Supplementary Material for "Minimum Resource Threshold Policy Under Partial Interference"

Supplementary Material

This document contains supplementary materials for "Minimum Resource Threshold Policy Under Partial Interference." Section A presents additional results related to the main paper. Section B proves lemmas and theorems stated in the paper.

A Additional Details of the Main Paper

A.1 Examples of Beneficial Intervention/Treatment for Most of the Population

We provide plausible examples where the intervention/treatment seems beneficial for almost all members of the population.

- Prior works have shown that improving household access to improved water, sanitation, and hygiene (WASH) resources are critical to reduce rates of diarrhea-related diseases (Esrey et al., 1985; Clasen et al., 2007), especially among children (Daniels et al., 1990; McMichael, 2019). Additionally, there is no biological rationale that well-managed WASH facilities cause diarrhea-related diseases.

- Devoto et al. (2012) studied the effect of getting easier access to piped water on various kinds of outcomes such as quality and quantity of water consumed, water-related time and financial costs incurred by the household. In particular, based on the results in Table 3 and related discussions, we can deduce that getting easier access to piped water results in a substantial increase in the quantity of water for most of the population.

- As discussed in page 10 of Cohen and Dupas (2010), higher insecticide-treated bed nets (ITN) coverage rates would be beneficial for the population because the use of ITN in a household may have positive health externalities for neighboring households.

- Feikin et al. (2022) conducted a meta-analysis of studying the effect of COVID-19 vaccines against SARS-CoV-2 infection. Their work and references therein suggest that COVID-19 vaccines are beneficial for most of the population.

- Miguel and Kremer (2004) studied the effect of school-level deworming projects on students' health status and academic achievements. They remarked that there were within- and across-school spillover effects, indicating that students who did not directly receive deworming treatment still benefited from those who did. Therefore, combined with the biological reasons, we can infer that deworming drugs are beneficial for most of a majority of students.

A.2 Examples of Real-world Applications Targeting a Certain Level of Outcomes

In this section, we present examples of real-world applications that target a specific level of outcomes rather than the maximum level of outcomes.

- High levels of protein in the urine, known as proteinuria, may be caused by diabetes, high blood pressure, autoimmune disorders, infections, and kidney diseases. Thus, for a normal adult, it is recommended to maintain the total urinary protein excretion less than 150 mg/day (Carroll and Temte, 2000).

- HDL cholesterol is commonly referred to as "good" cholesterol because it plays a crucial role in removing harmful cholesterol from the bloodstream. As a result, maintaining high levels of HDL cholesterol is associated with a lower risk of cardiovascular disease. It is generally recommended to maintain HDL cholesterol levels above 60 mg/dL (Grundy et al., 2002).

- Warfarin, a blood-thinning medication, is used to increase the international normalized ratio (INR), a measure of the time for the blood to clot, and it should be prescribed to keep patients' INR within the desired range, usually between 2 and 3, according to the recommendations from American Heart Association (January et al., 2014).

- Major medical associations recommend targeting proper ranges for chronic disease management measures such as hemoglobin level (male: 138-172 g/L; female: 121-151 g/L) (American Association of Clinical Endocrinologists and Others, 2019).

- UN has established the Sustainable Development Goals in 2015 (United Nations, 2016), which consists of 17 specific goals. In particular, Goal 1 is to eradicate extreme poverty for all people everywhere by 2030 and Goal 3 is to ensure healthy lives and promoting well-being at all ages. Some specifics of these goals target certain levels of outcomes of interest as follows:
  - 1.1: By 2030, eradicate extreme poverty for all people everywhere, currently measured as people living on less than \$1.25 a day
  - 1.2: By 2030, reduce at least by half the proportion of men, women and children of all ages living in poverty in all its dimensions according to national definitions
  - 3.1: By 2030, reduce the global maternal mortality ratio to less than 70 per 100,000 live births

- 3.2: By 2030, end preventable deaths of newborns and children under 5 years of age, with all countries aiming to reduce neonatal mortality to at least as low as 12 per 1,000 live births and under-5 mortality to at least as low as 25 per 1,000 live births

- The Global Technical Strategy for malaria 2016-2030 was adopted by the World Health Assembly in May 2015 (World Health Organization, 2021). It has set a target of reducing malaria incidence by 40, 75, and 90 percent by 2020, 2025, and 2030, respectively, compared with malaria incidence in 2015.

A.3 Application of Our Method to Other Real-World Examples

Motivated from the examples in Sections A.1 and A.2, we lay out some concrete examples where our approach can be used.

- Motivated by Cohen and Dupas (2010) and World Health Organization (2021), we can study the minimum ITN coverage necessary to meet the thresholds set by the Global Technical Strategy for malaria control Also, because an ITN is likely to reduce malaria incidence in both the household where it is installed and nearby (but not too far away) households, partial interference is a viable framework for modeling the effect of ITN installation on malaria incidence. Furthermore, there are biological reasons to believe that malaria incidence would exhibit a monotonic response to ITN coverage. Combined together, we can use our method to determine the Minimum Resource Threshold Policy (MRTP) of ITN coverage to achieve a desired malaria incidence level.

- Motivated by Devoto et al. (2012), we can study the minimum proportion of households with piped water that will meet or exceed the levels of existing hygiene and/or welfare indicators. For instance, as Tables 3 and 4 of Devoto et al. (2012) reported, one may use the numbers of baths and showers and the number of times a child fetched water in recent days as a basis for the hygiene and welfare indicators, respectively. Also, as suggested by Devoto et al. (2012), these indicators are likely to show monotonic response to water pipe installation. Finally, partial interference is reasonable in this context because the hygiene and welfare indicators of a household are affected by piped water in nearby (but not too far away) households. Therefore, we can use our method to determine the smallest water pipe coverage necessary to achieve the desired hygiene and/or welfare levels.

- We can consider a policy for allocating water, sanitation, and hygiene (WASH) in developing countries to achieve the Sustainable Development Goals 3.2 by targeting under-5 mortality being lower than 25 per 1000 live births. The context is similar to the application of the main paper except that the outcome is under-5 mortality. As before, partial interference and monotonicity are reasonable assumptions for the context, and investigators may determine the MRTPof the amount of WASH facilities that achieves the desired under-5 mortality rate.

A.4 A Graphical Illustration for the Setup

We provide a visual illustration for the setup in Figure A.1. For simplicity, we consider $N = 2$ clusters where each cluster has $n_i = 2$ study units. The black arrows from A_{ij} to Y_{ij} $(i, j = 1, 2)$ depict the direct effect of the treatment, and the red arrows from A_{ij} to $Y_{ij'}$ $(i, j, j' = 1, 2, j \neq j')$ depict the indirect effect of the treatment. No connection between two clusters illustrate the cluster-level independence.

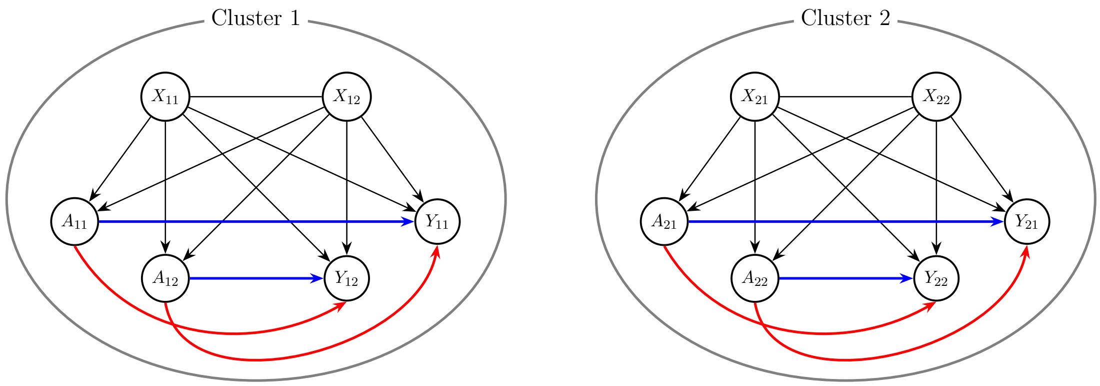

Figure A.1: A Graphical Illustration for the Setup. The blue arrows from A_{ij} to Y_{ij} $(i, j = 1, 2)$ depict the direct effect of the treatment, and the red arrows from A_{ij} to $Y_{ij'}$ $(i, j, j' = 1, 2, j \neq j')$ depict the indirect effect of the treatment.

A.5 The (Mostly) Wrong Approach: Analysis With Aggregated, Cluster-Level Data

We briefly discuss a tempting approach based on aggregating the data at the cluster-level. This aggregation approach has been discussed in the literature (e.g., Section 2.3 of Imbens and Wooldridge (2009) and Kilpatrick and Hudgens (2021)) as a simple way to deal with interference. While this approach will clearly not work for estimating MRTPs like $\theta^*_{\mathrm{SO}}(\boldsymbol{x}_i)$ or other MRTPs targeting spillover-specific outcomes, from a practitioner's point of view, it is worth asking whether this approach can be used to estimate, or at least approximate, MRTPs like $\theta^*_{\mathrm{OV}}(\boldsymbol{x}_i)$ which combine both the direct and spillover effects of treatment in a block. Unfortunately, as we illustrate below, this aggregation approach will lead to grossly misleading estimates of $\theta^*_{\mathrm{OV}}(\boldsymbol{x}_i)$ except in very restrictive settings.

Formally, following the above advice from the literature, suppose an investigator attempts to bypass the problem from interference at the unit/household-level by aggregating their data at the cluster/block-level. That is, for each cluster i, the investigator can consider $\overline{\boldsymbol{O}}_i = (\overline{Y}_i, \overline{A}_i, \overline{\boldsymbol{X}}_i)$ to be the available data and use existing techniques in the optimal treatment regime literature for a continuous treatment, such as Chen et al. (2016), to obtain the minimum proportion of WASH facilities necessary to achieve a certain target $\mathcal{T}$. For example, given a cluster-level outcome model for the expected value of $\overline{Y}_i$ as a function of cluster-level variables $\overline{A}_i$ and $\overline{\mathbf{X}}_i$, the investigator can find the smallest $\overline{A}_i \in [0,1]$ where the expected outcome exceeds $\mathcal{T}$.

Despite its simplicity, the above analysis is only appropriate in very restrictive settings, which we illustrate with an example. Suppose the treatment assignment depends on the measured covariates, and the outcome regression is given as $\mathrm{E}\{Y_{ij}^{(a_{ij},\boldsymbol{a}_{i(-j)})} \mid \boldsymbol{X}_i\} = \beta_1 a_{ij} + \beta_2 \overline{a}_{i(-j)} + \boldsymbol{\beta}_3^\intercal \boldsymbol{X}_{ij} a_{ij} + \boldsymbol{\beta}_4^\intercal \boldsymbol{X}_{ij} \overline{a}_{i(-j)}$ where $\beta_1, \ldots, \boldsymbol{\beta}_4$ are non-negative coefficients to guarantee Assumption (A5). Some algebra reveals the average potential outcome at the cluster-level is

$$\mathrm{E}\{\overline{Y}_i^{(\boldsymbol{a}_i)} \mid \boldsymbol{X}_i\} = \left(\beta_1 + \beta_2 + \frac{n_i \boldsymbol{\beta}_4^\intercal \overline{\boldsymbol{X}}_i}{n_i - 1}\right)\overline{a}_i + \left\{\frac{(n_i-1)\boldsymbol{\beta}_3 + \boldsymbol{\beta}_4}{n_i - 1}\right\}^\intercal \left(\frac{1}{n_i}\sum_{j=1}^{n_i} a_{ij}\boldsymbol{X}_{ij}\right). \tag{1}$$

If the investigator uses the aggregated, cluster-level data to estimate the MRTP, the resulting estimate will be biased because the cluster-level outcome model of $\overline{Y}_i$ given $\overline{A}_i$ and $\overline{\boldsymbol{X}}_i$ is mis-specified. Or equivalently, there is an omitted variable bias because of the term $\sum_{j=1}^{n_i} a_{ij}\boldsymbol{X}_{ij}$, which roughly measures the covariance between the unit-level treatment variable and the unit-level covariate. The magnitude and the direction of the bias will depend on (a) the magnitude of treatment effect heterogeneity, as measured by $\boldsymbol{\beta}_3$ and $\boldsymbol{\beta}_4$, and (b) the magnitude and the sign of the measured, unit-level confounding, as measured by the covariance of A_{ij} and $\boldsymbol{X}_{ij}$.

