAL FOR "RECONCILED ESTIMATES OF MONTHLY GDP IN THE US"

This appendix comprises 4 parts: Appendix A is a technical appendix, Appendix B is the data appendix, Appendix C contains additional empirical results, and Appendix D contains supplementary tables.

A Technical Appendix

A.1 Model details and priors

A.1.1 ADNSS model in structural VAR form

The model of Sub-section 2.2 of Aruoba, Diebold, Nalewaik, Schorfheide, and Song (2016) [ADNSS], sets Σ to be:

$$\begin{bmatrix} \sigma_{GG}^2 & 0 & 0 \\ 0 & \sigma_{EE}^2 & \sigma_{EI} \\ 0 & \sigma_{EI} & \sigma_{II}^2 \end{bmatrix}.$$

Take an LDL decomposition of Σ, where D is diagonal and L is lower triangular. If we then multiply both sides of equations (3) and (4) in ADNSS by L^{-1} and re-arrange so that the LHS of each equation contains all variables with t subscripts and RHS variables are all lags, we get the SVAR form with A being:

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 - a_{32} & a_{32} & 1 \end{bmatrix}, \tag{A.1}$$

where a_{32}, the 3,2-th element of A, is a function of the elements of Σ. The key property is that the two off-diagonal elements in the bottom row of A sum to -1.

A.1.2 Prior for quarterly VAR containing only GDP variables

The following prior is bounded to ensure $0.55 < \xi_E, \xi_I < 1.15$:

1. $a_{21}, a_{31}, a_{32} \sim N(0, 10)$.
2. $\mu \sim N(0, 100)$, $b_{11} \sim N(0, 10)$.
3. $\sigma_{GG}^2, \sigma_{EE}^2, \sigma_{II}^2 \sim IG(3.8, 8.4)$. The inverse gamma prior mean is 3 and variance is 5.

A.1.3 Prior for quarterly VAR containing unemployment and GDP variables

The following prior is bounded to ensure $0.55 < \xi_E, \xi_I < 1.15$:

1. Ta$a_{21} \sim N(0.5, 1)$,$a_{32}, a_{42} \sim N(-1, 0.1)$ and $a_{43} \sim N(0, 1)$.

2. $\mu, \mu_b \sim N(0, 100)$, $b_{11}, b_{12}, b_{21}, b_{22} \sim N(0, 10)$.

3. $\sigma^2_{UU}, \sigma^2_{GG}, \sigma^2_{EE}, \sigma^2_{II} \sim IG(3.8, 8.4)$. The inverse gamma prior mean is 3 and variance is 5.

A.1.4 Choice for A and priors for MF-VAR containing GDP variables, unemployment and other monthly variables

Our MF-VARs contain the GDP variables and the same eight monthly variables as in Schorfheide and Song (2015). The monthly variables are a measure of hours worked (awh_t), inflation (π_t), industrial production (ip_t), personal consumption expenditures (pce_t), short-term interest rates (r_t), long-term interest rates (r_t^{GS10}), stock prices (st_t), and the unemployment rate (U_t). Exact definitions and data transformations are given below in the Data Appendix.

The part of B defining a VAR for true GDP and these monthly variables is unrestricted. The part of B relating to the relationship between $\mathrm{GDP_E}$, $\mathrm{GDP_I}$ and true GDP is restricted as in Sub-section 4.2 of the main paper.

A.1.5 Model that imposes restriction that all monthly variables are instruments

The left-hand side of the MF-VAR for this model takes the following form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 1 & 0 & 0 & 0 & 0 \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & 1 & 0 & 0 & 0 \\ a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97} & a_{98} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{109} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{119} & a_{1110} & 1 \end{bmatrix} \begin{bmatrix} awh_t \\ \pi_t \\ ip_t \\ pce_t \\ r_t \\ r_t^{GS10} \\ st_t \\ U_t \\ GDP_t \\ GDP_{E,t} \\ GDP_{I,t} \end{bmatrix}$$

We use notation where $\hat{a} = (a_{21}, a_{31}, \ldots, a_{95}, a_{98})'$ and $\tilde{a}$ are all the remaining coefficients in A, all the free coefficients in B and the intercepts in the MF-VAR. σ^2_{ii} denotes the error variance in equation i. The prior is:

1. $a_{109}, a_{119} \sim N(-1, 0.1)$ and $a_{1110} \sim N(0, 1)$.

2. $\tilde{a} \sim DL(\alpha)$ - α is the hyperparameter on the DL priors and is set to $\alpha = 0.5$.

3. $\hat{a} \sim DL(\bar{\alpha})$- $\bar{\alpha}$ is the hyperparameter on the DL priors and is set to $\bar{\alpha} = 0.5$.

4. $\sigma^2_{ii} \sim IG(5, .01)$.

The prior is bounded to ensure $0.55 < \xi_E, \xi_I < 1.15$.

Model that imposes noise restriction and the restriction that all monthly variables are instruments

The left-hand side of the MF-VAR for this model takes the following form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 1 & 0 & 0 & 0 & 0 \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & 1 & 0 & 0 & 0 \\ a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97} & a_{98} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 - a_{1110} & a_{1110} & 1 \end{bmatrix} \begin{bmatrix} awh_t \\ \pi_t \\ ip_t \\ pce_t \\ r_t \\ r_t^{GS10} \\ st_t \\ U_t \\ GDP_t \\ GDP_{E,t} \\ GDP_{I,t} \end{bmatrix}$$

We use notation where $\hat{a} = (a_{21}, a_{31}, \ldots, a_{95}, a_{98})'$ and $\tilde{a}$ are all the remaining coefficients in A, all the free coefficients in B and the intercepts in the MF-VAR. σ_{ii}^2 denotes the error variance in equation i. The prior is:

1. $a_{1110} \sim N(0, 1)$.
2. $\tilde{a} \sim DL(\alpha)$ - α is the hyperparameter on the DL priors and is set to $\alpha = 0.5$.
3. $\hat{a} \sim DL(\bar{\alpha})$- $\bar{\alpha}$ is the hyperparameter on the DL priors and is set to $\bar{\alpha} = 0.5$.
4. $\sigma_{ii}^2 \sim IG(5, .01)$.

The prior is bounded to ensure $0.55 < \xi_E, \xi_I < 1.15$.

Model that only imposes the restriction that unemployment is an instrument

The left-hand side of the MF-VAR for this model takes the following form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 1 & 0 & 0 & 0 & 0 \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & 1 & 0 & 0 & 0 \\ a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97} & a_{98} & 1 & 0 & 0 \\ a_{101} & a_{102} & a_{103} & a_{104} & a_{105} & a_{106} & a_{107} & 0 & a_{109} & 1 & 0 \\ a_{111} & a_{112} & a_{113} & a_{114} & a_{115} & a_{116} & a_{117} & 0 & a_{119} & a_{1110} & 1 \end{bmatrix} \begin{bmatrix} awh_t \\ \pi_t \\ ip_t \\ pce_t \\ r_t \\ r_t^{GS10} \\ st_t \\ U_t \\ GDP_t \\ GDP_{E,t} \\ GDP_{I,t} \end{bmatrix}$$

We use notation where $\hat{a} = (a_{21}, a_{31}, \ldots, a_{116}, a_{117})'$ and $\tilde{a}$ are all the remaining coefficients in A, all the free coefficients in B and the intercepts in the MF-VAR. σ_{ii}^2 denotes the error variance in equation i. The prior is:

1. $a_{109}, a_{119} \sim N(-1, 0.1)$ and $a_{1110} \sim N(0, 1)$.
2. $\tilde{a} \sim DL(\alpha)$ - α is the hyperparameter on the DL priors and is set to $\alpha = 0.5$.
3. $\hat{a} \sim DL(\bar{\alpha})$- $\bar{\alpha}$ is the hyperparameter on the DL priors and is set to $\bar{\alpha} = 0.5$.
4. $\sigma_{ii}^2 \sim IG(5, .01)$.

The prior is bounded to ensure $0.55 < \xi_E, \xi_I < 1.15$.

Model that imposes the noise restriction and the restriction that unemployment is an instrument

The left-hand side of the MF-VAR for this model takes the following form:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{51} & a_{52} & a_{53} & a_{54} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1 & 0 & 0 & 0 & 0 & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 1 & 0 & 0 & 0 & 0 \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & 1 & 0 & 0 & 0 \\
a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97} & a_{98} & 1 & 0 & 0 \\
a_{101} & a_{102} & a_{103} & a_{104} & a_{105} & a_{106} & a_{107} & 0 & -1 & 1 & 0 \\
a_{111} & a_{112} & a_{113} & a_{114} & a_{115} & a_{116} & a_{117} & 0 & -1-a_{1110} & a_{1110} & 1
\end{bmatrix}
\begin{bmatrix}
awh_t \\ \pi_t \\ ip_t \\ pce_t \\ r_t \\ r_t^{GS10} \\ st_t \\ U_t \\ GDP_t \\ GDP_{E,t} \\ GDP_{I,t}
\end{bmatrix}
$$

We use notation where $\hat{a} = (a_{21}, a_{31}, \ldots, a_{116}, a_{117})'$ and $\tilde{a}$ are all the remaining coefficients in A, all the free coefficients in B and the intercepts in the MF-VAR. σ_{ii}^2 denotes the error variance in equation i. The prior is:

1. $a_{1110} \sim N(0, 1)$.
2. $\tilde{a} \sim DL(\alpha)$ - α is the hyperparameter on the DL priors and is set to $\alpha = 0.5$.
3. $\hat{a} \sim DL(\bar{\alpha})$- $\bar{\alpha}$ is the hyperparameter on the DL priors and is set to $\bar{\alpha} = 0.5$.
4. $\sigma_{ii}^2 \sim IG(5, .01)$.

The prior is bounded to ensure $0.55 < \xi_E, \xi_I < 1.15$.

A.2 MCMC algorithm without mixed frequencies

In this section, we provide the details of the MCMC algorithm for the quarterly model with a single quarterly predictor. This algorithm can be easily extended to the models with many additional variables. Specifically, we can expand equation (1) of the main paper as:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
a_{21} & 1 & 0 & 0 \\
0 & a_{32} & 1 & 0 \\
0 & a_{42} & a_{43} & 1
\end{bmatrix}
\begin{bmatrix}
U_t \\ GDP_t \\ GDP_{E,t} \\ GDP_{I,t}
\end{bmatrix}
=
\begin{bmatrix}
\mu_{UE} \\ \mu_{GDP} \\ 0 \\ 0
\end{bmatrix}
+
\begin{bmatrix}
b_{11} & b_{12} & 0 & 0 \\
b_{21} & b_{22} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
U_{t-1} \\ GDP_{t-1} \\ GDP_{E,t-1} \\ GDP_{I,t-1}
\end{bmatrix}
+
\begin{bmatrix}
\epsilon_{U,t} \\ \epsilon_{G,t} \\ \epsilon_{E,t} \\ \epsilon_{I,t}
\end{bmatrix},
\tag{A.2}
$$

where

$$
\begin{bmatrix}
\epsilon_{U,t} \\ \epsilon_{G,t} \\ \epsilon_{E,t} \\ \epsilon_{I,t}
\end{bmatrix}
\sim N(
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0
\end{bmatrix},
\begin{bmatrix}
\sigma_{UU}^2 & 0 & 0 & 0 \\
0 & \sigma_{GG}^2 & 0 & 0 \\
0 & 0 & \sigma_{EE}^2 & 0 \\
0 & 0 & 0 & \sigma_{II}^2
\end{bmatrix}
). \tag{A.3}
$$

The preceding sub-section described the priors for A and the error variances. For the remaining parameters and initial conditions, we make relatively non-informative choices of: $\mu_{UE}, \mu_{GDP} \sim N(0, V_4)$, $b_{11}, b_{12}, b_{21}, b_{22} \sim N(0, V_5)$, $GDP_1 \sim N(0, V_{GDP})$ and $\sigma^2_{ii} \sim IG(\nu, S)$. We set the following hyperparameters: $V_4 = 100$, $V_5 = 10$, $V_{GDP} = 10$, $\nu = 3.8$ and $S = 8.4$. We can use the equation by equation method of Carriero et al. (2019) to sample all the parameters, and the Gibbs sampler is specified below. Note, for the models with more monthly variables, we use Dirichlet–Laplace (DL) priors on the VAR coefficients and covariance terms. Since this prior is conditionally Gaussian, the Gibbs sampler described below is mostly unchanged. All that is required are the additional steps to draw the DL parameters and this is carried out as detailed in the appendix of Koop et al. (2020).