More generally, if the outcome model is nonlinear, which is often the case in popular epidemiological models (e.g., Magal and Ruan (2014)), no amount of modeling with aggregated, cluster-level data $(\overline{Y}_i, \overline{A}_i, \overline{\boldsymbol{X}}_i)$ will completely remove this bias as the cluster-level data cannot capture both unit-level treatment heterogeneity and unit-level confounding. As a concrete example, suppose the treatment is completely randomized and there are no interactions between the covariates and the treatment, but there exists non-linear relationship between the treatment on the outcome:

$$\mathrm{E}\{Y_{ij}^{(a_{ij},\boldsymbol{a}_{i(-j)})} \mid \boldsymbol{X}_i\} = \beta_0 + \beta_1 a_{ij} + \beta_2 \{(\overline{a}_{i(-j)} - q_a)^p\}_+ \tag{2}$$

where β_1 and β_2 are non-negative coefficients, $q_a \in [0,1]$, and p is a positive integer. Roughly speaking, the model states that the household's outcome can be affected by its peer households through a non-linear function $z \mapsto (z - q_a)^p$ if at least $(100 \times q_a)\%$ of their peers are treated; see Granovetter (1978), Watts (2002), and Kempe et al. (2003) and references therein for other types of threshold models in networks. As before, Assumptions (A4) and (A5) hold for this model and some algebra will reveal that the cluster-level outcome model will be mis-specified when using only aggregated, cluster-level data $(\overline{Y}_i, \overline{A}_i, \overline{\boldsymbol{X}}_i)$ due to the non-linearity of $\overline{a}_{i(-j)}$ in the household-level outcome model. Consequently, the resulting MRTP with the cluster-level data will be biased.

We provide a graphical illustration of model (2). To demonstrate, we fix the cluster size $n_i = 10$ and the coefficients $\beta_0 = \beta_1 = 0$, and choose β_2 so that the range of the outcome regression becomes $[0,1]$. We consider three levels for $q_a \in \{0.4, 0.6, 0.8\}$ and $p \in \{1,2,5\}$, respectively, and we choose the threshold $\mathcal{T} = 0.2$. Figure A.2 visually presents the differences between the MRTPs based on τ_{OV} and the aggregated cluster-level outcome regression. We find that the differences vary between

0.07 and 0.17. The toy example suggests that estimating the MRTP based on the aggregated outcome regression may yield significantly biased estimates of $\theta_{\rm OV}$ in equation (6) of the main paper.

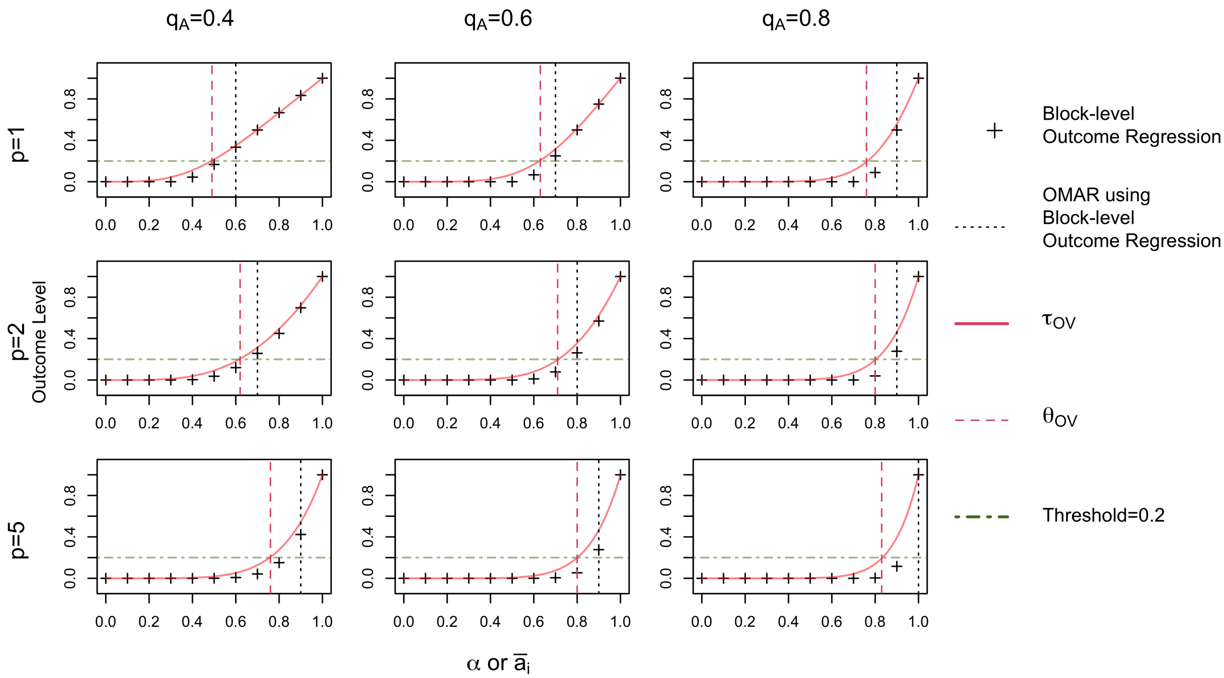

Figure A.2: Graphical Comparison between the MRTPs Based on the Aggregated Cluster-level Outcome Regression and $\tau_{\rm OV}(\alpha)$. Black dotted and red dashed lines indicate the MRTP based on the aggregated cluster-level outcome regression and that based on $\tau_{\rm OV}(\alpha)$ (i.e., $\theta_{\rm OV}$), respectively.

However, we mention that this simple aggregation approach may work under different assumptions. For instance, Kilpatrick and Hudgens (2021) assumed that the cluster-level potential outcome only depends on the total number of treatment implemented in a cluster, i.e., the cluster-level stratified interference. This implies that the average potential outcome at the cluster-level has a form of $\mathrm{E}\big\{\overline{Y}_i^{(\boldsymbol{a}_{ij})} \,\big|\, \overline{\boldsymbol{X}}_i\big\} = \mu^\dagger\big(\overline{a}_i, \overline{\boldsymbol{X}}_i\big)$ for some function $\mu^\dagger$, which can be identified as (nonparametric) regression models of $\overline{Y}_i$ on $(\overline{A}_i, \overline{\boldsymbol{X}}_i)$. In turn, using their g-formula approach and/or the indirect approach in Section 3.1 of the main paper, we can get a valid estimate of $\theta^*_{\rm OV}$. We remark that the cluster-level stratified interference assumption lacks the necessary flexibility to define $\theta^*_{\rm SO}$. This is because it eliminates the possibility of having distinct cluster-level outcomes based on the treatment recipients, which is a critical aspect of interference.

Next, we compare the classification performance measures of the true policy of the main manuscript and the policy obtained from the aggregated cluster-level outcome regression. We consider the following simple data generating process:

$$
\begin{aligned}
& n_i = 10 \ , \quad X_{ij} \sim \mathrm{Ber}(0.5) \ , \quad A_{ij} \,\big|\, X_{ij} \sim \mathrm{Ber}(0.5) \\
& Y_{ij} \,\big|\, (A_{ij}, \boldsymbol{A}_{i(-j)}, X_{ij}, \boldsymbol{X}_{i(-j)}) = A_{ij} + 0.5\overline{A}_{i(-j)} + 0.5 A_{ij} X_{ij} + \epsilon_{ij} , \quad \epsilon_{ij} \sim N(0,1) \ .
\end{aligned}
$$

The aggregated cluster-level outcome regression is given as $\mathrm{E}\big(\overline{Y}_i \,\big|\, \overline{A}_i, \overline{X}_i\big) = (1.5 + 0.5\overline{X}_i)\overline{A}_i$, and the simple policy is obtained as follows:

$$\theta^*_{\text{Simple}}(\overline{x}_i) = \min \Big\{ a \in \{0, 0.1, \ldots, 1\} \,\Big|\, \mathrm{E}\big(\overline{Y}_i \,\big|\, \overline{A}_i = a, \overline{X}_i = \overline{x}_i\big) \geq \theta \Big\} \ .$$

The true policy $\theta^*_{\text{OV}}(\alpha)$ is defined based on equation (6) of the main paper.

We generate $N = 10^5$ observations from the above data generating process, and compare the three classification performance measures of θ^*_{Simple} and θ^*_{OV} across $\mathcal{T} \in [0.8, 1.6]$. We use the true outcome regressions μ^* and $\mu^\dagger$ to construct θ^*_{OV} and θ^*_{Simple}. As a result, the discrepancies in the performance measures can be attributed to the use of the aggregated approach instead of the approach proposed of the main paper. The range of $\mathcal{T}$ has been chosen such that the lower bound of the interval is not significantly smaller than the average outcome of $\mathrm{E}\big(\overline{Y}_i\big) = 0.875$. Figure A.3 graphically summarizes the result. We find that θ^*_{OV} uniformly yields better classification performance measures compared to θ^*_{Simple}, suggesting that using the aggregated cluster-level outcome regression is suboptimal even in the simple model.

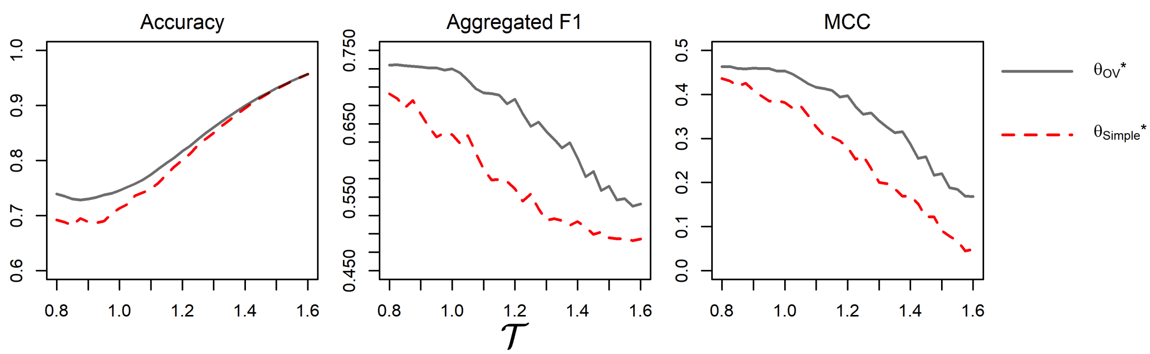

Figure A.3: Classification Performance Measures of θ^*_{OV} and θ^*_{Simple}.

In summary, an analysis based on aggregated, cluster-level data will often lead to biased estimates of MRTP. Also, if the investigator is interested in MRTPs based on spillover-specific outcomes such as $\theta^*_{\text{SO}}(\boldsymbol{x}_i)$, a cluster-level analysis is simply infeasible.

A.6 Inverse Probability-Weighted and Outcome Regression-based Loss Functions

We introduce the IPW and outcome regression-based loss functions. Specifically, we can replace ψ_{DR} in equation (9b) of the main paper with the following functions.

$$\psi_{\text{IPW}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) = \frac{Y_{ij} \mathbb{1}(A_{ij} = a, S_{i(-j)} = s)}{e^*(a, s \,|\, \boldsymbol{X}_i)} \ , \quad \psi_{\text{OR}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) = \mu^* \left(a, \frac{s}{n_i - 1}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)} \right) \ .$$

A.7 Machine Learning Methods Used for the Outcome Regression Estimation

As candidate machine learning methods, we include the following methods and R packages in our super learner library: linear regression (`glm`), Lasso/elastic net (`glmnet` (Friedman et al., 2010)), spline (`earth` (Friedman, 1991), `polspline` (Kooperberg, 2020)), generalized additive model (`gam` (Hastie and Tibshirani, 1986)), boosting (`xgboost` (Chen and Guestrin, 2016), `gbm` (Greenwell et al., 2019)), random forest (`ranger` (Wright and Ziegler, 2017)), and neural net (`RSNNS` (Bergmeir and Benítez, 2012)).

A.8 Computation: Training the Support Vector Machine in (10) of the Main Paper

This section presents the computational details on training the support vector machine in (10) of the main paper. The algorithm to train SVMs under this type of nonconvex function is already discussed in prior works (An and Tao, 1997; Chen et al., 2016) and we present a summary of it for completeness. Also, to keep the notation clear, the discussion below assumes that the nuisance functions are known, but the identical computation algorithm is used to train the SVM when the nuisance functions are estimated.

To start off, we can decompose the loss function for the overall outcome case in equation (9) of the main paper into the difference of two convex function $L_+(t, \boldsymbol{O}_i)$ and $L_-(t, \boldsymbol{O}_i)$, i.e., $L(t, \boldsymbol{O}_i) = L_+(t, \boldsymbol{O}_i) - L_-(t, \boldsymbol{O}_i)$ for any t and $\boldsymbol{O}_i$ where

$$
\begin{aligned}
L_+(t, \boldsymbol{O}_i) &= \begin{cases} \nu_+(0, \boldsymbol{O}_i) - 2\delta t & \text{if } -\infty < t < 0 \\ \nu_+(t, \boldsymbol{O}_i) & \text{if } 0 \le t \le 1 \\ \nu_+(1, \boldsymbol{O}_i) + \big(\overline{\delta} + 2\delta\big)(t-1) & \text{if } 1 < t < \infty \end{cases} \\
L_-(t, \boldsymbol{O}_i) &= \begin{cases} \nu_-(0, \boldsymbol{O}_i) - 2\delta t - \delta + \delta e^{t} & \text{if } -\infty < t < 0 \\ \nu_-(t, \boldsymbol{O}_i) & \text{if } 0 \le t \le 1 \\ \nu_-(1, \boldsymbol{O}_i) + \big(\overline{\delta} + 2\delta\big)(t-1) - \delta + \delta e^{1-t} & \text{if } 1 < t < \infty \end{cases} \\
\nu_\pm(t, \boldsymbol{O}_i) &= \frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i - 1} \binom{n_i - 1}{s} \psi_{\text{DR}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) \\
&\qquad\qquad \times \sum_{\ell=0}^{n_i - a - s} \binom{n_i - a - s}{\ell} \left\{ \frac{(-1)^\ell}{\ell + a + s + 1} \right\}_{\pm} t^{\ell + a + s + 1} + (\mathcal{T})_{\pm} t
\end{aligned}
$$

Here, $(a)_+ = \max(a, 0)$, $(a)_- = -\min(a, 0)$, and $\overline{\delta}$ is chosen as the maximum of the left derivatives of $\nu_+(t, \boldsymbol{O}_i)$ and $\nu_-(t, \boldsymbol{O}_i)$ at $t = 1$, i.e., $\overline{\delta} = \max\big\{ \lim_{\epsilon \downarrow 0} \nabla \nu_+(1-\epsilon, \boldsymbol{O}_i), \lim_{\epsilon \downarrow 0} \nabla \nu_-(1-\epsilon, \boldsymbol{O}_i) \big\}$ and $\nabla \nu_\pm(t, \boldsymbol{O}_i)$ is the derivative of $\nu_\pm(t, \boldsymbol{O}_i)$ with respect to t. Critically, the two loss functions L_+ and

L_- are convex and non-decreasing in t. For the spillover outcome case, we use

$$\nu_{\mathrm{SO},\pm}(t,\boldsymbol{O}_i) = \frac{1}{n_i}\sum_{j=1}^{n_i}\sum_{s=0}^{n_i-1}\binom{n_i-1}{s}\psi_{\mathrm{DR}}(0,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})\sum_{\ell=0}^{n_i-s}\binom{n_i-s}{\ell}\left\{\frac{(-1)^\ell}{\ell+s+1}\right\}_{\pm} t^{\ell+s+1} + (\mathcal{T})_{\pm}t$$

Given the decomposition of the loss function into the difference of two convex functions, we use the DC algorithm (An and Tao, 1997), which is an iterative algorithm, to solve the original non-convex optimization problem; see Algorithm 1 for details.

Algorithm 1 DC Algorithm

Require: Initialize values $\boldsymbol{\eta}^{(0)} \in \mathbb{R}^N$, $b^{(0)} \in \mathbb{R}$. Set iteration number to zero, $j \leftarrow 0$.

1: Precompute the gradient $\nabla L_-(t,\boldsymbol{O}_i)$ where

$$\nabla L_-(t,\boldsymbol{O}_i) = \begin{cases} \dfrac{\partial}{\partial t}L_-(t,\boldsymbol{O}_i) & t \neq 0,1 \\ \dfrac{1}{2}\lim_{\epsilon\downarrow 0}\left\{\dfrac{\partial}{\partial t}L_-(t+\epsilon,\boldsymbol{O}_i) + \dfrac{\partial}{\partial t}L_-(t-\epsilon,\boldsymbol{O}_i)\right\} & t = 0,1 \end{cases}$$

2: **repeat**

3: Let $\boldsymbol{\eta}^{(j+1)}$ and $b^{(j+1)}$ be the solution to the following convex optimization problem.

$$\begin{bmatrix}\boldsymbol{\eta}^{(j+1)} \\ b^{(j+1)}\end{bmatrix} \in \operatorname*{arg\,min}_{\boldsymbol{\eta},b}\left[\frac{1}{N}\sum_{i=1}^N\left\{\begin{array}{l} L_+\big(\boldsymbol{k}_i^\intercal\boldsymbol{\eta}+b,\boldsymbol{O}_i\big) \\ \quad -\nabla L_-\big(\boldsymbol{k}_i^\intercal\boldsymbol{\eta}^{(j)}+b^{(j)},\boldsymbol{O}_i\big)\big(b+\boldsymbol{k}_i^\intercal\boldsymbol{\eta}\big)\end{array}\right\} + \frac{\lambda_N}{2}\boldsymbol{\eta}^\intercal K\boldsymbol{\eta}\right]$$

4: $j \leftarrow j+1$

5: **until** convergence

6: **return** $\big(\widehat{\boldsymbol{\eta}},\widehat{b}\big) \leftarrow \big(\boldsymbol{\eta}^{(j)},b^{(j)}\big)$.

To initiate the DC algorithm, we choose the initial value as follows. First, for each i, let the solution be r_i, i.e., $r_i = \arg\min_{t\in[0,1]} L(t,\boldsymbol{O}_i)$ which can be obtained from a grid-search. In words, r_i is an approximate of $\widehat{\theta}(\boldsymbol{x}_i)$ that are found by a grid-search. But, since r_i is bounded in the unit interval, it may not be a suitable approximate of $\widetilde{\theta}(\boldsymbol{x}_i)$, the SVM solution before the winsorization. As a consequence, directly using r_i to construct initial points may lead to an estimate policy shrinking to a certain value, i.e., a policy does not reflect the heterogeneity induced by $\overline{\boldsymbol{x}}_i$. To stretch r_i outside of the unit interval, we consider the following steps.

(a) Let ϕ and φ be

$$\phi(a,a',\boldsymbol{X}_i) = \frac{1}{n_i}\sum_{j=1}^{n_i}\widehat{\mu}\Big(a,a',\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\Big)\ ,\ \varphi(\boldsymbol{X}_i) = \frac{1}{n_i}\sum_{j=1}^{n_i}\frac{Y_{ij}-\widehat{\mu}(A_{ij},\overline{A}_{i(-j)},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})}{\widehat{e}(A_{ij},S_{i(-j)}\,|\,\boldsymbol{X}_i)}\ .$$

(b) By only using the clusters with non-0 and non-1 r_is, i.e., $r_i \in (0,1)$, we fit linear regression models where $\phi(a,a',\boldsymbol{X}_i)$ and $\varphi(\boldsymbol{X}_i)$ are regressed on r_is. We choose (a,a') from $\{0,1\}\otimes\{0,0.2,0.4,0.6,0.8,1\} = \{(0,0),(0,0.2),\ldots,(1,0.8),(1,1)\}$, i.e., 12 levels. Let $(\widehat{\beta}_{0,\text{model }k},\widehat{\beta}_{1,\text{model }k})$ are the estimated regression coefficients from kth model.

(c) Let $\widehat{r}_i$ be the adjusted initial points which are defined as follows.

(c-1) If $r_i \in (0,1)$, no adjustment is required, i.e., $\widehat{r}_i = r_i$.

(c-2) For clusters having $r_i = 1$, we use the largest prediction values obtained from the 13 regression models and 1, i.e.,

$$\widehat{r}_i = \max\left\{ \frac{\phi(0,0,\boldsymbol{X}_i) - \widehat{\beta}_{0,\text{model 1}}}{\widehat{\beta}_{1,\text{model 1}}}, \ldots, \frac{\varphi(\boldsymbol{X}_i) - \widehat{\beta}_{0,\text{model 13}}}{\widehat{\beta}_{1,\text{model 13}}}, 1 \right\} .$$

(c-3) Similarly, for clusters having $r_i = 0$, we use the smallest prediction values obtained from the 13 regression models and 0, i.e.,

$$\widehat{r}_i = \min\left\{ \frac{\phi(0,0,\boldsymbol{X}_i) - \widehat{\beta}_{0,\text{model 1}}}{\widehat{\beta}_{1,\text{model 1}}}, \ldots, \frac{\varphi(\boldsymbol{X}_i) - \widehat{\beta}_{0,\text{model 13}}}{\widehat{\beta}_{1,\text{model 13}}}, 0 \right\} .$$

Second, we take $b^{(0)} = \sum_{i=1}^{N} \widehat{r}_i / N$ and $\boldsymbol{\eta}^{(0)}$ as a vector satisfying $\widehat{r}_i = \boldsymbol{k}_i^{\intercal} \boldsymbol{\eta}^{(0)} + b^{(0)}$ for all i; i.e., $\widehat{\boldsymbol{r}} = K\boldsymbol{\eta}^{(0)} + b^{(0)}\mathbf{1}$ where $\widehat{\boldsymbol{r}} = [\widehat{r}_1, \ldots, \widehat{r}_N]^{\intercal} \in \mathbb{R}^N$ and $\mathbf{1} = [1, \ldots, 1]^{\intercal} \in \mathbb{R}^N$. Even though the kernel matrix K is invertible due to the positive definiteness of the kernel function $\mathcal{K}$, the inverse of K cannot be obtained due to the numerical singularity. Under such case, we add a tiny value to diagonal of K until its inverse can be obtained. In line 1, ∇L_{-} is a subgradient of L_{-} that accounts for the non-differentiability of L_{-} at $t = 0$ and $t = 1$.

The convex optimization in line 3 can be solved by using many standard algorithms and softwares. The iteration stops when $\big\| (\boldsymbol{\eta}^{(j+1)}, b^{(j+1)}) - (\boldsymbol{\eta}^{(j)}, b^{(j)}) \big\|_2$ drops below some threshold value. We remark that because the objective function in (10) of the main paper is bounded below, the algorithm will always converge in finite steps (An and Tao, 1997; Chen et al., 2016).