A.3 Sample U_t equation

We can rewrite the first equation of the VAR in (A.2) as:

$$\mathbf{U} = \mathbf{X}\beta + \epsilon_U, \epsilon_U \sim N(0, \sigma^2_{UU}\mathbf{I}_T), \tag{A.4}$$

where

$$\mathbf{X} = \begin{bmatrix} 1 & U_0 & 0 \\ 1 & U_1 & GDP_1 \\ \vdots & \vdots & \vdots \\ 1 & U_{T-1} & GDP_{T-1} \end{bmatrix}.$$

Let $\mathbf{U} = (U_1, \ldots, U_T)'$ and $\beta = (\mu_{UE}, b_{11}, b_{12})'$. Combining (A.4) with the above specified priors and using the simple Bayesian linear regression formula, the conditional posterior for β is:

$$\beta | \bullet \sim N(\hat{\beta}, \mathbf{K}_\beta), \tag{A.5}$$

where $\mathbf{S}_1 = \text{diag}(V_4, V_5, V_5)$ and:

$$\mathbf{K}_\beta = (\tfrac{\mathbf{X}'\mathbf{X}}{\sigma^2_{UU}} + \mathbf{S}_1^{-1})^{-1}, \quad \hat{\beta} = \mathbf{K}_\beta(\tfrac{\mathbf{X}'\mathbf{U}}{\sigma^2_{UU}}).$$

Finally, the conditional posterior for σ^2_{UU} is:

$$\sigma^2_{UU} | \bullet \sim IG(\nu + \frac{T}{2}, S + \frac{(\mathbf{U} - \mathbf{X}\beta)'(\mathbf{U} - \mathbf{X}\beta)}{2}). \tag{A.6}$$

A.4 Sample GDP_t equation

We can rewrite the second equation of the VAR in (A.2) as:

$$GDP = \mathbf{Z}\theta + \epsilon_G, \epsilon_G \sim N(0, \sigma^2_{GG}\mathbf{I}_T), \tag{A.7}$$

where:

$$\mathbf{Z} = \begin{bmatrix} 1 & U_1 & GDP_1 & -U_2 \\ 1 & U_2 & GDP_2 & -U_3 \\ \vdots & \vdots & \vdots\vdots & \vdots \\ 1 & U_{T-1} & GDP_{T-1} & -U_T \end{bmatrix},$$

Let $GDP = (GDP_2, \ldots, GDP_T)'$ and $\theta = (\mu_{GDP}, b_{21}, b_{22}, a_{21})'$. Combining (A.7) with the above specified priors and using the simple Bayesian linear regression formula, the conditional posterior for θ is:

$$\theta|\bullet \sim N(\hat{\theta}, \mathbf{K}_\theta), \tag{A.8}$$

where $\mathbf{S}_2 = \text{diag}(V_4,V_5,V_5,V_1)$, $\delta = (0,0,0,\hat{a})'$ and:

$$\mathbf{K}_\theta = (\frac{\mathbf{Z}'\mathbf{Z}}{\sigma^2_{GG}} + \mathbf{S}_2^{-1})^{-1}, \quad \hat{\theta} = \mathbf{K}_\theta(\frac{\mathbf{Z}'GDP}{\sigma^2_{GG}} + \mathbf{S}_2^{-1}\delta).$$

Finally, the conditional posterior for σ^2_{GG} is:

$$\sigma^2_{GG}|\bullet \sim IG(\nu + \frac{T-1}{2}, S + \frac{(GDP - \mathbf{Z}\theta)'(GDP - \mathbf{Z}\theta)}{2}). \tag{A.9}$$

A.5 Sample $GDP_{E,t}$ equation

We can rewrite the third equation of the VAR in (A.2) as:

$$GDP_E = \mathbf{W}a_{32} + \epsilon_E, \epsilon_E \sim N(0, \sigma^2_{EE}\mathbf{I}_T), \tag{A.10}$$

where:

$$\mathbf{W} = \begin{bmatrix} -GDP_1 \\ -GDP_2 \\ \vdots \\ -GDP_T \end{bmatrix},$$

and $GDP_E = (GDP_{E,1}, \ldots, GDP_{E,T})'$. Combining (A.10) with the above specified priors and using the simple Bayesian linear regression formula, the conditional posterior for a_{32} is:

$$a_{32}|\bullet \sim N(\hat{a_{32}}, \mathbf{K}_{a_{32}}), \tag{A.11}$$

where:

$$\mathbf{K}_{a_{32}} = (\frac{\mathbf{W}'\mathbf{W}}{\sigma^2_{EE}} + V_2^{-1})^{-1}, \quad \hat{\theta} = \mathbf{K}_{a_{32}}(\frac{\mathbf{W}'GDP_E}{\sigma^2_{EE}} + V_2^{-1}\tilde{a}).$$

Finally, the conditional posterior for σ^2_{EE} is:

$$\sigma^2_{EE}|\bullet \sim IG(\nu + \frac{T}{2}, S + \frac{(GDP_E - \mathbf{W}a_{32})'(GDP_E - \mathbf{W}a_{32})}{2}). \tag{A.12}$$

A.6 Sample $GDP_{I,t}$ equation

We can rewrite the fourth equation of the VAR in (A.2) as:

$$GDP_I = \mathbf{M}\gamma + \epsilon_I, \epsilon_I \sim N(0, \sigma_{II}^2 \mathbf{I}_T), \tag{A.13}$$

where:

$$\mathbf{M} = \begin{bmatrix} -GDP_1 & -GDP_{E,1} \\ -GDP_2 & -GDP_{E,2} \\ \vdots & \vdots \\ -GDP_T & -GDP_{E,T} \end{bmatrix},$$

$GDP_I = (GDP_{I,1}, \ldots, GDP_{I,T})'$ and $\gamma = (a_{42}, a_{43})'$. Combining (A.13) with the above specified priors and using the simple Bayesian linear regression formula, the conditional posterior for γ is:

$$\gamma | \bullet \sim N(\hat{\gamma}, \mathbf{K}_\gamma), \tag{A.14}$$

where $\mathbf{S}_4 = \text{diag}(V_2, V_3)$, $\tilde{\delta} = (\tilde{a}, 0)'$ and:

$$\mathbf{K}_\gamma = (\tfrac{\mathbf{M}'\mathbf{M}}{\sigma_{II}^2} + \mathbf{S}_4^{-1})^{-1}, \quad \hat{\gamma} = \mathbf{K}_\gamma(\tfrac{\mathbf{M}'GDP_I}{\sigma_{II}^2} + \mathbf{S}_4^{-1}\tilde{\delta}).$$

Finally, the conditional posterior for σ_{II}^2 is:

$$\sigma_{II}^2 | \bullet \sim IG(\nu + \frac{T}{2}, S + \frac{(GDP_I - \mathbf{M}\gamma)'(GDP_I - \mathbf{M}\gamma)}{2}). \tag{A.15}$$

A.7 Sample GDP_t

In our model, GDP_t is an unobserved latent variable and here we provide details on sampling this latent variable. First, we rewrite (A.2) as a combination of state and measurement equations:

$$\tilde{\mathbf{y}} = \tilde{\mathbf{X}} GDP + \eta, \eta \sim N(0, \Omega), \tag{A.16}$$

where $\tilde{\mathbf{y}} = (GDP_{E,1}, GDP_{I,1} + a_{43}GDP_{E,1}, \ldots, GDP_{E,T}, GDP_{I,T} + a_{43}GDP_{E,T})'$, $GDP = (GDP_1, \ldots, GDP_T)'$, $\tilde{\mathbf{X}} = \mathbf{I}_T \otimes [-a_{32}, a_{42}]'$, and $\Omega = \mathbf{I}_T \otimes \begin{bmatrix} \sigma_{EE}^2 & 0 \\ 0 & \sigma_{EE}^2 \end{bmatrix}$. The state equations can be defined as:

$$\mathbf{H} GDP = \tilde{\alpha} + \epsilon_G, \epsilon_G \sim N(0, \mathbf{S}_5), \tag{A.17}$$

where:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & & \cdots & 0 \\ -b_{22} & 1 & 0 & & & \\ 0 & -b_{22} & 1 & & & 0 \\ 0 & 0 & -b_{22} & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 1 & 0 \\ 0 & 0 & \cdots & 0 & -b_{22} & 1 \end{bmatrix}, \quad \tilde{\alpha} = \begin{bmatrix} 0 \\ \mu_{GDP} + b_{21}U_1 - a_{21}U_2 \\ \mu_{GDP} + b_{21}U_2 - a_{21}U_3 \\ \vdots \\ \vdots \\ \mu_{GDP} + b_{21}U_{T-1} - a_{21}U_T \end{bmatrix},$$

and $\mathbf{S}_5 = \mathrm{diag}(V_{GDP}, \sigma^2_{GG}, \ldots, \sigma^2_{GG})$.

Next let:

$$\tilde{\mathbf{U}} = \tilde{\mathbf{H}} GDP + \epsilon_U, \epsilon_U \sim N(0, \sigma^2_{UU}\mathbf{I}_T), \tag{A.18}$$

where:

$$\tilde{\mathbf{U}} = \begin{bmatrix} U_1 - U_0 b_{11} - \mu_{UE} \\ U_2 - U_1 b_{11} - \mu_{UE} \\ \vdots \\ \vdots \\ U_T - U_{T-1} b_{11} - \mu_{UE} \end{bmatrix}, \quad \tilde{\mathbf{H}} = \begin{bmatrix} 0 & \cdots & & 0 & 0 \\ b_{12} & 0 & & & \\ 0 & b_{12} & 0 & 0 & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & b_{12} & 0 \end{bmatrix}.$$

Therefore, combining (A.16), (A.17) and (A.18), the conditional posterior for GDP is:

$$GDP|\bullet \sim N(G\hat{D}P, \mathbf{K}_{GDP}), \tag{A.19}$$

where:

$$\mathbf{K}_{GDP} = (\tilde{\mathbf{X}}'\Omega^{-1}\tilde{\mathbf{X}} + \mathbf{H}'\mathbf{S}_5^{-1}\mathbf{H} + \tfrac{\tilde{\mathbf{H}}'\tilde{\mathbf{H}}}{\sigma^2_{UU}})^{-1}, \quad G\hat{D}P = \mathbf{K}_{GDP}(\tilde{\mathbf{X}}'\Omega^{-1}\tilde{\mathbf{y}} + \mathbf{H}'\mathbf{S}_5^{-1}\mathbf{H}\mathbf{H}^{-1}\tilde{\alpha} + \frac{\tilde{\mathbf{H}}'\tilde{\mathbf{U}}}{\sigma^2_{UU}}).$$

Since the precision matrix $\mathbf{K}_{GDP}$ is a band matrix, one can sample this conditional posterior efficiently using the algorithm proposed by Chan and Jeliazkov (2009).

A.8 MCMC for mixed frequency models

When model (A.2) is in mixed frequency, that is U_t is a monthly variable, and $GDP_{E,t}$ and $GDP_{I,t}$ are quarterly variables, the Gibbs sampler is unchanged except for the blocks that draw GDP_t, $GDP_{E,t}$ and $GDP_{I,t}$.