A.9 Details of Cross-validation

We present the details on how to choose the SVM parameters γ and λ. We consider a set of candidate values for $(\gamma_\ell, \lambda_\ell)$ where $\ell = 1, \ldots, K$. Without loss of generality, let the estimation data fold be $\mathcal{D}_1 = \mathcal{D}_2^c$ and, as a consequence, observations in $\mathcal{D}_2$ is used to evaluate the estimated loss function $\widehat{L}_{(-1)}(t, \boldsymbol{O}_i)$ for $i \in \mathcal{D}_2$. We further split $\mathcal{D}_2$ into training and tuning sets, denoted by $\mathcal{D}_{2,\text{train}}$ and $\mathcal{D}_{2,\text{tuning}}$, respectively, based on the number of cross-validation folds. For each candidate parameter $(\gamma_\ell, \lambda_\ell)$, we estimate the direct MRTP $\widehat{\theta}_{\text{train}}(\boldsymbol{X}_i\,;\,\ell)$ by only using the training set $\mathcal{D}_{2,\text{train}}$ and obtain the empirical risk using the tuning set $\mathcal{D}_{2,\text{tuning}}$. The optimal parameters (γ^*, λ^*) are the minimizer of the average of the empirical risks across the tuning sets, i.e.

$$(\gamma^*, \lambda^*) = \operatorname*{arg\,min}_{\ell=1,\ldots,K} \frac{1}{\big|\mathcal{D}_{2,\text{tuning}}\big|} \sum_{i \in \mathcal{D}_{2,\text{tuning}}} \widehat{L}_{(-1)}\big(\widehat{\theta}_{\text{train}}(\boldsymbol{X}_i\,;\,\ell), \boldsymbol{O}_i\big) .$$

A.10 Details of Undersampling and Cross-fitting Procedures

We discuss the details on how to negate the impact of a particular realization of undersampling procedure. We randomly choose a subset of observations so that the cluster sizes are (nearly) balanced, and we repeat the undersampling for U times indexed by u. Let $\widehat{\mu}^{(u)}$ and $\widehat{e}^{(u)}$ be the estimated outcome regression and propensity score obtained from uth undersample. Then, we take the median-adjusted nuisance function across U estimated functions as the final estimate of the nuisance function, i.e., $\widehat{\mu} := \text{median}_{u=1,\ldots,U}\, \widehat{\mu}^{(u)}$ and $\widehat{e} := \text{median}_{u=1,\ldots,U}\, \widehat{e}^{(u)}$.

Next, we discuss the median-adjustment of cross-fitting procedure. Once we split the data into two folds $\mathcal{D}_1$ and $\mathcal{D}_2$, we obtain two directly estimated policies $\widehat{\theta}_{(-\ell)}$ for $k = 1, 2$ where $\mathcal{D}_\ell^c$ is used as the estimation data fold and $\mathcal{D}_\ell$ is used as the evaluation data fold. Investigators may use either $\widehat{\theta}_{(-1)}$ or $\widehat{\theta}_{(-2)}$ as the final estimate of the MRTP, denoted by $\widehat{\theta}^{(F)}$. However, we recommend to use $\widehat{\theta}^{(F)}(\boldsymbol{x}) = \mathcal{W}\big(\{\widehat{\theta}_{(-1)} + \widehat{\theta}_{(-2)}\}/2\big)(\boldsymbol{x})$, the winsorized policy of the average of two non-winsorized policies, for the new $\overline{\boldsymbol{x}}$ as the estimate of the MRTP to fully use the data. If the evaluation point is one of the points in the data, i.e., $\boldsymbol{x} = \boldsymbol{x}_i$ for some $i \in \mathcal{D}_\ell$, we recommend using $\widehat{\theta}^{(F)}(\boldsymbol{x}) = \widehat{\theta}_{(-\ell)}(\boldsymbol{x}_i)$ because $\widehat{\theta}_{(-\ell)}$ does not depend on i while $\widehat{\theta}_{(\ell)}$ depends on i which may lead to an overfitted value. Second, to construct a more robust estimate of the MRTP under cross-fitting, we use the recommendation in Chernozhukov et al. (2018) to our setting by taking the mean or the median of multiple MRTP estimates. Specifically, we repeat the estimation of $\widehat{\theta}^{(F)}$ multiple times, say T times, and obtain $\widehat{\theta}_t^{(F)}$ $(t = 1, \ldots, T)$ where the sample partitions are randomly done across splits. We define the mean-MRTP estimate $\widehat{\theta}^{(F,\text{mean})}(\boldsymbol{x}) = \sum_{t=1}^{T} \widehat{\theta}_t^{(F)}(\boldsymbol{x})/T$ and the median-MRTP estimate $\widehat{\theta}^{(F,\text{median})}(\boldsymbol{x}) = \text{median}_{t=1,\ldots,T}\, \widehat{\theta}_t^{(F)}(\boldsymbol{x})$.

A.11 Details of the Data Generating Process of the Simulation

We provide details of the data generating process of the simulation in Section 4 of the main paper. First, we provide the distribution of the cluster size n_i, which is the same as the empirical distribution of n_i in the dataset used in Section 4 of the main paper.

n_i	3	4	5	6	7	8	9	10	11	12
Frequency	4	4	11	22	30	55	61	76	80	88
Probability	0.004	0.004	0.011	0.021	0.029	0.054	0.059	0.074	0.078	0.086

n_i	13	14	15	16	17	18	19	20	21	22
Frequency	92	90	113	86	72	71	39	22	6	5
Probability	0.090	0.088	0.110	0.084	0.070	0.069	0.038	0.021	0.006	0.005

Next, in Figure A.4, we provide graphical summaries of the distributions of $\overline{A}_i$ in the 2014-2017 Senegal DHS and the simulated datasets. The two distributions are similar to each other with the common support of $[0, 1]$.

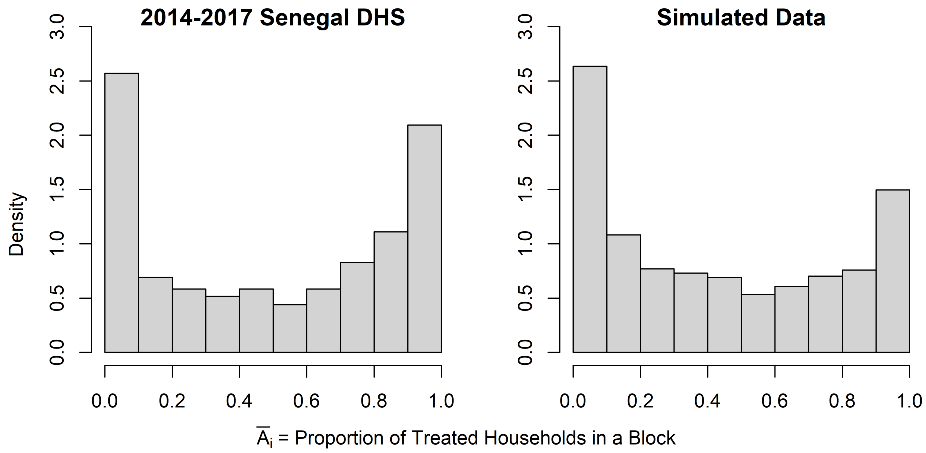

Figure A.4: Histograms of the treated households (i.e., $\overline{A}_i$) in the 2014-2017 Senegal DHS (left) and simulated datasets (right). The distribution of $\overline{A}_i$ in the simulation shows the distribution across 50 repetitions.

Lastly, we provide the details of the outcome regression model:

$$Y_{ij} \,\Big|\, (\boldsymbol{A}_i, \boldsymbol{X}_i) \sim \text{Ber}\left(\text{expit} \left[\begin{array}{l} -0.35 + \big\{0.1 + 0.25(C_i + W_{ij1})^2\big\} A_{ij} \\ +\big\{0.05 + 0.15(\overline{W}_{i(-j)2} + \overline{W}_{i(-j)3})^2\big\} \overline{A}_{i(-j)} \\ +0.1(C_i + \sum_{k=1}^3 W_{ijk}) + 0.25(C_i^2 + \sum_{k=1}^3 W_{ijk}^2) + 0.05(\sum_{k=1}^3 \overline{W}_{i(-j)k}) \end{array} \right] \right) .$$

Here, $\overline{W}_{i(-j)k} = \sum_{\ell \neq j} W_{ijk}/(n_i - 1)$. We remark that the outcome model satisfies Assumptions (A1)-(A5) of the main paper.

A.12 Details of Classification Performance Measures

For an given MRTP θ, we define the *true positives* (TP), *true negatives* (TN), *false positives* (FP), and *false negatives* (FN) as follows:

$$\begin{aligned} \text{TP} &= \sum_{i \in \mathcal{D}_{\text{test}}} \mathbb{1}\big\{\overline{Y}_i > \mathcal{T}, \overline{A}_i > \theta(\boldsymbol{X}_i)\big\} , & \text{TN} &= \sum_{i \in \mathcal{D}_{\text{test}}} \mathbb{1}\big\{\overline{Y}_i \leq \mathcal{T}, \overline{A}_i \leq \theta(\boldsymbol{X}_i)\big\} , \\ \text{FP} &= \sum_{i \in \mathcal{D}_{\text{test}}} \mathbb{1}\big\{\overline{Y}_i \leq \mathcal{T}, \overline{A}_i > \theta(\boldsymbol{X}_i)\big\} , & \text{FN} &= \sum_{i \in \mathcal{D}_{\text{test}}} \mathbb{1}\big\{\overline{Y}_i > \mathcal{T}, \overline{A}_i \leq \theta(\boldsymbol{X}_i)\big\} . \end{aligned} \tag{3}$$

Given these definitions, we use the following classification performance measures: accuracy, two-sided F1 score, and the Matthews correlation coefficient (MCC) (Matthews, 1975) which are defined as follows:

$$\begin{aligned} &\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}} , \ \text{F1} = \frac{2\text{TP}}{2\text{TP} + \text{FP} + \text{FN}} + \frac{2\text{TN}}{2\text{TN} + \text{FP} + \text{FN}} , \\ &\text{MCC} = \frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\{(\text{TP} + \text{FP}) \times (\text{TP} + \text{FN}) \times (\text{TN} + \text{FP}) \times (\text{TN} + \text{FN})\}^{1/2}} . \end{aligned}$$

The usual F1 score does not use true negatives in its score, i.e., 2TP/(2TP+FP+FN), and is sensitive to the definition of a positive label. For example, if we were to define the positive label as the opposite of the definition in equation (3), i.e., positive label if $\overline{Y}_i \le \mathcal{T}$, the F1 score changes. To avoid this, we consider the two-sided F1 score, the average of the usual F1 score and the "opposite" F1 score, 2TN/(2TN+FP+FN).

A.13 Assessment of Assumptions of the Main Paper

We take a moment to discuss the plausibility of the bounded cluster size n_i assumption and Assumptions (A1)-(A5) in the Senegal DHS.

(Bounded n_i) The bounded block size assumption is plausible in the Senegal DHS because the data was collected based on a stratified sampling design where a fixed number of households were sampled from each block (ANSD and ICF, 2020). Also, the maximum number of households among $N = 1027$ census blocks in the 2014-2017 Senegal DHS is $M = 22$, and the small value of $M/N = 0.021$ (i.e., an upper bound on n_i/N) suggests that the "large N, small n_i" asymptotic regime is a reasonable approximation for our data.

(A1) Assumption (A1) is plausible as long as households in different census blocks do not interact with each other. In the data, 99.15% of the census blocks are geographically far apart from each other. The average and median distances among 22,578 pairs of census blocks in the 2018 Senegal DHS are 245.04km and 230.17km, respectively; only 192 (0.85%) pairs of census blocks have distance smaller than 10km. Given this, we find that the partial interference assumption is plausible where interference likely occurs between households in the same census block and not across different census blocks.

(A2) To check Assumptions (A2) and (A3), we check covariate balance and overlap by using the binning approach in Hirano and Imbens (2004), Kluve et al. (2012) and Flores et al. (2012) for a continuous treatment variable. Algorithm 2 shows the details on the covariate balance assessment.

We use the median of the propensity score estimates from 100 cross-fitting procedures. As a consequence, we obtain the unadjusted/adjusted t-statistics in Figure A.5, which suggests covariate balance was satisfied for all cases.

(A3) Next, we assess the overlap assumption based on Algorithm 3. Again, we use the median of the propensity score estimates from 100 cross-fitting procedures.

Figure A.6 shows histograms that visually assess the overlap assumption. Based on the histograms, the overlap assumption seems to be satisfied or to be not severely violated.

Overall, all 9 observed covariates are balanced across different bins of treatment values and overlap is reasonable.

Algorithm 2 Assessment of Covariate Balance

1: Divide $\sum_{i=1}^{N} n_i = 13556$ units into four groups:

$$\mathcal{A}_k = \big\{ (i,j) \,\big|\, (A_{ij}, \overline{A}_{i(-j)}) \in R_k \big\} \ , \ R_k = \begin{cases} \{0\} \times [0,\alpha_0] \in \{0,1\} \times [0,1] & \text{if } k=1 \\ \{0\} \times (\alpha_0,1] \in \{0,1\} \times [0,1] & \text{if } k=2 \\ \{1\} \times [0,\alpha_1] \in \{0,1\} \times [0,1] & \text{if } k=3 \\ \{1\} \times (\alpha_1,1] \in \{0,1\} \times [0,1] & \text{if } k=4 \end{cases}$$

where α_0 and α_1 are chosen so that $\mathcal{A}_1, \ldots, \mathcal{A}_4$ have similar sizes.

2: **for** $k = 1,2,3,4$ **do**

3: Obtain unadjusted t-statistics that compare the distribution of $\boldsymbol{X}_i$ between $\mathcal{A}_k$ and $\mathcal{A}_k^c$, i.e.,

$$\left\{ \widetilde{\boldsymbol{X}}_{i,k} \,\middle|\, \widetilde{\boldsymbol{X}}_{i,k} = \frac{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \in \mathcal{A}_k\} \boldsymbol{X}_{ij}}{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \in \mathcal{A}_k\}} \right\} \quad \text{v.} \quad \left\{ \widetilde{\boldsymbol{X}}_{i,k^c} \,\middle|\, \widetilde{\boldsymbol{X}}_{i,k^c} = \frac{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \notin \mathcal{A}_k\} \boldsymbol{X}_{ij}}{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \notin \mathcal{A}_k\}} \right\}$$

4: Calculate the estimated propensity score $\widehat{e}_{ij,k} = \widehat{P}\big\{ (A_{ij}, \overline{A}_{i(-j)}) \in R_k \,\big|\, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)} \big\}$.

5: Let $-\infty = q_0 \le q_1 \le \ldots \le q_9 \le q_{10} = \infty$ be the deciles of $\big\{ \widehat{e}_{ij,k} \,\big|\, (i,j) \in \mathcal{A}_k \big\}$.

6: Let $\mathcal{E}_{b,k} = \Big\{ (i,j) \,\Big|\, \widehat{e}_{ij,k} \in (q_{b-1}, q_b] \Big\}$ $(b = 1, \ldots, 10)$.

7: Obtain t-statistics that compare the distribution of $\boldsymbol{X}_i$ between $\mathcal{E}_{b,k} \cap \mathcal{A}_k$ and $\mathcal{E}_{b,k} \cap \mathcal{A}_k^c$, i.e.,

$$\left\{ \widetilde{\boldsymbol{X}}_{i,k} \,\middle|\, \widetilde{\boldsymbol{X}}_{i,k} = \frac{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \in \mathcal{E}_{b,k} \cap \mathcal{A}_k\} \boldsymbol{X}_{ij}}{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \in \mathcal{E}_{b,k} \cap \mathcal{A}_k\}} \right\} \quad \text{v.} \quad \left\{ \widetilde{\boldsymbol{X}}_{i,k^c} \,\middle|\, \widetilde{\boldsymbol{X}}_{i,k^c} = \frac{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \in \mathcal{E}_{b,k} \cap \mathcal{A}_k^c\} \boldsymbol{X}_{ij}}{\sum_{j=1}^{n_i} \mathbb{1}\{(i,j) \in \mathcal{E}_{b,k} \cap \mathcal{A}_k^c\}} \right\}$$

8: Aggregate the t-statistics obtained in Step 7 with weights from the size of $\mathcal{E}_{1,k}, \ldots, \mathcal{E}_{10,k}$.

9: Obtain adjusted t-statistics by taking the median of t-statistics in Step 6 across multiple cross-fitting procedures.

10: **end for**

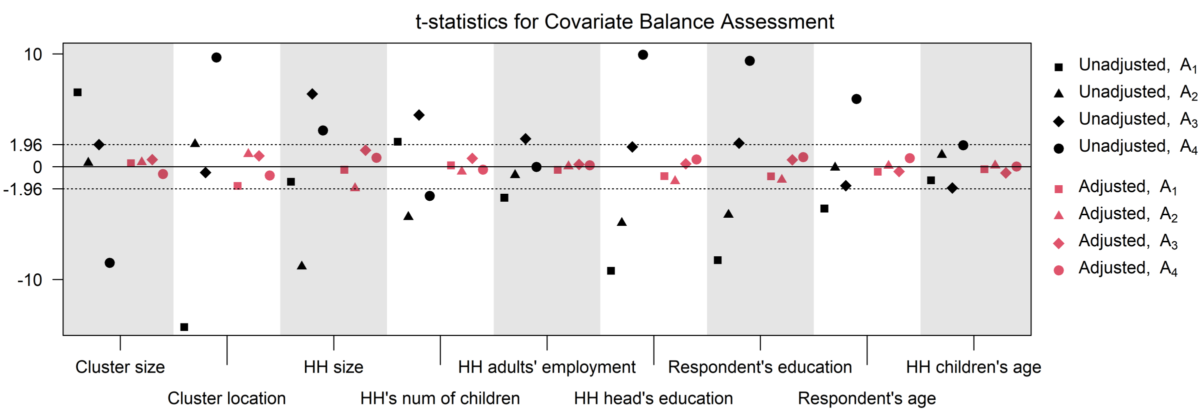

Figure A.5: Covariate Balance Assessment

(A4) Assumption (A4) is plausible if the number of diarrhea-free children in a household can be reasonably approximated by a summary of peers' WASH status. However, the assumption may fail if a few households' presence (or absence) of WASH facilities is driving the incidence

Algorithm 3 Assessment of Overlap

1: Divide $\sum_{i=1}^{N} n_i = 13556$ units into four groups:

$$\mathcal{A}_k = \left\{ (i,j) \,\middle|\, (A_{ij}, \overline{A}_{i(-j)}) \in R_k \right\} \ , \ R_k = \begin{cases} \{0\} \times [0,\alpha_0] \in \{0,1\} \times [0,1] & \text{if } k=1 \\ \{0\} \times (\alpha_0,1] \in \{0,1\} \times [0,1] & \text{if } k=2 \\ \{1\} \times [0,\alpha_1] \in \{0,1\} \times [0,1] & \text{if } k=3 \\ \{1\} \times (\alpha_1,1] \in \{0,1\} \times [0,1] & \text{if } k=4 \end{cases}$$

where α_0 and α_1 are chosen so that $\mathcal{A}_1, \ldots, \mathcal{A}_4$ have similar sizes.

2: Calculate the median of the estimated propensity scores obtained from multiple cross-fitting procedures, i.e.,

$$\widehat{e}_{ij,k}^{(\text{median})} = \underset{s=1,\ldots,S}{\text{median}} \, \widehat{P}^{(s)} \left\{ (A_{ij}, \overline{A}_{i(-j)}) \in R_k \,\middle|\, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)} \right\}$$

where the conditional probability $\widehat{P}^{(s)}$ is calculated from the estimated propensity score obtained from the sth cross-fitting procedure.

3: Compare histograms of $\widehat{e}_{ij,k}^{(\text{median})}(\boldsymbol{X}_i)$ for $\mathcal{A}_k$ and $\mathcal{A}_k^c$ for each k.

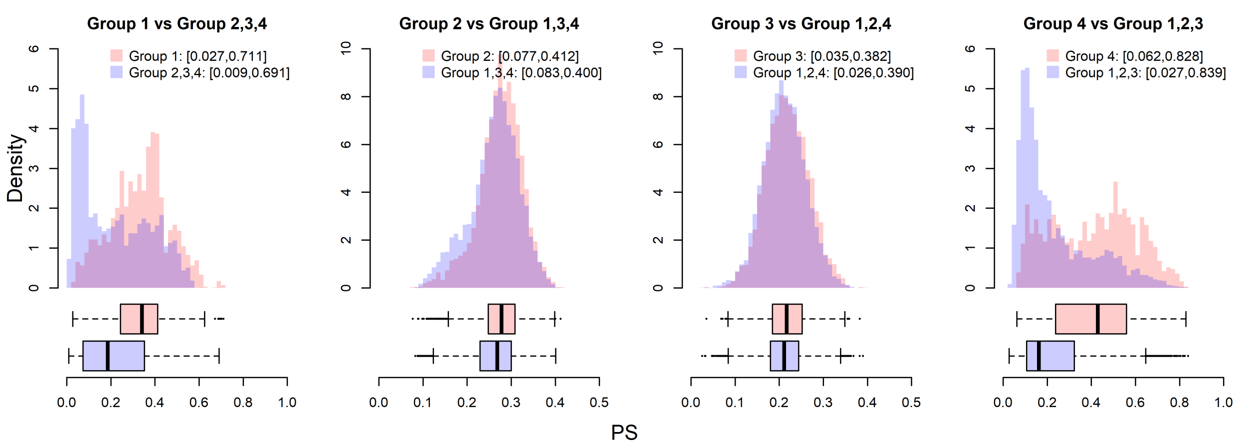

Figure A.6: Overlap Assessment. The numbers in brackets show the range of the estimated propensity scores for each group.

of diarrhea in the entire block, say if a few WASH-less households are located near communal water sources and they are primarily responsible for the diarrhea in the entire block. For example, if the census block has 20 households and the true response model for each household is $\text{E}(Y_{ij} \,|\, \boldsymbol{A}_i, \boldsymbol{X}_i) = \beta_0 + \beta_1 A_{i1} + \beta_2 A_{ij} + \boldsymbol{\beta}_3^\intercal \boldsymbol{X}_{ij}$, i.e., every household j's outcome depends on household 1's treatment status, then $\text{E}(\overline{Y}_i \,|\, \boldsymbol{A}_i, \boldsymbol{X}_i) = \beta_0 + \beta_1 A_{i1} + \beta_2 \overline{A}_i + \boldsymbol{\beta}_3^\intercal \overline{\boldsymbol{X}}_i$ and Assumption (A4) is violated because the average response of block i depends on the treatment status of household 1. Unfortunately, the data does not contain information about the location of households to test these hypothesized violations of Assumption (A4). Instead, we visually diagnose the assumption by using a residual plot of the predicted values of the mean block-level response versus the observed block-level response. Specifically, let

$\widehat{\epsilon}_{ij}^{(\text{median})} = Y_{ij} - \widehat{\mu}^{(\text{median})}(A_{ij}, \overline{A}_{i(-j)}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)})$ be the residuals where $\widehat{\mu}^{(\text{median})}$ is the median of the outcome regression from 100 cross-fitting procedures. We compare the residuals across the outcome regression estimate and the regressors $(A_{ij}, \overline{A}_{i(-j)}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)})$ and check whether the residuals deviate from zero in Figure A.7. Since the dimension of $\boldsymbol{X}_{i(-j)}$ varies, we use the average of $\boldsymbol{X}_{i(-j)}$, i.e., $\overline{\boldsymbol{X}}_{i(-j)} = \sum_{\ell \neq j} \boldsymbol{X}_{i\ell}/(n_i - 1)$. In general, the residuals are close to zero across the regressors, implying that the outcome regression under Assumption (A4) is not severely violated. That is, while the diagnostic is not perfect, we find the predicted means do not show trends across the x-axis and Assumption (A4) could be plausible, subject to inherent limitations of the diagnostic plot.