To draw the unobserved monthly GDP_t variable, we reparameterize the VAR in (A.2) in a state-space representation:

$$\mathbf{y}_t = \tilde{\mathbf{c}} + \tilde{\mathbf{B}}\mathbf{y}_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \Sigma), \tag{A.20}$$

where $\tilde{\mathbf{c}} = \mathbf{A}^{-1}\mathbf{c}$, $\tilde{\mathbf{B}} = \mathbf{A}^{-1}\mathbf{B}$, $\Lambda = \mathrm{diag}(\sigma^2_{UU}, \sigma^2_{GG}, \sigma^2_{EE}, \sigma^2_{II})$, and $\Sigma = \mathbf{A}^{-1}\Lambda\mathbf{A}^{-1\prime}$. Let m denote the number of monthly variables and q denote the number of quarterly variables. For example, in the ADNSS MF-VAR $m = 1$ and $q = 3$. We can further partition (A.20) into:

$$\mathbf{y}_t = \begin{bmatrix} \tilde{\mathbf{c}}_m \\ \tilde{\mathbf{c}}_q \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{B}}_{mm} & \tilde{\mathbf{B}}_{mq} \\ \tilde{\mathbf{B}}_{qm} & \tilde{\mathbf{B}}_{qq} \end{bmatrix} \mathbf{y}_{t-1} + \epsilon_t, \epsilon_t \sim N(0, \begin{bmatrix} \Sigma_{mm} & \Sigma_{mq} \\ \Sigma_{qm} & \Sigma_{qq} \end{bmatrix}). \tag{A.21}$$

Then our state-space representation is:

$$\mathbf{s}_t = \mathbf{F}_0 + \mathbf{F}_1\mathbf{s}_{t-1} + \Phi_{qm} + \zeta_t, \zeta_t \sim N(0, \Theta_1), \tag{A.22}$$

where $\Phi_{qm} = \begin{bmatrix} \tilde{\mathbf{B}}_{qm} y^m_{t-1} \\ \mathbf{0}_{(s-q)\times 1} \end{bmatrix}$,$y^m_{t-1} = (U_0, \ldots, U_{T-1})'$ is a vector that consists of all the lagged monthly variables, $\mathbf{s}_t = (GDP_t, GDP_{E,t}, GDP_{I,t}, \ldots, GDP_{t-4}, GDP_{E,t-4}, GDP_{I,t-4})'$ is $s \times 1$ vector, $\Theta_1 = \text{blkdiag}(\Sigma_{qq}, \mathbf{0}_{(s-q)\times(s-q)})$,[1] and:

$$\mathbf{F}_0 = \begin{bmatrix} \tilde{\mathbf{c}}_q \\ \mathbf{0}_{(s-q)\times 1} \end{bmatrix}, \mathbf{F}_1 = \begin{bmatrix} \tilde{\mathbf{B}}_{qq} & 0 & \cdots & 0 \\ \mathbf{I}_{(s-q)\times 1} & 0 & & \vdots \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & \mathbf{I}_{(s-q)\times 1} & 0 \end{bmatrix}.$$

Then we have two measurement equations, when both the monthly and quarterly variables are observed:

$$\hat{\mathbf{y}}_t = \mathbf{M}_1 \mathbf{s}_t + \Phi_{mm} + \upsilon_t, \upsilon_t \sim N(0, \Theta_2), \tag{A.23}$$

where $\Phi_{qm} = \begin{bmatrix} \tilde{\mathbf{B}}_{mm} y^m_{t-1} + \tilde{\mathbf{c}}_m \\ \mathbf{0}_{(q-1)\times 1} \end{bmatrix}$, $\Theta_2 = \text{blkdiag}(\Sigma_{mm}, \mathbf{0}_{(q-1)\times(q-1)})$,

$$\hat{\mathbf{y}}_t = \begin{bmatrix} U_t \\ GDP_{E,t} \\ GDP_{I,t} \end{bmatrix},$$

and:

$$\mathbf{M}_1 = \begin{bmatrix} \mathbf{0}_{1\times q} & \tilde{\mathbf{B}}_{mq} & 0 & 0 & \cdots & & & & 0 & 0 \\ \mathbf{0}_{1\times(q-1)} & \frac{1}{3}\mathbf{I}_{(q-1)} & \mathbf{0}_{1\times(q-1)} & \frac{2}{3}\mathbf{I}_{(q-1)} & \mathbf{0}_{1\times(q-1)} & \mathbf{I}_{(q-1)} & \mathbf{0}_{1\times(q-1)} & \frac{2}{3}\mathbf{I}_{(q-1)} & \mathbf{0}_{1\times(q-1)} & \frac{1}{3}\mathbf{I}_{(q-1)} \end{bmatrix}.$$

However, when only the monthly variable is observed, the measurement equation becomes:

$$\hat{\mathbf{y}}_t = \mathbf{M}_2 \mathbf{s}_t + \Phi_{mm} + \upsilon_t, \upsilon_t \sim N(0, \Theta_2), \tag{A.24}$$

where:

$$\mathbf{M}_2 = \begin{bmatrix} \mathbf{0}_{1\times q} & \tilde{\mathbf{B}}_{mq} & 0 & 0 & \cdots & & 0 & 0 \end{bmatrix}.$$

Finally, we can run the standard Kalman filtering and Carter and Kohn smoothing algorithm through (A.22), (A.23) and (A.24) to draw the monthly latent estimates for GDP_t, $GDP_{E,t}$ and $GDP_{I,t}$.

A.9 ADNSS+SS model with revisions

Here we set out the ADNSS+SS model when modeling both the first and second releases of GDP_E and GDP_I. The left-hand side of the MF-VAR for this model takes the following form:

[1] blkdiag denotes a block diagonal matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{91} & a_{92} & a_{93} & a_{94} & a_{95} & a_{96} & a_{97} & a_{98} & 1 & 0 & 0 & 0 & 0 \\ a_{101} & a_{102} & a_{103} & a_{104} & a_{105} & a_{106} & a_{107} & 0 & a_{109} & 1 & 0 & 0 & 0 \\ a_{111} & a_{112} & a_{113} & a_{114} & a_{115} & a_{116} & a_{117} & 0 & a_{119} & a_{1110} & 1 & 0 & 0 \\ a_{121} & a_{122} & a_{123} & a_{124} & a_{125} & a_{126} & a_{127} & 0 & a_{129} & a_{1210} & a_{1211} & 1 & 0 \\ a_{131} & a_{132} & a_{133} & a_{134} & a_{135} & a_{136} & a_{137} & 0 & a_{139} & a_{1310} & a_{1311} & a_{1312} & 1 \end{bmatrix} \begin{bmatrix} awh_t \\ \pi_t \\ ip_t \\ pce_t \\ r_t \\ r_t^{GS10} \\ st_t \\ U_t \\ GDP_t \\ GDP^1_{E,t} \\ GDP^2_{E,t} \\ GDP^1_{I,t} \\ GDP^2_{I,t} \end{bmatrix}.$$

In this model, we include both the first and the second release of $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$. $GDP^1_{E,t}$ and $GDP^2_{E,t}$ are denoted as the first and second release of $\mathrm{GDP_E}$, respectively. We denote the first and second estimates of $\mathrm{GDP_I}$ similarly. We use notation where $\hat{a} = (a_{21}, a_{31}, \dots, a_{116}, a_{137})'$ and $\tilde{a}$ are all the remaining coefficients in A, all the free coefficients in B and the intercepts in the MF-VAR. σ^2_{ii} denotes the error variance in equation i.

The prior is:

1. $a_{109}, a_{119}, a_{129}, a_{139} \sim N(-1, 0.1)$ and $a_{1110}, a_{1210}, a_{1211}, a_{1310}, a_{1311}, a_{1312} \sim N(0, 1)$.

2. $\tilde{a} \sim DL(\alpha)$ - α is the hyperparameter on the DL priors and is set to $\alpha = 0.5$.

3. $\hat{a} \sim DL(\bar{\alpha})$- $\bar{\alpha}$ is the hyperparameter on the DL priors and is set to $\bar{\alpha} = 0.5$.

4. $\sigma^2_{ii} \sim IG(5, .01)$.

Since we have two releases of $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$ in the model, we now have four $\xi^1_E, \xi^2_E, \xi^1_I, \xi^2_I$ and we accept each MCMC draw that satisfies the restriction:

$$0.55 < \frac{\xi^1_E + \xi^2_E + \xi^1_I + \xi^2_I}{4} < 2.$$

B Data Appendix

B.1 Data set for models with 8 monthly variables

All data were gathered from the McCracken and Ng (2016) FRED-MD database. The real time data were sourced from the FRED-MD and ALFRED databases.

Table B1: Data set for models with 8 monthly variables (plus the 2 quarterly variables)

Variables	FRED mnemonic	Frequency	Transformation
Avg weekly hours: Manufacturing	AWHMAN	Monthly	Level divided by 10
CPI: All Items	CPIAUCSL	Monthly	$\triangle lnx_t \times 100$
Industrial production	INDPRO	Monthly	$\triangle lnx_t \times 100$
Real personal consumption expenditures	DPCERA3M086SBEA	Monthly	$\triangle lnx_t \times 100$
Effective federal funds rate	FEDFUNDS	Monthly	Level
10-year Treasury rate	GS10	Monthly	Level
S&P's Common stock price index: Composite	S&P 500	Monthly	$\triangle lnx_t \times 100$
Civilian unemployment Rate	UNRATE	Monthly	$\triangle x_t \times 100$
Real gross domestic income	A261RX1Q020SBEA	Quarterly	$\triangle lnx_t \times 400$
Real gross domestic product	GDPC1	Quarterly	$\triangle lnx_t \times 400$

B.2 Data set for models with 48 monthly variables

All data were gathered from the McCracken and Ng (2016) FRED-MD database. In regard to the real-time data, we sourced them from both the FRED-MD and the ALFRED database. The 48 monthly variables were designed to span 6 categories: 1) industrial production/economic activity indicators - 19 variables; 2) employments indicators - 10 variables; 3) inflation indicators - 9 variables; 4) financial indicators - 5 variables; 5) stock market indicators - 3 variables; 6) exchange rate - 2 variables. Along with the 2 quarterly GDP variables, GDP_E and GDP_I, listed in Table B1, these 48 monthly variables comprise the 50-variable big data VAR model.

Table B2: Data set for models with 48 monthly variables

Variables	FRED mnemonic	Transformation
Industrial Production	INDPRO	$\triangle ln x_t \times 100$
Real personal consumption expenditures	DPCERA3M086SBEA	$\triangle ln x_t \times 100$
Real Personal Income	RPI	$\triangle ln x_t \times 100$
Real Manu. and Trade Industries Sales	CMRMTSPLx	$\triangle ln x_t \times 100$
Retail and Food Services Sales	RETAILx	$\triangle ln x_t \times 100$
IP: Final Products and Nonindustrial Supplies	IPFPNSS	$\triangle ln x_t \times 100$
IP: Final Products (Market Group)	IPFINAL	$\triangle ln x_t \times 100$
IP: Consumer Goods	IPCONGD	$\triangle ln x_t \times 100$
IP: Durable Consumer Goods	IPDCONGD	$\triangle ln x_t \times 100$
IP: Nondurable Consumer Goods	IPNCONGD	$\triangle ln x_t \times 100$
IP: Business Equipment	IPBUSEQ	$\triangle ln x_t \times 100$
IP: Materials	IPMAT	$\triangle ln x_t \times 100$
IP: Durable Materials	IPDMAT	$\triangle ln x_t \times 100$
IP: Nondurable Materials	IPNMAT	$\triangle ln x_t \times 100$
IP: Manufacturing	IPMANSICS	$\triangle ln x_t \times 100$
IP: Residential Utilities	IPB51222S	$\triangle ln x_t \times 100$
IP: Fuels	IPFUELS	$\triangle ln x_t \times 100$
Avg Weekly Hours : Manufacturing	AWHMAN	Level divided by 10
Capacity Utilization: Manufacturing	CUMFNS	$\triangle ln x_t \times 100$
Civilian Labor Force	CLF16OV	$\triangle ln x_t \times 100$
Civilian Employment	CE16OV	$\triangle ln x_t \times 100$
Civilians Unemployed - Less Than 5 Weeks	UEMPLT5	$\triangle ln x_t \times 100$
Civilians Unemployed for 5-14 Weeks	UEMP5TO14	$\triangle ln x_t \times 100$
Civilians Unemployed - 15 Weeks & Over	UEMP15OV	$\triangle ln x_t \times 100$
Civilians Unemployed for 15-26 Weeks	UEMP15T26	$\triangle ln x_t \times 100$
Civilians Unemployed for 27 Weeks and Over	UEMP27OV	$\triangle ln x_t \times 100$

Table B2: Data set for models with 48 monthly variables (cont.)

Variables	FRED mnemonic	Transformation
Initial Claims	CLAIMSx	$\triangle lnx_t \times 100$
PAYEMS	PAYEMS	$\triangle lnx_t \times 100$
PPI: Metals and metal products:	PPICMM	$\triangle lnx_t \times 100$
CPI : All Items	CPIAUCSL	$\triangle lnx_t \times 100$
CPI : Apparel	CPIAPPSL	$\triangle lnx_t \times 100$
CPI : Transportation	CPITRNSL	$\triangle lnx_t \times 100$
CPI : Medical Care	CPIMEDSL	$\triangle lnx_t \times 100$
CPI : Commodities	CUSR0000SAC	$\triangle lnx_t \times 100$
CPI : Durables	CUSR0000SAD	$\triangle lnx_t \times 100$
CPI : Services	CUSR0000SAS	$\triangle lnx_t \times 100$
Personal Cons. Expend.: Chain Index	PCEPI	$\triangle lnx_t \times 100$
Real M2 Money Stock	M2REAL	$\triangle lnx_t \times 100$
Effective Federal Funds Rate	FEDFUNDS	Level
10-Year Treasury Rate	GS10	Level
Moody's Aaa Corporate Bond Minus FEDFUNDS	AAAFFM	Level
Moody's Baa Corporate Bond Minus FEDFUNDS	BAAFFM	Level
US / UK Foreign Exchange Rate	EXUSUKx	$\triangle lnx_t \times 100$
Canada / US Foreign Exchange Rate	EXCAUSx	$\triangle lnx_t \times 100$
S&P's Common Stock Price Index: Industrials	S&P: indust	$\triangle lnx_t \times 100$
S&P's Composite Common Stock: Price-Earnings Ratio	S&P PE ratio	$\triangle lnx_t \times 100$
S&P's Common Stock Price Index: Composite	S&P 500	$\triangle lnx_t \times 100$
Civilian Unemployment Rate	UNRATE	$\triangle x_t \times 100$

C Further Empirical Results

C.1 Model comparison and properties of monthly GDP

The main goal of the paper is to produce historical monthly estimates of true GDP growth. Given that the BEA does not produce estimates of monthly true GDP, against which we might evaluate our estimates, we compare the estimates produced by the seven models of Table 1 (in the main paper) in various ways. We begin by taking the posterior median of historical monthly estimates of true GDP, $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$ from each estimated model and calculate various summary descriptive statistics. These are given in Table C1. The overall impression is that the different models produce monthly GDP estimates that have very similar time-series properties. This broadly holds true for all three of our monthly GDP estimates - true GDP, $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$ - when comparing across models.