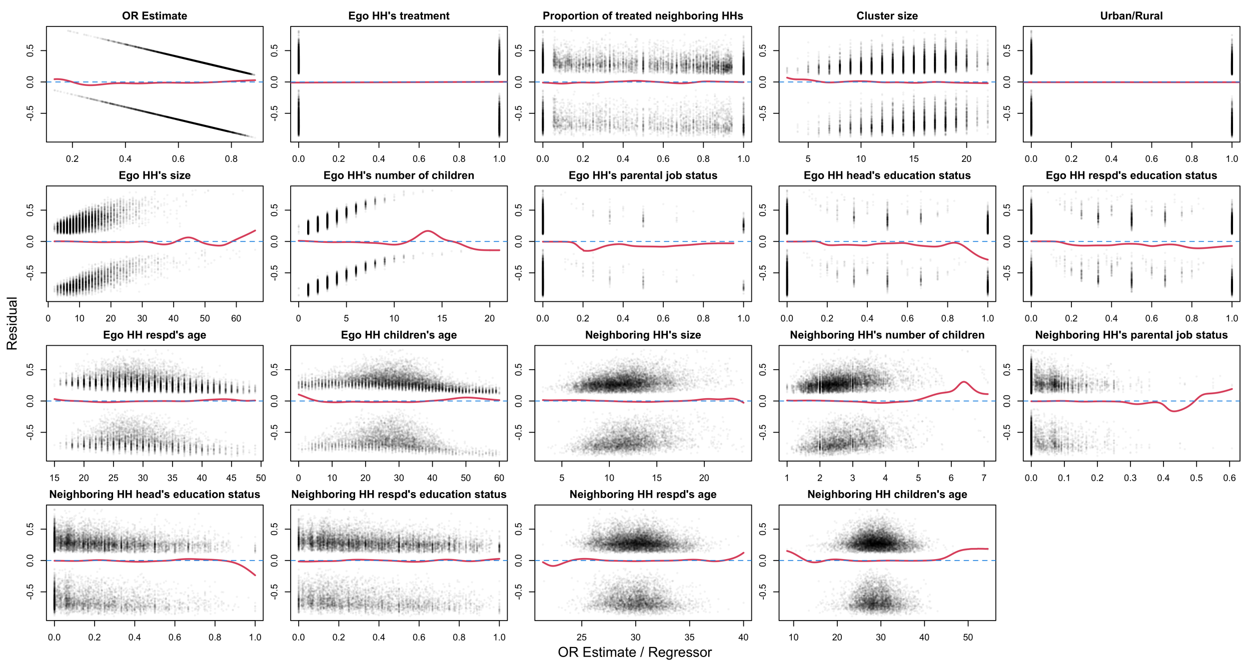

Figure A.7: Residual Plots. The x-axis shows the outcome regression estimate $\widehat{\mu}^{(\text{median})}$ (top left) and the regressors $(\overline{A}_i, \overline{\boldsymbol{X}}_i)$. The y-axis shows the residuals $\widehat{\epsilon}_i^{(\text{median})}$. The red curves are smoothing lines drawn for visual guidance. The blue dashed lines show the zero residual.

(A5) Finally, for Assumption (A5), many prior works (Esrey et al., 1985; Daniels et al., 1990; Clasen et al., 2007; Ejemot-Nwadiaro et al., 2015; McMichael, 2019) suggest that installing WASH facilities will not have a negative impact on incidence of diarrhea; however, it may have a negative effect on other, non-health outcomes. Also, when we empirically assess Assumption (A5), we find that the monotonicity assumption is rarely violated in the Senegal DHS and if violated, the deviation from monotonicity is small. Specifically, we first consider the difference between two cluster-level outcome regressions:

$$V(a, a', s, s') = \overline{\mu}\left(a', \frac{s'}{n_i - 1}, \boldsymbol{X}_i\right) - \overline{\mu}\left(a, \frac{s}{n_i - 1}, \boldsymbol{X}_i\right) \ , \ \overline{\mu}(a, a', \boldsymbol{X}_i) = \frac{1}{n_i} \sum_{j=1}^{n_i} \mu(a, a', \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)}) \ .$$

In particular, we focus on the variations contrasting two adjacent outcome regressions. As a consequence, there are $3n_i - 2$ finest variations where $n_i - 1$ variations have the form $V(0, 0, s, s+1)$, $n_i - 1$ variations have the form $V(1, 1, s, s+1)$, and n_i variations have the form $V(0, 1, s, s)$; see the diagram below.

$$\begin{array}{cccccc}
\overline{\mu}\big(0, \frac{0}{n_i-1}, \boldsymbol{X}_i\big) & \overset{V_i(0,0,0,1)}{\rightarrow} & \overline{\mu}\big(0, \frac{1}{n_i-1}, \boldsymbol{X}_i\big) & \overset{V_i(0,0,1,2)}{\rightarrow} & \cdots & \overline{\mu}\big(0, \frac{n_i-1}{n_i-1}, \boldsymbol{X}_i\big) \\
{\scriptstyle V_i(0,1,0,0)} \downarrow & & {\scriptstyle V_i(0,1,1,1)} \downarrow & & & {\scriptstyle V_i(0,1,n_i-1,n_i-1)} \downarrow \\
\overline{\mu}\big(1, \frac{0}{n_i-1}, \boldsymbol{X}_i\big) & \overset{V_i(1,1,0,1)}{\rightarrow} & \overline{\mu}\big(1, \frac{1}{n_i-1}, \boldsymbol{X}_i\big) & \overset{V_i(1,1,1,2)}{\rightarrow} & \cdots & \overline{\mu}\big(1, \frac{n_i-1}{n_i-1}, \boldsymbol{X}_i\big)
\end{array}$$

In the Senegal DHS, we have $\sum_{i=1}^{N}(3n_i - 2) = 38614$ variations in total. Let $\widehat{V}_i^{(t)}(a, a', s, s')$ be the estimated variation of cluster i obtained from the tth cross-fitting procedure, and let $\widehat{V}_i^{(m)}(a, a', s, s')$ be the median of the variation, i.e.,

$$\widehat{V}_i^{(m)}(a, a', s, s') = \text{median}\Big\{\widehat{V}_i^{(1)}(a, a', s, s'), \ldots, \widehat{V}_i^{(S)}(a, a', s, s')\Big\}$$

Assumption (A5) can be empirically assessed by two means. First, out of 38614 variations, we count the number of times monotonicity is violated. Second, we measure the worst-case slope of the estimated μ as follows. Let $TV_i(a, a', s, s')$ be the absolute value of $V_i(a, a', s, s')$. Thus, the sum of 38614 $TV_i(a, a', s, s')$ is the total variation of the cluster-level outcome regression. We compute the relative magnitude of the slopes that are decreasing compared to the total variation, i.e., $\sum \mathbb{1}(V < 0)TV / \sum TV$. Overall, under the first assessment, we found that the monotonicity is violated 1.11% of the time and under the second assessment, the relative magnitude of decreasing slopes is 6.01×10^{-4}. In short, the empirical validations show that the monotonicity assumption is rarely violated in the Senegal DHS and if violated, the deviation from monotonicity is small.

A.14 Details of Figures 5.2-5.4 of the Main Paper

We additionally describe how we draw Figures 5.2-5.4 of the main paper. The reported estimated MRTPs in Figures 5.2 and 5.4 are weighted average of the estimated MRTPs in each administrative region where weights are the number of households in a census block, i.e., census block size n_i. That is, the values represent $\overline{\theta}_g$s, which are defined as

$$\overline{\theta}_g = \frac{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\} \cdot n_i \cdot \widehat{\theta}(\boldsymbol{x}_i)}{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\} \cdot n_i}$$

where $\mathcal{D}_{2018}$ is the collection of census blocks in the 2018 Senegal DHS. In words, $\overline{\theta}_g$ is the proportion of households in administrative area g that require WASH facilities. Similarly, the average household sizes in Figure 5.4 of the main paper represent

$$\overline{\overline{x}}_{g,\text{Household Size}} = \frac{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\} \cdot n_i \cdot \bar{x}_{i,\text{Household Size}}}{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\} \cdot n_i}$$
$$= \frac{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\} \sum_{j=1}^{n_i} x_{ij,\text{Household Size}}}{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\} \cdot n_i} .$$

Again, $\overline{\overline{x}}_{g,\text{Household Size}}$ is the average household wise in administrative area g. The proportions of rural area in Figure 5.4 of the main paper represent

$$\overline{c}_{g,\text{Rural}} = \frac{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\} \cdot c_{i,\text{Rural}}}{\sum_{i \in \mathcal{D}_{2018}} \mathbb{1}\{i \in \text{administrative area } g\}} .$$

Here $\overline{c}_{g,\text{Rural}}$ is the proportion of the rural census blocks in administrative area g. Note that $\overline{\theta}_g$, $\overline{\overline{x}}_{g,\text{Household Size}}$, and $\overline{c}_{g,\text{Rural}}$ do not address the geographical distance between census regions in different administrative areas. But, we believe that these statistics are geographically meaningful summaries to highlight the heterogeneity across administrative areas; see Figures 1 and 2 of Houngbonon et al. (2021) for similar summary statistics aggregated at Senegalese administrative areas.

Lastly, Figure 5.3 of the main paper shows the weighted average of the estimated MRTPs across all 45 administrative areas where weights are the number of households in a census block, i.e., census block size n_i. That is, the y-axis represents $\overline{\theta}$, which is defined as

$$\overline{\theta} = \frac{\sum_{i \in \mathcal{D}_{2018}} n_i \cdot \widehat{\theta}(\boldsymbol{x}_i)}{\sum_{i \in \mathcal{D}_{2018}} n_i} .$$

In words, $\overline{\theta}$ is the proportion of households in Senegal that require WASH facilities.

B Proof of Lemmas and Theorems

B.1 Useful Lemmas

Lemma B.1. *Suppose that θ^* belongs to a Besov space on $\mathbb{R}^d$ with smoothness parameter $\beta > 0$, i.e., $\mathcal{B}^{\beta}_{1,\infty}(\mathbb{R}^d) = \{\theta \in L_\infty(\mathbb{R}^d) \mid \sup_{t>0} t^{-\beta}\{\omega_{r,L_1(\mathbb{R}^d)}(\theta,t)\} < \infty, r > \beta\}$ where ω_r is the modulus of continuity of order r. Then, for any positive ϵ, p, τ satisfying $d/(d+\tau) < p < 1$, we have the following excess risk bound of $\widehat{\theta}$ with probability not less than $1 - 3e^{-\tau}$:*

$$R\big(\widehat{\theta}\big) - R\big(\theta^*\big) \le c_1 \lambda_N \gamma_N^{-d} + c_2 \gamma_N^{\beta} + c_3 \Big\{ \gamma_N^{(1-p)(1+\epsilon)d} \lambda_N^{p} N \Big\}^{-\frac{1}{2-p}} + c_4 N^{-1/2} \tau^{1/2} + c_5 N^{-1} \tau$$

Lemma B.2. *Let $\widehat{R}_{(-\ell)}(\theta) = \mathrm{E}\big\{\widehat{L}_{(-\ell)}\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big) \,\big|\, \mathcal{D}_\ell^c\big\}$ be the estimated risk function where the expectation is taken with respect to $\boldsymbol{O}_i$ while $\widehat{L}_{(-\ell)}$ is considered as a fixed function which is clarified by denoting $\mathcal{D}_\ell^c$ in the conditioning statement. Let $\Theta^{[0,1]} = \big\{ f \,\big|\, f(\boldsymbol{x}_i) \in [0,1] \big\}$ be the collection of policies ranging over the unit interval. Under Assumption (A1)-(A5) and (E1)-(E3) of the main* *paper, we have* $\big|R(\theta) - \widehat{R}_{(-\ell)}(\theta)\big| \le 0.5 c_6 r_N$ *with probability greater than* $1-\Delta_N$ *for any* $\theta \in \Theta^{[0,1]}$ *where* c_6 *is a fixed constant,* $r_N = r_{e,N}$ *if the inverse probability-weighted loss function is used,* $r_N = r_{\mu,N}$ *if the outcome regression loss function is used, and* $r_N = r_{e,N} r_{\mu,N}$ *if the doubly robust loss function is used.*

See Sections B.3 and B.4 for the proof.

B.2 Proof of Lemma 3.1 of the Main Paper

We only show the result about the overall outcome case because the result about the spillover outcome case is obtained from a similar manner. Let $\Theta^{[0,1]} = \big\{ f \,\big|\, f(\boldsymbol{x}_i) \in [0,1] \big\}$ be the collection of policies ranging over the unit interval and $\mathcal{W}(\theta) \in \Theta^{[0,1]}$ be the winsorized function of $\theta \in \Theta$ over the unit interval, i.e.

$$\mathcal{W}(\theta)(\boldsymbol{x}_i) = 0 \cdot \mathbb{1}\big\{\theta(\boldsymbol{x}_i) < 0\big\} + \theta(\boldsymbol{x}_i) \cdot \mathbb{1}\big\{0 \le \theta(\boldsymbol{x}_i) \le 1\big\} + 1 \cdot \mathbb{1}\big\{1 < \theta(\boldsymbol{x}_i)\big\} \ .$$

From the definition of L, we find $L(0, \boldsymbol{O}_i) \le L(t, \boldsymbol{O}_i)$ for any $t \in (-\infty, 0)$ and $L(1, \boldsymbol{O}_i) \le L(t, \boldsymbol{O}_i)$ for any $t \in (1, \infty)$. As a consequence, for any policy $\theta \in \Theta$ satisfies $L\big(\mathcal{W}(\theta)(\boldsymbol{X}_i), \boldsymbol{O}_i\big) \le L\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big)$ and $R(\mathcal{W}(\theta)) \le R(\theta)$. This implies that θ', the minimizer of R, must belong to $\Theta^{[0,1]}$.

For any function $\theta \in \Theta^{[0,1]}$, we find $L\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big) = \nu\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big)$ and $R(\theta) = R^{[0,1]}(\theta) = \mathrm{E}\big\{\nu\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\big\}$ due to the constructions of L and R. Combining the above results, we observe the following relationship.

$$\operatorname*{arg\,min}_{\theta \in \Theta} R(\theta) = \operatorname*{arg\,min}_{\theta \in \Theta^{[0,1]}} R(\theta) = \operatorname*{arg\,min}_{\theta \in \Theta^{[0,1]}} R^{[0,1]}(\theta)$$

Thus, it suffices to show that θ^* defined in equation (6) of the main paper minimizes $R^{[0,1]}$, which is represented as follows.

$$\begin{aligned}
& R^{[0,1]}(\theta) - C_0 \\
& = \mathrm{E}\big\{\nu\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\big\} - C_0 \\
& = \mathrm{E}\Bigg[\frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i-1} \binom{n_i-1}{s} \psi_{\mathrm{DR}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) \sum_{\ell=0}^{n_i-a-s} \binom{n_i-a-s}{\ell} \frac{(-1)^\ell \big\{\theta(\boldsymbol{X}_i)\big\}^{\ell+a+s+1}}{\ell+a+s+1} - \mathcal{T}\big\{\theta(\boldsymbol{X}_i)\big\} \Bigg] \\
& = \mathrm{E}\Bigg[\int_0^1 \Bigg[\frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i-1} \binom{n_i-1}{s} \mathrm{E}\big\{\psi_{\mathrm{DR}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) \,\big|\, \boldsymbol{X}_i\big\} \alpha^s (1-\alpha)^{n_i-a-s} - \mathcal{T} \Bigg] \mathbb{1}\big\{\alpha \le \theta(\boldsymbol{X}_i)\big\} \, d\alpha \Bigg] \\
& = \mathrm{E}\Bigg[\int_0^1 \Bigg\{ \frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i-1} \binom{n_i-1}{s} \mu^*\Big(a, \frac{s}{n_i-1}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)}\Big) \alpha^s (1-\alpha)^{n_i-a-s} - \mathcal{T} \Bigg\} \mathbb{1}\big\{\alpha \le \theta(\boldsymbol{X}_i)\big\} \, d\alpha \Bigg] \ . \qquad (4)
\end{aligned}$$

The first and second identities are trivial from the definition of $R^{[0,1]}$ and ν. The third identity is from the law of iterated expectation and the following algebra:

$$\begin{aligned}
& H(\boldsymbol{O}_i) \sum_{\ell=0}^{n_i-a-s} \binom{n_i-a-s}{\ell} \frac{(-1)^\ell t^{\ell+a+s+1}}{\ell+a+s+1} - \mathcal{T} t \\
& = \int_0^t \Big\{ H(\boldsymbol{O}_i) \alpha^s (1-\alpha)^{n_i-a-s} - \mathcal{T} \Big\} \, d\alpha \\
& = \int_0^1 \Big\{ H(\boldsymbol{O}_i) \alpha^s (1-\alpha)^{n_i-a-s} - \mathcal{T} \Big\} \mathbb{1}\{\alpha \le t\} \, d\alpha \ , \ {}^\forall H(\boldsymbol{O}_i) \ , \ {}^\forall t \in [0,1] \ .
\end{aligned}$$

The fourth identity is from $\mathrm{E}\big\{\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) \,\big|\, \boldsymbol{X}_i\big\} = \mu^*(a, \frac{s}{n_i-1}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)})$ for $s=0,1,\ldots,n_i-1$; we remark that any ψ' satisfying $\mathrm{E}\big\{\psi'(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) \,\big|\, \boldsymbol{X}_i\big\} = \mu^*(a, \frac{s}{n_i-1}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)})$ (e.g., inverse probability-weighted or outcome regression-based) can be used instead of ψ_{DR}. From the monotonicity condition (A5), it is straightforward to check that the following sets are intervals if they are non-empty:

$$\mathcal{S}_-(\boldsymbol{X}_i) := \left\{ \alpha \,\middle|\, \frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i-1} \binom{n_i-1}{s} \mathrm{E}\big\{\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) \,\big|\, \boldsymbol{X}_i\big\} \alpha^s (1-\alpha)^{n_i-a-s} < \mathcal{T} \right\}$$

$$\mathcal{S}_+(\boldsymbol{X}_i) := \left\{ \alpha \,\middle|\, \frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i-1} \binom{n_i-1}{s} \mathrm{E}\big\{\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) \,\big|\, \boldsymbol{X}_i\big\} \alpha^s (1-\alpha)^{n_i-a-s} \ge \mathcal{T} \right\} .$$

Since $\mathcal{S}_-(\boldsymbol{X}_i)$ and $\mathcal{S}_+(\boldsymbol{X}_i)$ are non-overlapping intervals, we establish that $\sup \mathcal{S}_-(\boldsymbol{X}_i)$ and $\inf \mathcal{S}_+(\boldsymbol{X}_i)$ are equivalent. If $\mathcal{S}_-(\boldsymbol{X}_i)$ and $\mathcal{S}_+(\boldsymbol{X}_i)$ are empty, we define $\mathcal{S}_-(\boldsymbol{X}_i) = \{0\}$ and $\mathcal{S}_+(\boldsymbol{X}_i) = \{1\}$, respectively. In these cases, we also establish that $\sup \mathcal{S}_-(\boldsymbol{X}_i)$ and $\inf \mathcal{S}_+(\boldsymbol{X}_i)$ are equivalent as 0 or 1, respectively. We remark that, without the monotonicity condition (A5), these two sets may be disconnected sets, i.e., not intervals, and we cannot establish that $\sup \mathcal{S}_-(\boldsymbol{X}_i)$ and $\inf \mathcal{S}_+(\boldsymbol{X}_i)$ are equivalent.

The last representation (4) suggests that that $R^{[0,1]}(\theta)$ is minimized at θ' where $\theta'(\boldsymbol{X}_i) = \sup \mathcal{S}_-(\boldsymbol{X}_i) = \inf \mathcal{S}_+(\boldsymbol{X}_i)$, i.e.,

$$\frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i-1} \binom{n_i-1}{s} \mathrm{E}\big\{\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) \,\big|\, \boldsymbol{X}_i\big\} \alpha^s (1-\alpha)^{n_i-a-s} \le \mathcal{T} \ \text{ for all } \alpha \in \big[0, \theta'(\boldsymbol{X}_i)\big] \ ,$$

$$\frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i-1} \binom{n_i-1}{s} \mathrm{E}\big\{\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) \,\big|\, \boldsymbol{X}_i\big\} \alpha^s (1-\alpha)^{n_i-a-s} \ge \mathcal{T} \ \text{ for all } \alpha \in \big[\theta'(\boldsymbol{X}_i), 1\big] \ .$$

As a consequence, θ' agrees with θ^*_{OV} defined in (6) of the main paper. We can establish the result about θ^*_{SO} by fixing $a=0$. This concludes the proof.

B.3 Proof of Lemma B.1

We only show the result about the overall outcome case because the result about the spillover outcome case is obtained from a similar manner. The proof of B.1 is similar to that of Theorem 2 of Chen et al. (2016) which use Theorem 7.23 of Steinwart and Christmann (2008) and Theorem 2.2 and 2.3 of Eberts and Steinwart (2013). For completeness, we present a full exposition to our setting below. We first introduce Theorem 7.23 of Steinwart and Christmann (2008).

Theorem 7.23. (*Oracle Inequality for SVMs Using Benign Kernels*; Steinwart and Christmann (2008)) Let L be a loss function having non-negative value. Also, let $\mathcal{H}_{\mathcal{K}}$ be a separable RKHS of a measurable kernel $\mathcal{K}$ over $\mathcal{X} = \text{supp}(\boldsymbol{X}_i) \subset \mathbb{R}^d$. Let P be a distribution on $\boldsymbol{O}_i$. Furthermore, suppose the following conditions are satisfied.

(C1) For all $(t, \boldsymbol{o}_i)$, there exists a constant $B > 0$ satisfying $L(t, \boldsymbol{o}_i) \leq B$.

(C2) $L(t, \boldsymbol{o}_i)$ is locally Lipschitz continuous with respect to t.

(C3) For all $(t, \boldsymbol{o}_i)$, we have $L(\mathcal{W}_{c_0}(t), \boldsymbol{o}_i) \leq L(t, \boldsymbol{o}_i)$ where $\mathcal{W}_{c_0}(t) = t \cdot \mathbb{1}\big\{|t| \leq c_0\big\} + \text{sign}(t) c_0 \cdot \mathbb{1}\big\{c_0 < |t|\big\}$.

(C4) $\text{E}\big[\big\{L\big(\mathcal{W}_{c_0}(\theta)(\boldsymbol{X}_i), \boldsymbol{O}_i\big) - L\big(\theta^*(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\big\}^2\big] \leq V \cdot \big[\text{E}\big\{L\big(\mathcal{W}_{c_0}(\theta)(\boldsymbol{X}_i), \boldsymbol{O}_i\big) - L\big(\theta^*(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\big\}\big]^v$ is satisfied for constant $v \in [0,1]$, $V \geq B^{2-v}$, and for all $\theta \in \mathcal{H}_{\mathcal{K}}$.

(C5) For fixed $N \geq 1$, there exists constants $p \in (0,1)$ and $a \geq B$ such that the dyadic entropy number $\text{E}_{D_X \sim P_X^N}\big[e_i\big(\text{identity map} : \mathcal{H}_{\mathcal{K}} \to L_2(D_{\boldsymbol{X}})\big)\big] \leq a \cdot i^{-\frac{1}{2p}}$ $(i \geq 1)$ where $e_i(A)$ is the entropy number of A.

We fix $\theta_0 \in \mathcal{H}_{\mathcal{K}}$ and a constant $B_0 \geq B$ such that $L\big(\theta_0(\boldsymbol{x}_i), \boldsymbol{o}_i\big) \leq B_0$ for any $\boldsymbol{o}_i$. Then, for all fixed $\tau > 0$ and λ_N, the SVM using $\mathcal{H}_{\mathcal{K}}$ and L satisfies

$$
\begin{aligned}
& \lambda_N \big\|\theta\big\|_{\mathcal{H}_{\mathcal{K}}}^2 + R\big(\mathcal{W}_{c_0}(\theta)\big) - R\big(\theta^*\big) \\
& \leq 9\big\{\lambda_N \big\|\theta_0\big\|_{\mathcal{H}_{\mathcal{K}}}^2 + R\big(\theta_0\big) - R\big(\theta^*\big)\big\} + K_0 \left(\frac{a^{2p}}{\lambda_N^p N}\right)^{\frac{1}{2-p-v+vp}} + 3\left(\frac{72V\tau}{N}\right)^{\frac{1}{2-v}} + \frac{15 B_0 \tau}{N} \ .
\end{aligned} \tag{5}
$$

with probability P^N not less than $1 - 3e^{-\tau}$, where $K_0 \geq 1$ is a constant only depending on p, c_0, B, v, and V.

We verify Assumptions (C1)-(C5) as follows:

Verification of Assumption (C1): From Assumptions (A1)-(A5) and $\overline{Y}_i \in [0,1]$, we find that ψ_{IPW}, ψ_{OR}, and ψ_{DR} are bounded and, as a consequence, ν in equation (9a) of the main paper is bounded. As a result, L in equation (9) of the main paper is bounded as well.