One interesting difference between models can be seen in the means and medians they produce. In models that impose the noise restriction, true GDP must lie between $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$. However, without the noise restriction, this does not necessarily occur. We also see that true GDP is always less volatile than both $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$, except in the ADNSS $+SS^{+}$ model that includes 48 monthly indicators. The ADNSS+SS model, with just 8 monthly indicators, also delivers true GDP estimates with volatility closer to $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$ than the ADNSS model that considers unemployment only. In other words, consideration of additional monthly indicators does increase the relative volatility of the true GDP estimates. This is what we should expect, if these monthly indicators provide information about within-quarter economic dynamics.

In the ADNSS $+SS^{+}$ model with 48 additional monthly predictors, it can also be seen that true GDP, on average, is slightly higher than both $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$. In the ADNSS+SS model, which is the same as ADNSS $+SS^{+}$ except that these additional monthly predictors are excluded, in contrast, true GDP on average lies between $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$. Clearly, the additional monthly predictors are having an impact on our monthly GDP estimates. Inspection of the posterior median estimates for σ^2_{GG} also reveals the benefits of moving beyond consideration of monthly unemployment data alone: the ADNSS models offer poorer fit for the underlying monthly GDP equation than the SS models, with the exception of SS+. This provides tentative evidence to suggest that, in-sample at least, consideration of 48 rather than just 8 additional monthly indicators may not provide informational value-added for underlying GDP. However, we re-emphasize the clear conclusion from Table C1 that inference about historical GDP growth across the different models is very similar.

Comparing true GDP with $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$ we see from Table C1 that, across models, true GDP is always more negatively skewed than either $\mathrm{GDP_E}$ or $\mathrm{GDP_I}$. The dynamics of monthly GDP are also similar across models, with true GDP and $\mathrm{GDP_I}$ exhibiting slightly more persistence (as measured by the sample autocorrelations) than $\mathrm{GDP_E}$. True GDP and $\mathrm{GDP_I}$ have smaller AR(1) innovation variance and greater predictability as measured by the R^2 than $\mathrm{GDP_E}$. The final column of Table C1 reveals that our monthly estimates of true GDP are more highly correlated with our estimates of monthly $\mathrm{GDP_I}$ than with our estimates of monthly $\mathrm{GDP_E}$. This is understood by Table 1 (in the main paper) confirming that $\mathrm{GDP_I}$ is more important than $\mathrm{GDP_E}$ in explaining true GDP, explaining up to two-thirds of its

Table C1: Descriptive statistics for the posterior median monthly GDP estimates, by model

Model	Mean	Median	$\hat{\sigma}$	Skew	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	Q_{12}	$\hat{\sigma}_e$	R^2	$\widehat{\sigma}^2_{GG}$	corr
ADNSS+SS(IV)													
GDP_t	2.86	2.91	2.87	-0.39	0.92	0.74	0.53	0.39	1672.01	1.13	0.93	1.79	1.00
$GDP_{E,t}$	2.96	2.97	3.09	-0.28	0.90	0.67	0.43	0.30	1309.52	1.36	0.90		0.94
$GDP_{I,t}$	2.95	3.10	3.07	-0.37	0.92	0.73	0.52	0.38	1656.15	1.21	0.92		0.98
ADNSS+SS(IV+N)													
GDP_t	2.96	3.01	2.96	-0.38	0.92	0.74	0.53	0.39	1651.82	1.17	0.93	1.90	1.00
$GDP_{E,t}$	2.96	2.97	3.09	-0.28	0.90	0.67	0.44	0.30	1310.49	1.36	0.90		0.95
$GDP_{I,t}$	2.95	3.11	3.07	-0.37	0.92	0.73	0.52	0.38	1657.12	1.21	0.92		0.97
ADNSS+SS													
GDP_t	3.02	3.08	3.04	-0.39	0.92	0.74	0.53	0.39	1673.83	1.19	0.93	2.00	1.00
$GDP_{E,t}$	2.96	2.97	3.09	-0.28	0.90	0.67	0.44	0.30	1310.38	1.36	0.91		0.94
$GDP_{I,t}$	2.95	3.11	3.07	-0.37	0.92	0.73	0.52	0.38	1656.61	1.21	0.92		0.98
ADNSS+SS(N)													
GDP_t	2.96	3.01	2.96	-0.39	0.92	0.74	0.53	0.39	1666.00	1.17	0.93	1.89	1.00
$GDP_{E,t}$	2.96	2.97	3.09	-0.28	0.90	0.67	0.44	0.30	1309.86	1.36	0.90		0.94
$GDP_{I,t}$	2.95	3.11	3.07	-0.37	0.92	0.73	0.52	0.38	1656.80	1.21	0.92		0.98
ADNSS													
GDP_t	2.75	2.79	2.77	-0.44	0.92	0.74	0.54	0.40	1681.63	1.08	0.93	2.23	1.00
$GDP_{E,t}$	2.97	2.98	3.09	-0.32	0.90	0.67	0.44	0.30	1318.79	1.36	0.91		0.93
$GDP_{I,t}$	2.95	3.09	3.08	-0.41	0.92	0.73	0.52	0.38	1653.78	1.21	0.92		0.98
ADNSS(N)													
GDP_t	2.96	3.02	2.97	-0.43	0.92	0.74	0.53	0.39	1654.73	1.17	0.93	2.63	1.00
$GDP_{E,t}$	2.97	2.97	3.10	-0.32	0.90	0.67	0.44	0.30	1317.40	1.36	0.91		0.95
$GDP_{I,t}$	2.95	3.09	3.08	-0.41	0.92	0.73	0.52	0.38	1654.50	1.21	0.92		0.97
ADNSS+SS^+													
GDP_t	3.08	3.21	3.11	-0.40	0.92	0.74	0.53	0.39	1674.82	1.23	0.92	2.32	1.00
$GDP_{E,t}$	2.96	2.96	3.10	-0.29	0.90	0.67	0.43	0.29	1300.50	1.37	0.90		0.92
$GDP_{I,t}$	2.95	3.15	$+SS^+$3.08	-0.38	0.92	0.73	0.52	0.37	1643.02	1.23	0.92		0.99

Notes: The models and their features are summarized in Table 1. The sample period is 1960m1-2019m12. $\hat{\sigma}$ is the sample standard deviation. $\widehat{\rho}_1 - \widehat{\rho}_4$ are the sample autocorrelations at displacements of 1 to 4 months. Q_{12} is the Ljung–Box serial correlation test statistic calculated using $\widehat{\rho}_1, \ldots, \widehat{\rho}_{12}$. $R^2 = 1 - \frac{\hat{\sigma}_e^2}{\hat{\sigma}^2}$, where $\hat{\sigma}_e$ is the estimated disturbance standard deviation from a fitted AR(1) model. $\widehat{\sigma}^2_{GG}$ are the posterior median estimates for σ^2_{GG}. corr is the correlation coefficient against GDP_t.

variation.

Posterior evidence relating to the noise restriction can also be found in the models that do not impose it. Table C2 shows, for ξ_E, there is virtually no probability that it is above one. Thus, the noise restriction is found to hold for $\mathrm{GDP_E}$. However, for ξ_I in the unrestricted model, there is an appreciable probability that it is greater than one. This evidence that the measurement error in monthly $\mathrm{GDP_I}$ is at least in part news is consistent with the quarterly analysis in Fixler and Nalewaik (2010). Table C2 thus raises some doubts about whether it is sensible to impose the noise restriction. Thus our preference is for a model that allows for both news and noise.

Following Nalewaik (2010), we next calculate the correlations between our estimates of monthly true GDP growth and various other monthly business cycle indicators that should be correlated with true GDP but that are measured independently. These indicators are the industrial production index (IPI), the change in the unemployment rate, the Institute for Supply Management's Purchasing Managers Index (PMI) for manufacturing, employment growth, the S&P500 index and the Aruoba, Diebold, and Scotti (ADS) business conditions index (aggregated to a monthly frequency from the underlying daily index data). Again, for those indicators that are revised, we use June 2021 vintage data, and all monthly indicators are converted to quarter-on-quarter annualized changes except the PMI, which is analyzed in levels (as it is a balance statistic).

In addition, we consider the correlations against four alternative direct estimates of monthly GDP computed by Stock and Watson (2014), IHS Markit, the OECD, and BBK's estimates published at the Federal Reserve Bank of Chicago. All four monthly estimates are considered, like y_t^Q, as quarter-on-quarter annualized log changes. Stock and Watson's (2014) GDP estimates, available monthly through 2010m6, are computed as the geometric average of their monthly estimates of $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$. As real-time estimates are unavailable, we use the estimates accompanying their 2014 paper. IHS Markit, the global information provider, produces monthly GDP estimates from 1992m4 designed to be "an indicator of real aggregate output" and "whose variation at the quarterly frequency mimics that of official GDP" (see https://ihsmarkit.com/products/us-monthly-gdp-index.html), although we are unaware of the formal details of their methodology and any temporal aggregation constraints imposed. The OECD's monthly estimates of US real GDP are a leading indicator normalized to US GDP.[2] BBK uses a collapsed dynamic panel model of over 500 monthly indicators and quarterly $\mathrm{GDP_E}$ and, just like equation (6) in the main paper, ensures that the monthly GDP estimates temporally aggregate to the observed $\mathrm{GDP_E}$ data.[3]

From Table C3 it can be seen that the historical correlations with a given indicator are virtually identical across the seven models. This again points to the robustness of our historical estimates of monthly GDP. Reassuringly, we find an especially high correlation of our estimates of true GDP growth with the estimates produced by Stock and Watson (2014). While these estimates are also highly correlated, we see a slight drop in the correlation of our estimates with the monthly GDP estimates produced by BBK. This is understood when we recall that BBK focuses on consideration of $\mathrm{GDP_E}$ and neither exploits $\mathrm{GDP_I}$ data nor seeks to provide

[2]The OECD's monthly indicator for US GDP is available from FRED at https://fred.stlouisfed.org/series/USALORSGPNOSTSAM.

[3]See Box 2 of https://www.chicagofed.org/publications/economic-perspectives/2019/1.

Table C2: News versus noise by model: Posterior probabilities that ξ_E and ξ_I are greater than one, implying news

	$p(\xi_E > 1)$	$p(\xi_I > 1)$	$p(\xi_E > 1 \text{ and } \xi_I > 1)$
ANDSS+SS(IV)	0.00	0.01	0.00
ADNSS+SS(IV+N)	0.00	0.00	0.00
ADNSS+SS	0.01	0.37	0.01
ADNSS+SS(N)	0.00	0.00	0.00
ADNSS $+SS^{+}$	0.00	0.51	0.00
ADNSS	0.00	0.01	0.00
ADNSS(N)	0.00	0.00	0.00

Table C3: Correlation by model of the posterior median of monthly GDP_t growth with selected business cycle indicators and alternative estimates of monthly GDP growth (1960m1-2019m12)

	OECD	S&P500	IPI	Unemployment	PMI
ADNSS+SS(IV)	0.84	0.27	0.81	-0.68	0.68
ADNSS+SS(IV+N)	0.84	0.27	0.81	-0.68	0.68
ADNSS+SS	0.84	0.27	0.81	-0.68	0.68
ADNSS+SS(N)	0.84	0.27	0.81	-0.68	0.68
ADNSS	0.84	0.27	0.81	-0.68	0.68
ADNSS(N)	0.84	0.27	0.81	-0.68	0.68
ADNSS+SS^{+}	0.84	0.27	0.81	-0.68	0.68

	Employment	Stock Watson	IHS Markit	ADS Index	BBK
ADNSS+SS(IV)	0.70	0.95	0.52	0.77	0.93
ADNSS+SS(IV+N)	0.70	0.96	0.52	0.77	0.95
ADNSS+SS	0.70	0.96	0.52	0.77	0.93
ADNSS+SS(N)	0.70	0.96	0.52	0.77	0.94
ADNSS	0.70	0.95	0.53	0.78	0.92
ADNSS(N)	0.71	0.96	0.53	0.77	0.95
ADNSS+SS^{+}	0.71	0.95	0.53	0.77	0.95

Notes: All monthly indicators except PMI are analyzed in quarterly (quarter-over-quarter) annualized percent changes. PMI is analyzed in levels. Due to data availability, the correlations reported for Stock-Watson and IHS Markit are over the shorter sample periods of 1960m1-2010m6 and 1992m4-2019m12, respectively.

reconciled GDP estimates. Tables D4 and D5 in online Appendix D demonstrate that our models produce, reassuringly, monthly estimates of GDP_E that are almost perfectly correlated with those of BBK; and our monthly estimates of GDP_I are less strongly correlated (around 0.84) with BBK's estimates than our estimates of true GDP, which, as shown in Table C3, are correlated at least 0.93 with BBK's estimates.

Supplementary Table D6 in Appendix D shows that our monthly estimates, when aggregated to the quarterly frequency, correlate highly with the quarterly GDPplus estimates published by the Federal Reserve Bank of Philadelphia. Interestingly, our reconciled estimates of true GDP (when aggregated to a quarterly frequency to match GDPplus) are less correlated at 0.93 with GDPplus than our estimates of $\mathrm{GDP_I}$. This suggests that consideration of monthly indicators about the state of the economy is in effect lowering the weight on $\mathrm{GDP_I}$ in true GDP. Comparison of Table 1 (in the main paper) with ADNSS's reported estimate of $\lambda = 0.29$ on (quarterly) $\mathrm{GDP_E}$ confirms the view that when measuring monthly GDP, a higher weight should be placed on $\mathrm{GDP_E}$ and expenditure-side components of economic activity (as discussed in the main paper, this is when full-sample information is used - the narrative changes when the ragged edge is accommodated).

Table C3 indicates that our estimates of true GDP growth are less strongly correlated with the alternative estimates of monthly GDP produced by the OECD and IHS Markit, suggesting that these latter estimates are not designed to be consistent with quarterly GDP data. Turning to the correlations reported against the other macroeconomic indicators, we see that our monthly GDP estimates are highly positively correlated with industrial production and employment, and negatively correlated with unemployment. This is again reasonable and supports the plausibility of our estimates and, in turn, of our identification strategy.

C.2 Empirics: Further perspectives on monthly GDP

Having established that historical inference about true GDP is fairly robust to which model we consider, here we present additional results from our preferred model, the ADNSS+SS model, which imposes neither the noise restriction nor the restriction that all the monthly variables are instruments (but does impose the restriction that unemployment is an instrument). We choose this as the preferred model to focus on, given the empirical findings noted above. That is, the evidence in favor of the restrictions is not overwhelming. Since we are finding a high degree of robustness in that our seven models are producing similar estimates, we choose not to impose the restrictions. However, there is little evidence that moving from 8 to 48 monthly predictors improves our estimates and it does increase the computational burden substantially and, hence, we do not use the ADNSS+SS+ model.

Figures 4 and 5 (in the main paper) plot the monthly estimates of the three GDP variables. It can be seen that the lines tend to follow each other, with true GDP tending to lie between the estimates of $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$, but there are some exceptions to this pattern. Note also that the credible intervals are quite narrow, indicating accurate estimation.

The ADNSS+SS model has a large number of parameters. For the sake of brevity, we do not present posterior information about all of them. We are particularly interested in the noise restriction and the restriction that all the monthly variables are instruments. Given the way we have ordered the variables in the MF-VAR, these restrictions relate to its tenth and eleventh equations. Thus, Table C4 presents results for these two rows in the A matrix. The noise restriction implies $a_{10,9} = -1$ and $a_{11,9} = -1 + a_{11,10}$. It can be seen that the point estimates are not too far from both restrictions. But, particularly for the noise restriction, the posterior allocates enough weight away from the restriction, which accounts for the small differences between the restricted and unrestricted models noted in the preceding sub-section.

Table C4: Posterior estimates of key parameters in A from the ADNSS+SS model

Parameter	Median	16th quantile	84th quantile
$a_{10,1}$	0.00	-0.01	0.01
$a_{10,2}$	0.00	-0.07	0.04
$a_{10,3}$	0.00	-0.03	0.03
$a_{10,4}$	0.00	-0.06	0.03
$a_{10,5}$	0.00	-0.01	0.00
$a_{10,6}$	0.00	0.00	0.01
$a_{10,7}$	0.00	-0.01	0.01
$a_{11,1}$	0.00	0.00	0.00
$a_{11,2}$	0.00	-0.01	0.01
$a_{11,3}$	0.00	0.00	0.00
$a_{11,4}$	0.00	0.00	0.00
$a_{11,5}$	0.00	0.00	0.00
$a_{11,6}$	0.00	0.00	0.00
$a_{11,7}$	0.00	0.00	0.00
$a_{10,9}$	-0.96	-1.08	-0.88
$a_{11,9}$	-1.51	-1.74	-1.32
$a_{11,10}$	0.56	0.42	0.71

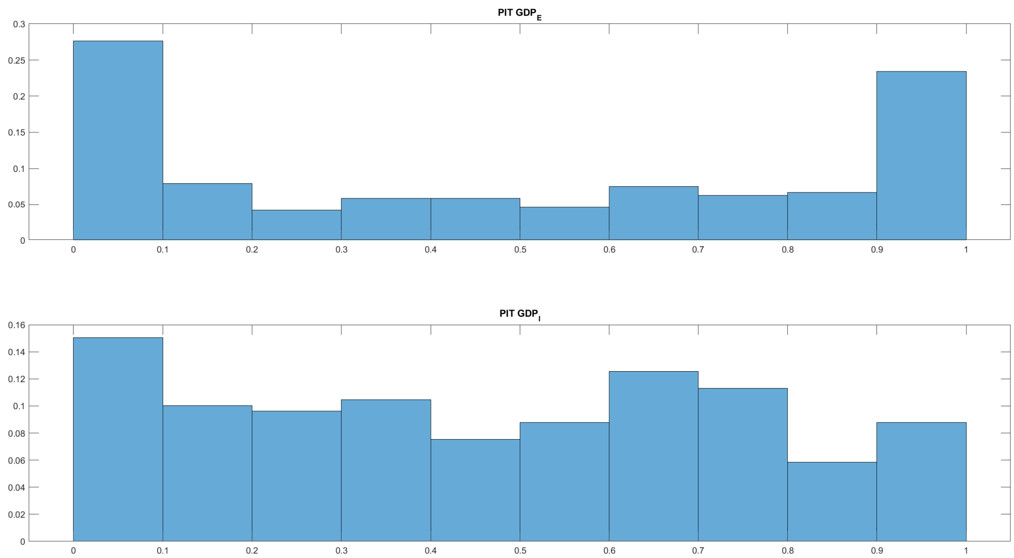
The posterior medians of all coefficients on the monthly variables in these two equations are zero to two decimal places, indicating that these variables are not strong instruments. Many of them have been shrunk to be extremely close to zero by the Dirichlet-Laplace prior. However, a small number of them have some posterior weight away from zero. Overall, these findings indicate that it is sensible to work with a monthly VAR involving true GDP and the monthly indicators. Adding GDP_E and GDP_I to this VAR will not improve its explanatory power.

It is, of course, not possible to compare our estimates of true monthly GDP directly to monthly GDP (or monthly GDP_E or GDP_I), since none of these concepts are measured by the BEA. But it is informative to turn our monthly posterior density estimates for true GDP into quarterly posteriors, and then see how well they match with (the observed) quarterly GDP_E and GDP_I data from the BEA.[4] This is achieved using the probability integral transform (PIT) histograms shown in Figure C1. These PITs are computed by integrating the posterior density for true GDP at time t up to the realized value of GDP_{Et} and GDP_{It}. The PITs will be uniformly distributed when the densities for true GDP_t equal those of GDP_{Et} or GDP_{It}. It can be seen that for GDP_E, the PITs depart from uniformity, placing extra weight in the tails. This sheds some light on the dispersion of the posteriors for true GDP at each point in time. Because observed GDP_E is often found to be in the tails of the posterior density of true GDP, this indicates its volatility is greater than that for true GDP (as supported by Table C1 above). This is not true to the same extent for GDP_I. This is again as we should expect, given our findings relating to the noise restriction (seen in Table C2). That is, GDP_E satisfies

[4] Again we continue to use recent vintage data for this historical analysis.

the noise restriction, which implies that it should be more volatile than true GDP. However, for $\mathrm{GDP_I}$ there is less evidence in favor of the noise restriction. Hence the densities of true GDP and $\mathrm{GDP_I}$ are more similar.

Figure C1: Probability integral transforms (PIT) histograms at a quarterly frequency for the true GDP density from the ADNSS+SS model, using quarterly $\mathrm{GDP_E}$ (top panel) and $\mathrm{GDP_I}$ (bottom panel) data from the BEA as the realizations



C.3 ADNSS+SS model with stochastic volatility

In this appendix we present: i) graphical output from the ADNSS+SS model extended to accommodate stochastic volatility (SV) and ii) tabular output summarizing the posterior median estimates of $p(\xi_E > 1)$ and $p(\xi_I > 1)$ from the ADNSS+SS model. This output is referenced in the main body of the paper.

First, we summarize how we extend the ADNSS+SS model to accommodate stochastic volatility (SV). Then we report some graphical evidence supporting our claim in the main paper that the historical properties of the monthly GDP estimates from the ADNSS+SS model are little affected by the inclusion of SV (see, in particular, Figure C3). In turn, this implies little time-variation in the relative importance of $\mathrm{GDP_E}$ (top panel) and $\mathrm{GDP_I}$, as shown in Figure C4. However, Figure C6 (building on Figure C5) shows that accommodating SV does introduce time variation in the posterior estimates for $p(\xi_E > 1)$ and $p(\xi_I > 1)$. Finally, in Table C5 we show that estimation of the ADNSS+SS model (without SV) over more recent samples of data tends to increase the news component to $\mathrm{GDP_E}$, even though the properties of true monthly GDP (our focus) are indistinguishable.

We preserve all model assumptions of the ADNSS+SS model, as stated above and in the main paper, and extend to allow for SV as follows. The $i-th$ equation of the VAR becomes:

$$y_{i,t} = X_{i,t}\beta_i + \epsilon_{i,t}, \epsilon_t \sim N(0, e^{h_{i,t}}),$$

$$h_{i,t} = h_{i,t-1} + v_t, v_t \sim N(0, \sigma^2_{h_i}),$$

where $y_{i,t}$ is the $i-th$ variable of the VAR and $h_{i,t}$ is the $i-th$ variable log-volatility, which follows a standard random walk process. We impose an inverse-gamma prior for $\sigma^2_{h_i} \sim IG(5, .01)$ across all the variables. We implement the precision-based auxiliary-mixture sampler algorithm of Chan and Hsiao (2014) to draw the log-volatilities of each variable within the Gibbs sampler. We do not impose SV in the errors of either the $GDP_{E,t}$ or the $GDP_{I,t}$ equations, that is, we maintain the assumption that $\epsilon_{GDP_{E,t}} \sim N(0, \sigma^2_{EE})$ and $\epsilon_{GDP_{I,t}} \sim N(0, \sigma^2_{II})$. All other variables, including true GDP_t, in the model assume SV processes for their errors. As a result, the variance of true GDP_t, $\sigma^2_{GG,t}$, will be time varying, and this will result in both $\xi_{E,t}$ and $\xi_{I,t}$ being time varying too. We achieve set identification by accepting each MCMC draw that satisfies the restriction:

$$0.55 < \bar{\xi}_E, \bar{\xi}_I < 1.15,$$

where $\bar{\xi}_E = \frac{1}{T}\sum_{t=1}^{T} \xi_{E,t}$ and $\bar{\xi}_I = \frac{1}{T}\sum_{t=1}^{T} \xi_{I,t}$.

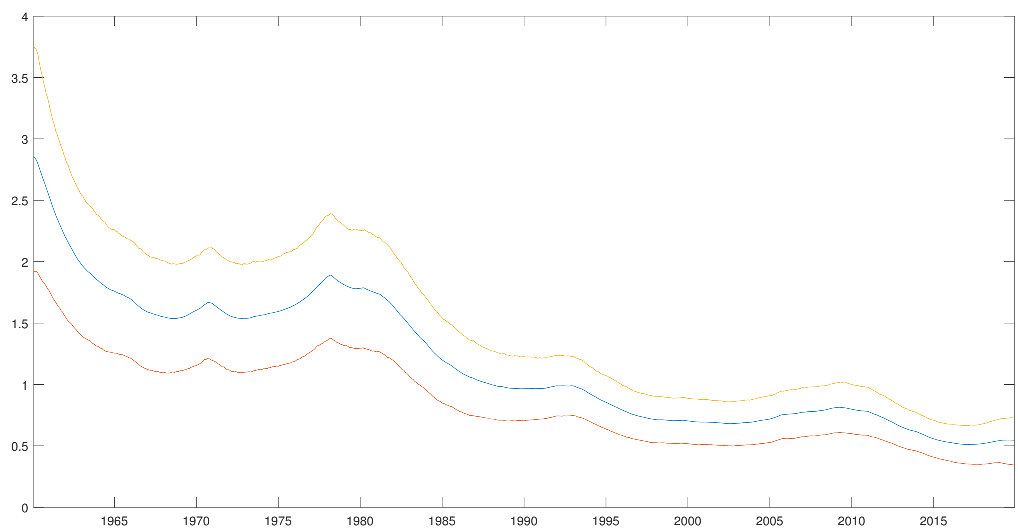


Figure C2: Posterior median (and 68 percent credible interval) estimates of SV for true GDP

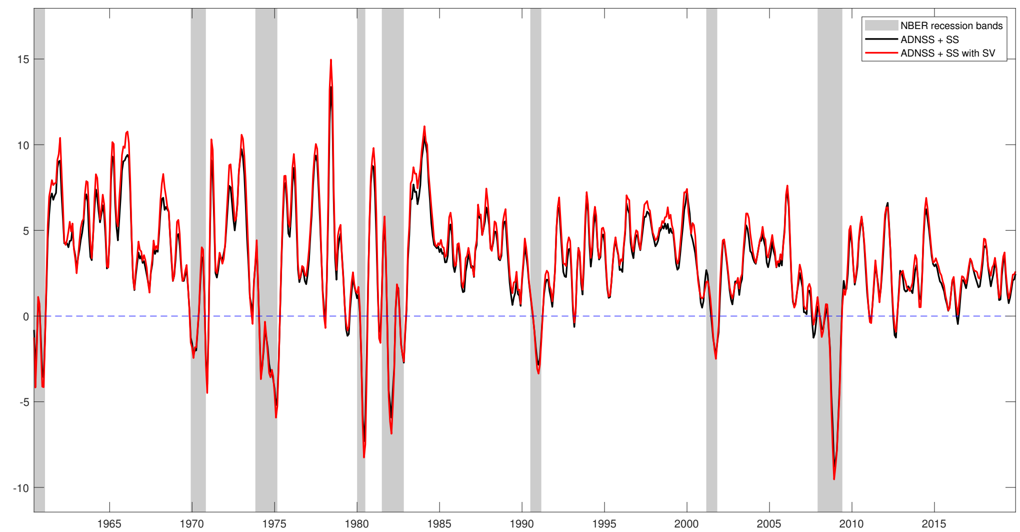


Figure C3: Posterior median estimates for monthly true GDP from the ADNSS+SS model with (and without) SV

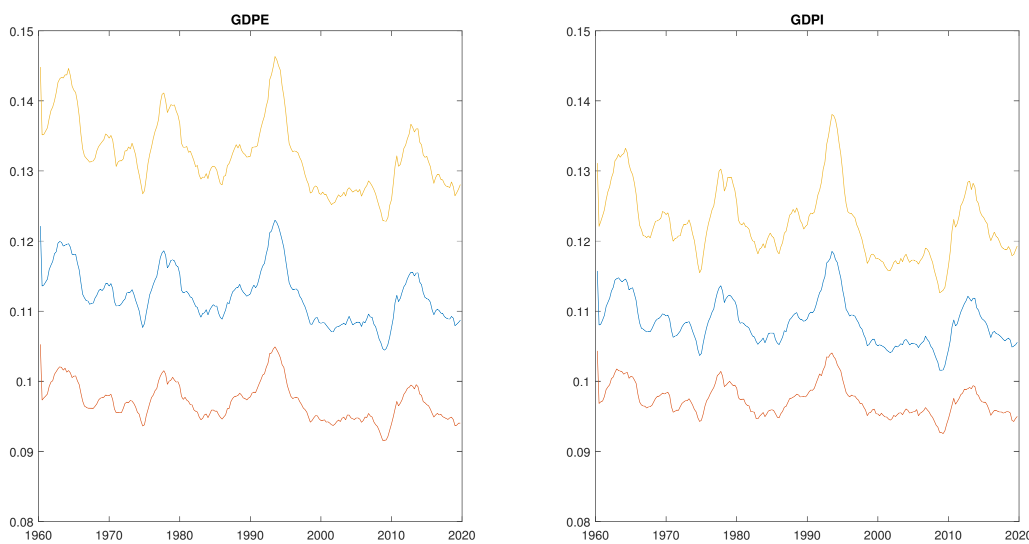


Figure C4: Time-varying Kalman gains for GDP_E and GDP_I from the ADNSS+SS model with SV: posterior medians (in blue) with 68 percent credible intervals (in yellow and red)

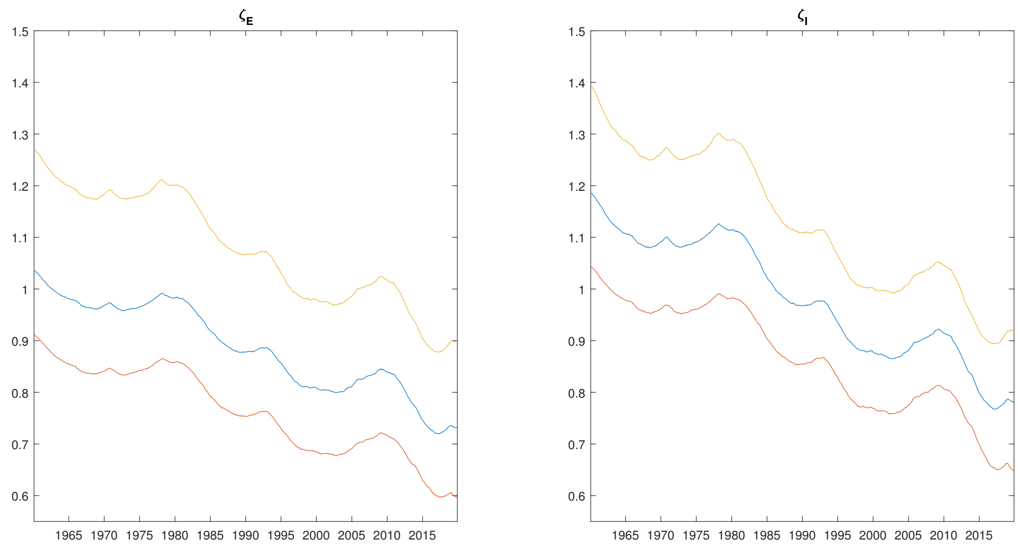


Figure C5: Time-varying posterior medians in blue (with 68 percent credible intervals) of ζ_E and ζ_I from the ADNSS+SS model with SV

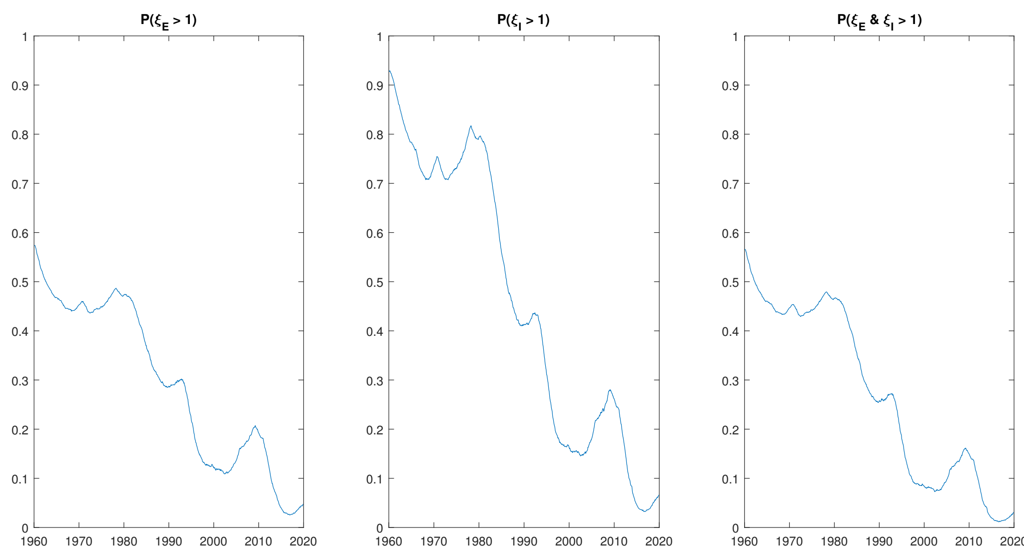


Figure C6: Time-varying probabilities $p(\xi_E > 1)$, $p(\xi_I > 1)$ and $p(\xi_E > 1 \text{and} \xi_I > 1)$ from the ADNSS+SS model with SV

C.4 Historical business cycles

To illustrate further the utility of our monthly estimates of true GDP, we consider their ability to capture historical US business cycles as assessed by the NBER Business Cycle

Table C5: Posterior median estimates of $p(\xi_E > 1)$ and $p(\xi_I > 1)$ from the ADNSS+SS model when estimated over different sample periods

	$p(\xi_E > 1)$	$p(\xi_I > 1)$
1960-2019	0.01	0.37
1985-2007	0.30	0.07
2008-2019	0.25	0.02
1960-1999	0.00	0.29
2000-2019	0.52	0.00

Dating Committee. An attraction of our Bayesian modeling approach is that probabilities of recession can be readily computed from our density estimates of monthly GDP. We proceed as follows. For each MCMC draw, we focus on monthly predictive estimates for GDP growth, y_t^Q (expressed as a quarterly change via (6)) and use these draws for y_t^Q to date business cycle turning points. Specifically, we classify "recessions" and "expansions" non–parametrically like Berge and Jordà (2011) and Brave et al. (2019). This involves relating our historical estimates of monthly GDP, y_t^Q, from 1960m1 through 2019q4, to the NBER recession dates and finding the "optimal" threshold, c, such that a recession is declared for month t when $y_t^Q < c$. We define the optimal threshold value as that c that maximizes the area under the receiver operating characteristic curve (AUC) giving equal weight to false positive and false negative signals.[5] By performing this exercise across MCMC draws for y_t^Q, and computing the fraction of draws where $y_t^Q < c$, we produce full-sample recession probabilities acknowledging the uncertainty about y_t^Q. We do so using our monthly estimates for GDP_{Et} and GDP_{It} as well as true GDP_t.

We plot these recession probabilities for the ADNSS+SS model in Figure C7. For expositional parsimony, and to reflect the empirical findings in favor of a model that allows for noise and news, we focus here on the ADNSS+SS model. Alongside, for comparison, we plot the recession probability estimates maintained by Jeremy Piger.[6] For consistency with our estimates, we use Piger's end of March 2020 vintage estimates that date back to 1967m6. Rather than using a non–parametric dating algorithm to define recessions and expansions, Piger calculates probability estimates from a dynamic factor Markov-switching model developed by Chauvet (1998) applied to four monthly variables. Chauvet and Piger (2008) analyze the performance of this model for dating recessions.

Figure C7 shows that the recession signals from the monthly ADNSS+SS model align well with NBER recessions. While the probabilities of recession do rise during NBER recessions (although they fall somewhat short of one in 1991 and 2001 and they also experience some volatility, sometimes falling after rising), what is relevant for our purposes is that the strength of the signal varies depending on whether one consults true GDP, $\mathrm{GDP_I}$, or $\mathrm{GDP_E}$. The recessionary probabilities based on true GDP, $\mathrm{GDP_I}$, and $\mathrm{GDP_E}$ often differ, with false signals most evident when one consults $\mathrm{GDP_I}$ or $\mathrm{GDP_E}$ alone. This impression is confirmed when in Table C6 we follow Berge and Jordà (2011) and Brave et al. (2019) and formally evaluate

[5]This corresponds to choosing c to maximize the Youden index.

[6]See https://pages.uoregon.edu/jpiger/us_recession_probs.htm/.

the classification ability of the different recession probability estimates seen in Figure C7 by reporting their AUC values.

Table C6 shows that dating NBER recessions using true GDP delivers 94 percent accuracy. Only 6 percent of estimates of true monthly GDP are *ambiguous*, that is, consistent with both a recessionary and an expansionary classification. But accuracy drops to 92 percent when using $\mathrm{GDP_I}$ or $\mathrm{GDP_E}$ alone: reconciling the information in these two proxies of GDP provides better classification ability. These improvements for true GDP in dating recessions and expansions are statistically significant at the 2 percent and 8 percent levels against $\mathrm{GDP_I}$ and $\mathrm{GDP_E}$, respectively.[7] BBK's monthly GDP estimates perform similarly to $\mathrm{GDP_I}$ and $\mathrm{GDP_E}$, with again true GDP providing superior classification performance. A comparison in Table C6 against broader measures of economic activity as captured by BBK's index and the ADS index does show, as anticipated, that when focused exclusively on dating the NBER business cycle, information beyond monthly GDP helps.[8] Piger's set of recession probabilities, calibrated specifically to signal NBER recessions, provide near perfect classification.

Our conclusion from Table C6 is therefore not that information beyond headline GDP growth does not provide additional value-added when dating business cycles; it is simply that our reconciled measures of true GDP are the most informative single measure - with a clear "economic interpretation." As made in Mariano and Murasawa (2003), an argument for producing measures of GDP itself, rather than construction of indices of economic activity, is that the size of movements in GDP has a direct economic interpretation, in contrast to the levels of indices. A further advantage is that, once a quarter when aggregated, measures of monthly GDP, at least as measured by $\mathrm{GDP_I}$, or $\mathrm{GDP_E}$, can be evaluated through comparison with the BEA's own quarterly estimates.

C.4.1 Real-time recession probabilities: Looking back at the 2007-2009 recession

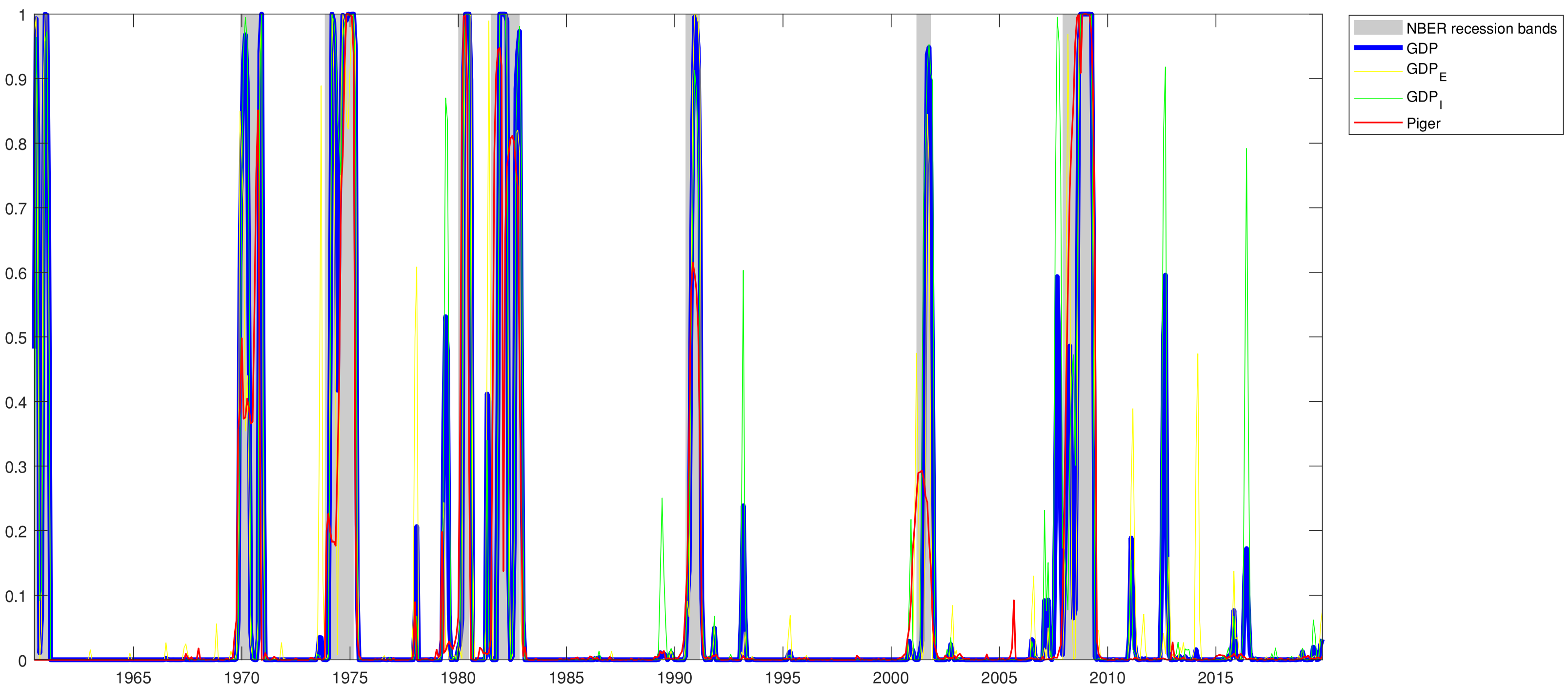
To showcase the use of our models in real time we revisit the 2007-2009 recession, as identified by the NBER, and assess our models' ability - when used monthly as if in real time - to date this recession. The NBER classifies the recession, due to the global financial crisis, as starting in January 2008 and ending in June 2009. As Nalewaik (2012) has emphasized, $\mathrm{GDP_I}$ has a track record of detecting recessions earlier than $\mathrm{GDP_E}$, although it is published more slowly. This raises the possibility that a model exploiting and reconciling both $\mathrm{GDP_E}$ and $\mathrm{GDP_I}$, along with additional monthly indicator variables as they accrue in real time, may be better able to anticipate recessions.

To mimic use of these models in real time, for all of these monthly variables and in all of our models we make use of the real-time monthly data vintages. And we acknowledge the

[7] The AUC statistics are compared using DeLong et al.'s (1988) test as implemented in the R package, https://cran.r-project.org/web/packages/pROC/pROC.pdf. This test is only illustrative, however, given it does not accommodate serial dependence in the data. In Appendix D, Table D7, we show that true GDP also yields higher AUC values when we date the business cycle not using the recession probability estimates computed across MCMC draws but using the posterior mean estimates of GDP. Table D7 also shows this result to be robust to which of the seven models of Table 1 we consider.

[8] As explained above, in Table C6 we analyze the ADS index when aggregated to represent quarter-on-quarter annualized growth rates. When analyzed in its underlying and original form, the ADS index achieves an AUC statistic of 0.99 (standard error of 0.007).

Figure C7: Historical US recession probabilities from the ADNSS+SS model's monthly density estimates of true GDP, GDP_I and GDP_E versus Piger's estimates



Notes: Sample: 1960m1-2019m12, except for Piger, which is only available from 1967m6. All data are end of March 2020 vintage. Shaded areas represent NBER-defined recessions.

Table C6: Business cycles features

Variables	ADNSS+SS	BBK: MGDP	Piger	BBK: Index	ADS
AUC estimates					
True GDP	0.94 (0.018)				
GDP_E	0.92 (0.020)	0.92 (0.020)	0.99 (0.003)	0.99 (0.008)	0.96 (0.014)
GDP_I	0.92 (0.020)				
Optimal threshold parameter					
True GDP	40%				
GDP_E	76%	-1.24	91%	-0.76	-2.05
GDP_I	63%				

Notes: Classification ability of the monthly GDP estimates from the ADNSS+SS MF-VAR model compared with the Brave, Butters, and Kelley (BBK) coincident index and monthly GDP (MGDP) estimates, estimates maintained by Jeremy Piger and a monthly aggregation of the Aruoba, Diebold, and Scotti (ADS) index. Area under the receiver operating characteristic curve (AUC) values and threshold estimates that optimize classification ability when hits and misses are given equal weight. Sample: 1960m1-2019m12, except for Piger, which is only available from 1967m6. Standard errors reported in parentheses. The BBK index and the ADS index are not to be interpreted as direct estimates of monthly GDP as they are broader indices of economic activity.

staggered release of data in real time (the so-called ragged edge) due to differing publication lags. These monthly variables are aligned with real-time monthly data vintages of quarterly GDP_I and GDP_E. Data vintages are organized so that our recession probability estimates for month t are produced near the end of month $t+1$, using monthly and quarterly indicator data available at this point in time. Given GDP_I data are published more slowly than GDP_E, this means that in the first month of each calendar quarter while the last quarter's GDP_E estimate is known, the BEA has yet to publish GDP_I.

We estimate the seven models of Table 1 (in the main paper) recursively from January 2007 through December 2009 and produce estimates of true monthly GDP, y_t^Q. For each MCMC draw, as in the historical business cycle analysis but now focusing on true GDP to again facilitate cross-model comparisons, we compute recession probabilities by comparing y_t^Q with the optimal estimates c. To acknowledge the fact that the NBER announces recessions with at least a 12-month lag, when using this strategy to classify in real time whether y_t^Q is a recession or not, we only use NBER data up to month $t-12$ to estimate c. We note how the estimates for c are recursively updated through our out–of-sample window.

Figure C8 plots the recursively computed estimates of a recession in month t from each of the seven models from January 2007 through December 2009. Alongside, for comparison, we plot the real time recession probability estimates maintained by Jeremy Piger.[9] These estimates are real-time and exploit the vintage data maintained by Piger. We note that, over this period, Piger's recessionary estimates for month t are produced not near the end of month $t+1$, but a month later and so have an informational advantage (or timing disadvantage)

[9] See https://pages.uoregon.edu/jpiger/us_recession_probs.htm/.

relative to our estimates.

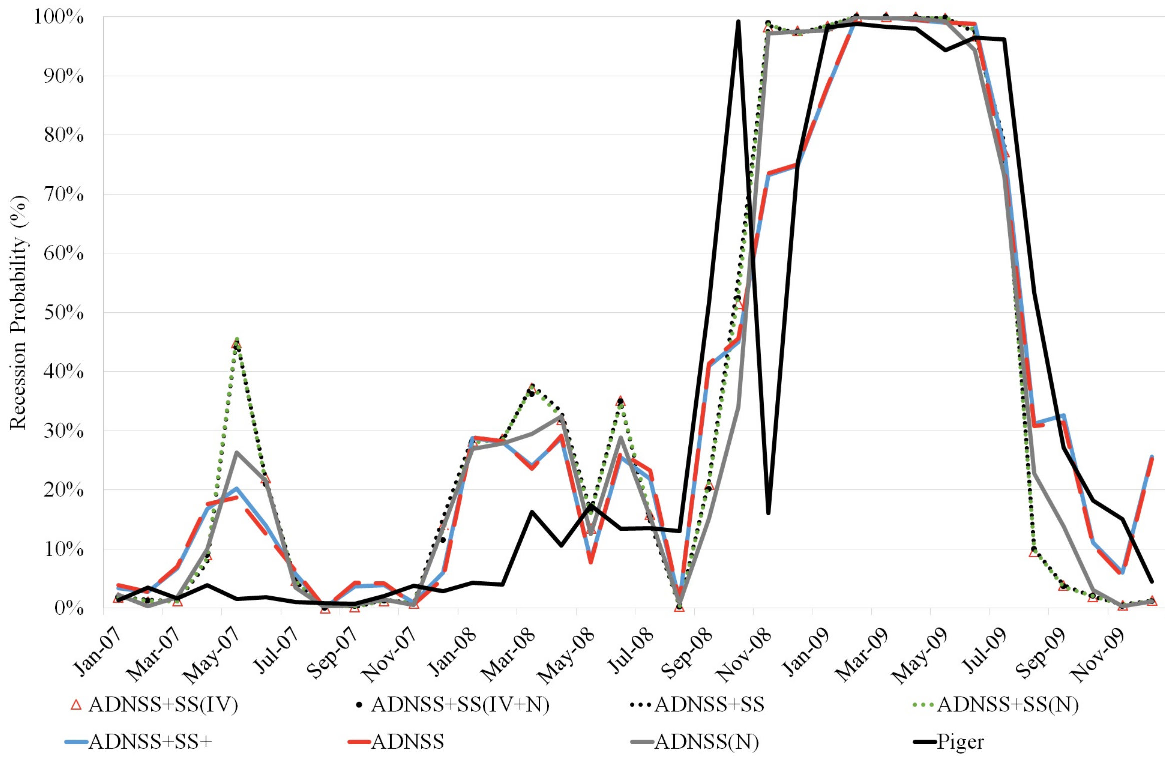
Figure C8 reveals that all of our models identify increasing recessionary risks from the beginning of 2008, well ahead of the NBER announcing in December 2008 that the recession did begin in January 2008. But, especially for the smaller SS models, there are (according to the NBER) false recessionary signals in mid-2007, with a local spike in the recession probabilities. This spike is driven by the negative estimates for true monthly GDP growth seen in Figure 4 (for the ADNSS+SS model) in May 2007, explained by larger negative estimates for $\mathrm{GDP_I}$. Interestingly, the big data model (ADNSS+SS^+) down weights the odds of a recession in mid-2007. The recessionary probabilities from all of the models do not approach unity until almost a year later, when the NBER does classify a recession. And they decline sharply from June 2009, well ahead of the NBER (in late September 2010) classifying June 2009 as the final month of the recession.

Again we conclude from this real-time exercise that our monthly estimates of true GDP do a good job of tracking NBER business cycles out-of-sample, as well as in-sample. Indeed, visually Figure C8 provides some evidence that these estimates of true GDP provide a sharper signal of the recession than Piger's real-time recession probabilities. Piger's estimates spike before the beginning of the recession and then fall more slowly at its conclusion. We also appear to find more variation across our models out-of-sample than in-sample. Conditioning on the published quarterly estimates of $\mathrm{GDP_I}$ and $\mathrm{GDP_E}$ from the BEA disciplines our models' monthly GDP estimates in-sample and helps explain their similarities. But out-of-sample, absent knowledge of these quarterly realizations, the monthly indicator variables appear to play a heightened role in shaping the probabilistic path of true GDP.

We emphasize that Figure C8 shows the *real-time* recession probabilities from our models. Arguably these are of most interest to policymakers, making decisions without the benefit of hindsight or revised data. But comparison against the full-sample (final vintage) recession probability estimates (cf. Figure C7) indicates the *unreliability* (in the sense of Orphanides and van Norden (2002)) of these real-time estimates. Data revisions explain much of this; for example, as we move across data vintages, the 2007q4 estimate of $\mathrm{GDP_E}$ switches from being a positive, to a negative, and back to a positive growth rate relative to 2007q3. Indeed, the April 30, 2020 vintage data show that while the 2007q4 value of real $\mathrm{GDP_E}$ is higher than the 2007q3 value, the reverse holds for $\mathrm{GDP_I}$.[10] This is an additional reason why we might be uncertain as to whether "true" GDP was expanding during 2007q4: the two BEA estimates of GDP disagree.

[10] Available at https://apps.bea.gov/histdata/fileStructDisplay.cfm?HMI=7&DY=2020&DQ=Q1&DV=Advance&dNRD=April-30-2020.

Figure C8: Real-time recession probabilities over the period of the global financial crisis from the seven models of Table 1 compared against Piger's real-time estimates



D Supplementary Tables

Table D1: Parameters from quarterly VAR, (1), estimated in only GDP_E and GDP_I

Parameters	**Median**	**16th quantile**	**84th quantile**
b_{11}	0.56	0.48	0.65
μ	1.32	1.06	1.59
a_{21}	-0.96	-1.08	-0.83
a_{31}	-0.72	-1.03	-0.49
a_{32}	-0.27	-0.44	-0.04
σ^2_{GG}	5.30	4.64	6.51
σ^2_{EE}	3.21	2.32	4.25
σ^2_{II}	1.96	1.44	2.42
ξ_E	0.63	0.57	0.77
ξ_I	0.71	0.62	0.86

Notes: μ is the intercept.

Table D2: Parameters from quarterly VAR, (1), estimated in GDP_E, GDP_I with U as an instrument

Parameters	**Median**	**16th quantile**	**84th quantile**
μ_1	2.26	1.91	2.67
μ_2	0.39	0.28	0.50
b_{11}	0.46	0.40	0.53
b_{12}	-0.14	-0.17	-0.10
b_{21}	0.48	0.27	0.70
b_{22}	0.22	0.12	0.32
a_{21}	0.22	0.12	0.32
a_{32}	-1.00	-1.11	-0.90
a_{42}	-1.01	-1.21	-0.81
a_{43}	0.00	-0.17	0.17
σ^2_{UU}	0.91	0.83	1.00
σ^2_{GG}	7.00	5.52	9.04
σ^2_{EE}	2.24	1.72	2.90
σ^2_{II}	1.47	1.09	1.92
ξ_E	0.75	0.61	0.94
ξ_I	0.81	0.66	1.01

Notes: Following ADNSS we put an intercept only in the equations for U_t and GDP_t and these are labeled μ_1 and μ_2 in this table. The error variance in the equation for unemployment is labelled σ^2_{UU}.

Table D3: Properties of quarterly GDP estimates

	Mean	Median	$\hat{\sigma}$	Skewness	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	Q_4	corr. GDPplus
GDP_E	2.96	2.98	3.24	-0.26	0.32	0.28	0.11	0.11	57.02	0.78
GDP_I	2.95	3.15	3.15	-0.36	0.45	0.29	0.23	0.11	98.07	0.96
GDPplus	2.98	3.13	2.47	-0.46	0.74	0.50	0.34	0.18	239.57	1.00
ADNSS_B1	3.05	3.16	2.60	-0.44	0.64	0.42	0.27	0.14	169.41	0.97
ADNSS_B2	2.96	3.04	2.72	-0.45	0.53	0.34	0.22	0.10	119.93	0.94

Notes: The sample period is 1960q1-2019q4. $\hat{\sigma}$ is the sample standard deviation. $\widehat{\rho}_1 - \widehat{\rho}_4$ are the sample autocorrelations at displacements of 1 to 4 quarters. Q_4 is the Ljung-Box serial correlation test statistic calculated using $\widehat{\rho}_1, \ldots, \widehat{\rho}_4$. corr. is the correlation coefficient against GDPplus as maintained by the Federal Reserve Bank of Philadelphia. ADNSS_B1 and ADNSS_B2 are the posterior median estimates of true GDP from the two Bayesian quarterly econometric models considered in Sections 4.1 and 4.2, respectively.

Table D4: Correlation by model of the posterior median of monthly GDP_E growth with selected business cycle indicators and alternative estimates of monthly GDP growth (1960m1-2019m12)

	OECD	S&P500	IPI	Unemp.	PMI
ADNSS+SS(IV)	0.84	0.26	0.76	-0.63	0.64
ADNSS+SS(IV+N)	0.84	0.26	0.76	-0.63	0.64
ADNSS+SS	0.84	0.26	0.76	-0.63	0.64
ADNSS+SS(N)	0.84	0.26	0.76	-0.63	0.64
ADNSS	0.84	0.26	0.77	-0.64	0.65
ANDSS(N)	0.84	0.26	0.77	-0.64	0.65
ADNSS+SS^{+}	0.84	0.25	0.77	-0.63	0.64

	Employ.	Stock Watson	IHS Markit	ADS Index	BBK
ADNSS+SS(IV)	0.67	0.93	0.45	0.71	1.00
ADNSS+SS(IV+N)	0.67	0.93	0.45	0.71	1.00
ADNSS+SS	0.67	0.93	0.45	0.71	1.00
ADNSS+SS(N)	0.67	0.93	0.45	0.71	1.00
ADNSS	0.67	0.93	0.46	0.72	1.00
ADNSS(N)	0.67	0.93	0.46	0.72	1.00
ADNSS+SS^{+}	0.67	0.93	0.47	0.72	0.99

Notes: The models are summarized in Table 1. All monthly indicators except PMI are analyzed in quarterly (quarter-over-quarter) annualized percent changes. PMI is analyzed in levels. Due to data availability, the correlations reported for Stock-Watson and IHS Markit are over the shorter sample periods of 1960m1-2010m6 and 1992m4-2019m12, respectively.

Table D5: Correlation by model of the posterior median of monthly GDP_I growth with selected business cycle indicators and alternative estimates of monthly GDP growth (1960m1-2019m12)

	OECD	S&P500	IPI	Unemp.	PMI
ADNSS+SS(IV)	0.77	0.26	0.79	-0.66	0.67
ADNSS+SS(IV+N)	0.77	0.26	0.79	-0.66	0.67
ADNSS+SS	0.77	0.26	0.79	-0.66	0.67
ADNSS+SS(N)	0.77	0.26	0.79	-0.66	0.67
ADNSS	0.77	0.27	0.79	-0.67	0.67
ANDSS(N)	0.77	0.27	0.79	-0.67	0.67
ADNSS+ SS^+	0.77	0.26	0.79	-0.66	0.67

	Employ.	Stock Watson	IHS Markit	ADS Index	BBK
ADNSS+SS(IV)	0.68	0.92	0.50	0.76	0.84
ADNSS+SS(IV+N)	0.68	0.92	0.50	0.76	0.84
ADNSS+SS	0.68	0.92	0.50	0.76	0.84
ADNSS+SS(N)	0.68	0.92	0.50	0.76	0.84
ADNSS	0.68	0.92	0.51	0.76	0.84
ANDSS(N)	0.68	0.92	0.51	0.76	0.84
ADNSS+ SS^+	0.68	0.92	0.52	0.76	0.84

Notes: The models are summarized in Table 1. All monthly indicators except PMI are analyzed in quarterly (quarter-over-quarter) annualized percent changes. PMI is analyzed in levels. Due to data availability, the correlations reported for Stock-Watson and IHS Markit are over the shorter sample periods of 1960m1-2010m6 and 1992m4-2019m12, respectively.

Table D6: Properties of monthly GDP estimates, by model, when aggregated to a quarterly frequency

	Mean	Median	$\hat{\sigma}$	Skew	$\hat{\rho_1}$	$\hat{\rho_2}$	$\hat{\rho_3}$	$\hat{\rho_4}$	Q_4	corr. GDP-plus
ADNSS+SS(IV)										
GDP	2.86	2.89	2.94	-0.38	0.47	0.31	0.22	0.11	100.95	0.93
GDPE	2.96	3.01	3.23	-0.26	0.33	0.28	0.11	0.11	58.08	0.78
GDPI	2.95	3.11	3.15	-0.36	0.46	0.30	0.24	0.11	99.43	0.96
ADNSS+SS(IV+N)										
GDP	2.96	3.06	3.04	-0.37	0.46	0.31	0.21	0.11	98.10	0.92
GDPE	2.96	3.01	3.23	-0.26	0.33	0.28	0.11	0.11	58.10	0.78
GDPI	2.95	3.11	3.15	-0.36	0.46	0.30	0.24	0.11	99.48	0.96
ADNSS+SS										
GDP	3.03	3.05	3.12	-0.37	0.47	0.31	0.22	0.11	101.12	0.93
GDPE	2.96	3.01	3.23	-0.26	0.33	0.28	0.11	0.11	58.09	0.78
GDPI	2.95	3.11	3.15	-0.36	0.46	0.30	0.24	0.11	99.48	0.96
ADNSS+SS(N)										
GDP	2.96	3.01	3.04	-0.37	0.47	0.31	0.21	0.11	100.03	0.93
GDPE	2.96	3.01	3.23	-0.26	0.33	0.28	0.11	0.11	58.08	0.78
GDPI	2.95	3.11	3.15	-0.36	0.46	0.30	0.24	0.11	99.46	0.96
ADNSS										
GDP	2.75	2.81	2.83	-0.38	0.47	0.31	0.22	0.11	102.61	0.94
GDPE	2.96	3.01	3.23	-0.26	0.33	0.28	0.11	0.11	58.55	0.78
GDPI	2.95	3.14	3.15	-0.36	0.46	0.30	0.24	0.11	99.92	0.96
ADNSS(N)										
GDP	2.96	3.06	3.04	-0.37	0.46	0.31	0.21	0.11	98.76	0.92
GDPE	2.96	3.01	3.23	-0.26	0.33	0.28	0.11	0.11	58.53	0.78
GDPI	2.95	3.14	3.15	-0.36	0.46	0.30	0.24	0.11	99.94	0.96
ADNSS+SS^+										
GDP	3.08	3.22	3.18	-0.38	0.47	0.31	0.23	0.11	103.29	0.94
GDPE	2.96	3.01	3.23	-0.26	0.33	0.28	0.11	0.11	58.07	0.78
GDPI	2.95	3.11	3.15	-0.36	0.46	0.30	0.24	0.11	99.41	0.96

Notes: The models are summarized in Table 1. The sample period is 1960q1-2019q4. $\hat{\sigma}$ is the sample standard deviation. $\widehat{\rho}_1 - \widehat{\rho}_4$ are the sample autocorrelations at displacements of 1 to 4 quarters. Q_4 is the Ljung-Box serial correlation test statistic calculated using $\widehat{\rho}_1, \ldots, \widehat{\rho}_4$. corr. is the correlation coefficient against GDPplus as maintained by the Federal Reserve Bank of Philadelphia.

Table D7: Business cycle features: Classification ability of the monthly GDP posterior mean estimates from the 7 MF-VAR models. Area under the receiver operating characteristic curve (AUC) values and threshold estimates that optimize classification ability when hits and misses are given equal weight

Variables	ADNSS+SS(IV)	ADNSS+SS(IV+N)	ADNSS+SS	ADNSS+SS(N)	ADNSS	ADNSS(N)	ADNSS+SS^{+}
AUC estimates							
True GDP	0.92	0.92	0.92	0.92	0.92	0.92	0.92
GDP_E	0.90	0.91	0.90	0.90	0.90	0.90	0.90
GDP_I	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Optimal threshold parameter							
True GDP	-0.90	-0.89	-0.95	-0.91	-0.91	-0.94	-1.01
GDP_E	-1.00	-0.97	-1.00	-0.99	-1.11	-1.07	-0.99
GDP_I	-1.04	-1.07	-1.03	-1.05	-1.11	-1.15	-1.04

Notes: The 7 models are summarized in Table 1. Sample: 1960m1-2019m12

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