Verification of Assumption (C2): We find the derivative of L in equation (9) of the main paper is

$$\nabla L(t, \boldsymbol{o}_i) = \begin{cases} \delta e^t & t \in (-\infty, 0) \\ \sum_{s=0}^{n_i} \psi_\ell(s, \boldsymbol{o}_i) t^s (1-t)^{n_i - s} - \mathcal{T} & t \in (0,1) \\ \delta e^{1-t} & t \in (0, \infty) \end{cases}$$

and we find that $\nabla L(t, \boldsymbol{O}_i)$ is bounded for all t except $t = 0, 1$. Moreover, $L(t, \boldsymbol{o}_i)$ is continuous at $t = 0$ and $t = 1$. Thus, $L(t, \boldsymbol{o}_i)$ is locally Lipschitz continuous with Lipschitz constant $B' = \sup_{(t, \boldsymbol{o}_i)} \nabla L(t, \boldsymbol{o}_i)$.

Verification of Assumption (C3): We take $c_0 = 1$. It is trivial that $L(\mathcal{W}_{c_0}(\theta), \boldsymbol{o}_i) \le L(\theta, \boldsymbol{o}_i)$ from the form of L in equation (9) of the main paper.

Verification of Assumption (C4): Note that $L(t, \boldsymbol{o}_i) \le B$. Thus, we find

$$\mathrm{E}\Big[\big\{L\big(\mathcal{W}_{c_0}(\theta)(\boldsymbol{X}_i), \boldsymbol{O}_i\big) - L\big(\theta^*(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\big\}^2\Big] \le 2\mathrm{E}\Big[\big\{L\big(\mathcal{W}_{c_0}(\theta)(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\big\}^2 + \big\{L\big(\theta^*(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\big\}^2\Big] \le 4B^2 \ .$$

We take $v = 0$ and $V = 4B^2$ and the condition is satisfied.

Verification of Assumption (C5): Since we use the Gaussian kernel, we can directly use Theorem 7.34 of Steinwart and Christmann (2008) which is given below.

Theorem 7.34. (*Entropy Numbers for Gaussian Kernels*; Steinwart and Christmann (2008)) Let ν be a distribution on $\mathbb{R}^d$ having tail exponent $\tau \in (0, \infty]$. Then, for all $\epsilon > 0$ and $d/(d+\tau) < p < 1$, there exists a constant $c_{\epsilon,p} \ge 1$ such that

$$e_i\big(\text{identity map} : \mathcal{H}_{\mathcal{K}} \to L_2(\nu)\big) \le c_{\epsilon,p} \gamma^{-\frac{(1-p)(1+\epsilon)d}{2p}} i^{-\frac{1}{2p}}$$

for all $i \ge 1$ and $\gamma \in (0, 1]$.

Therefore, Assumption (C5) holds with $a = c_{\epsilon,p} \gamma_N^{-\frac{(1-p)(1+\epsilon)d}{2p}}$.

As a consequence, the result in equation (5) holds with $c_0 = 1$, $v = 1$, $V = 4B^2$, $B_0 = B$, $a = c_{\epsilon,p} \gamma_N^{-\frac{(1-p)(1+\epsilon)d}{2p}}$. Moreover, we find $L(\mathcal{W}(\theta)(\boldsymbol{x}_i), \boldsymbol{o}_i) \le L(\mathcal{W}_{c_0=1}(\theta)(\boldsymbol{x}_i), \boldsymbol{o}_i)$ since $L(0, \boldsymbol{o}_i) \le L(t, \boldsymbol{o}_i)$ for $t \in [-1, 0]$ and this leads $R(\mathcal{W}(\theta)) \le R(\mathcal{W}_{c_0=1}(\theta))$. Thus, we find the following result holds with probability P^N not less than $1-3e^{-\tau}$.

$$
\begin{aligned}
&R(\widehat{\theta}) - R(\theta^*) \qquad (6)\\
&\leq R\big(\mathcal{W}_{c_0=1}(\widetilde{\theta})\big) - R(\theta^*)\\
&\leq \lambda_N \big\|\widehat{\theta}\big\|_{\mathcal{H}_\mathcal{K}}^2 + R\big(\mathcal{W}_{c_0=1}(\widetilde{\theta})\big) - R(\theta^*)\\
&\leq 9\big\{\lambda_N \big\|\theta_0\big\|_{\mathcal{H}_\mathcal{K}}^2 + R(\theta_0) - R(\theta^*)\big\} + K_0\left\{\frac{\gamma_N^{-(1-p)(1+\epsilon)d}}{\lambda_N^p N}\right\}^{\frac{1}{2-p}} + 36\sqrt{2}B\sqrt{\frac{\tau}{N}} + 15B\frac{\tau}{N}\ .
\end{aligned}
$$

The above result holds for any $\theta_0 \in \mathcal{H}_\mathcal{K}$, so we can further bound the approximation error $\lambda_N\big\|\theta_0\big\|_{\mathcal{H}_\mathcal{K}}^2 + R(\theta_0) - R(\theta^*)$ by choosing θ_0 in a specific way which is presented in Eberts and Steinwart (2013). We first define a function $Q_{r,\gamma}:\mathbb{R}^d \rightarrow \mathbb{R}$ as

$$
Q_{r,\gamma}(\boldsymbol{z}) = \sum_{j=1}^{r} \binom{r}{j} (-1)^{1-j} \frac{1}{j^d} \left(\frac{2}{\gamma^2}\right)^{\frac{d}{2}} \mathcal{K}_{j\gamma/\sqrt{2}}(\boldsymbol{z})\ ,\ \mathcal{K}_\gamma(\boldsymbol{z}) = \exp\big\{-\gamma^2 \big\|\boldsymbol{z}\big\|_2^2\big\} \tag{7}
$$

for $r \in \{1,2,\ldots\}$ and $\gamma > 0$. Since the range of θ^* is bounded between $[0,1]$, we find $\theta^* \in L_2(\mathbb{R}^d) \cap L_\infty(\mathbb{R}^d)$. Thus, we can define θ_0 by convolving $Q_{r,\gamma}$ with θ^* as follows (Eberts and Steinwart, 2013).

$$
\theta_0(\boldsymbol{x}_i) = \big(Q_{r,\gamma} * \theta^*\big)(\boldsymbol{x}_i) = \int_{\mathbb{R}^d} Q_{r,\gamma}(\boldsymbol{x}_i - \boldsymbol{z})\theta^*(\boldsymbol{z})\, d\boldsymbol{z}\ .
$$

Next, we introduce theorem 2.2 and 2.3 of Eberts and Steinwart (2013).

Theorem 2.2. Let us fix some $q \in [1,\infty)$. Furthermore, assume that $P_{\boldsymbol{X}}$ is a distribution on $\mathbb{R}^d$ that has a Lebesgue density $f_{\boldsymbol{X}} \in L_p(\mathbb{R}^d)$ for some $p \in [1,\infty]$. Let $\theta : \mathbb{R}^d \rightarrow \mathbb{R}$ be such that $\theta \in L_q(\mathbb{R}^d) \cap L_\infty(\mathbb{R}^d)$. Then, for $r \in \{1,2,\ldots\}$, $\gamma > 0$, and $s \geq 1$ with $1 = s^{-1} + p^{-1}$, we have

$$
\big\|Q_{r,\gamma} * \theta - \theta\big\|_{L_q(P_{\boldsymbol{X}})}^q \leq C_{r,q} \cdot \big\|f_{\boldsymbol{X}}\big\|_{L_p(\mathbb{R}^d)} \cdot \omega_{r,L_{qs}(\mathbb{R}^d)}^q(\theta, \gamma/2)
$$

where $C_{r,q}$ is a constant only depending on r and q.

Theorem 2.3. Let $\theta \in L_2(\mathbb{R}^d)$, $\mathcal{H}_\mathcal{K}$ be the RKHS of the Gaussian kernel $\mathcal{K}$ with parameter $\gamma > 0$, and $Q_{r,\gamma}$ be defined by (7) for a fixed $r \in \{1,2,\ldots\}$. Then we have $Q_{r,\gamma} * \theta \in \mathcal{H}_\mathcal{K}$ with

$$
\big\|Q_{r,\gamma} * \theta\big\|_{\mathcal{H}_\mathcal{K}} \leq \big(\gamma\sqrt{\pi}\big)^{-\frac{d}{2}} \big(2^r - 1\big) \big\|\theta\big\|_{L_2(\mathbb{R}^d)}\ .
$$

Moreover, if $\theta \in L_\infty(\mathbb{R}^d)$, we have $|Q_{r,\gamma} * \theta| \leq (2^r - 1)\big\|\theta\big\|_{L_\infty(\mathbb{R}^d)}$.

As a result, we obtain

$$
\begin{aligned}
&\lambda_N \|\theta_0\|_{\mathcal{H}_{\mathcal{K}}}^2 + R(\theta_0) - R(\theta^*) \\
&= \lambda_N \|Q_{r,\gamma_N} * \theta^*\|_{\mathcal{H}_{\mathcal{K}}}^2 + R(Q_{r,\gamma_N} * \theta^*) - R(\theta^*) \\
&\le \lambda_N (\gamma_N \sqrt{\pi})^{-d} (2^r - 1)^2 \|\theta^*\|_{L_2(\mathbb{R}^d)}^2 + R(Q_{r,\gamma_N} * \theta^*) - R(\theta^*) \\
&\le \lambda_N (\gamma_N \sqrt{\pi})^{-d} (2^r - 1)^2 \|\theta^*\|_{L_2(\mathbb{R}^d)}^2 + B' \cdot \|Q_{r,\gamma_N} * \theta^* - \theta^*\|_{L_1(P_{\boldsymbol{X}})} \\
&\le \lambda_N (\gamma_N \sqrt{\pi})^{-d} (2^r - 1)^2 \|\theta^*\|_{L_2(\mathbb{R}^d)}^2 + B' \cdot C_{r,1} \cdot \|f_{\boldsymbol{X}}\|_{L_\infty(\mathbb{R}^d)} \cdot \omega_{r,L_1(\mathbb{R}^d)}(\theta, \gamma_N/2) \\
&\le \lambda_N (\gamma_N \sqrt{\pi})^{-d} (2^r - 1)^2 \|\theta^*\|_{L_2(\mathbb{R}^d)}^2 + B'c' \cdot C_{r,1} \cdot \|f_{\boldsymbol{X}}\|_{L_\infty(\mathbb{R}^d)} \gamma_N^\beta
\end{aligned}
\tag{8}
$$

The first equality is from the construction of θ_0. The first inequality is from Theorem 2.3 of Eberts and Steinwart (2013). The second inequality is from Lipschitz continuity of L. The third inequality is from Theorem 2.2 of Eberts and Steinwart (2013) with $q = s = 1$ and $p = \infty$. The last inequality holds for some constant c' since $\theta^* \in \mathcal{B}_{1,\infty}^\beta(\mathbb{R}^d)$ implies $\omega_{r,L_1(\mathbb{R}^d)}(\theta^*, \gamma_N/2) \le c'\gamma_N^\beta$ from the definition of a Besov space. Combining the results in (6) and (8), we have the following result

$$
\begin{aligned}
R(\widehat{\theta}) - R(\theta^*) &\le 9\big\{\lambda_N (\gamma_N \sqrt{\pi})^{-d} (2^r - 1)^2 \|\theta^*\|_{L_2(\mathbb{R}^d)}^2 + B'c' \cdot C_{r,1} \cdot \|f_{\boldsymbol{X}}\|_{L_\infty(\mathbb{R}^d)} \gamma_N^\beta \big\} \\
&\qquad + K_0 \left\{ \frac{\gamma_N^{-(1-p)(1+\epsilon)d}}{\lambda_N^p N} \right\}^{\frac{1}{2-p}} + 36\sqrt{2} B \sqrt{\frac{\tau}{N}} + 15 B \frac{\tau}{N} \\
&\le c_1 \lambda_N \gamma_N^{-d} + c_2 \gamma_N^\beta + c_3 \left\{ \gamma_N^{(1-p)(1+\epsilon)d} \lambda_N^p N \right\}^{-\frac{1}{2-p}} + c_4 N^{-1/2} \tau^{1/2} + c_5 N^{-1} \tau \ .
\end{aligned}
$$

B.4 Proof of Lemma B.2

We only show the result about the overall outcome case because the result about the spillover outcome case is obtained from a similar manner. We find the following result for $t \in [0, 1]$.

$$
\begin{aligned}
&\left| \widehat{L}_{(-\ell)}(t, \boldsymbol{O}_i) - L(t, \boldsymbol{O}_i) \right| \\
&= \left| \widehat{\nu}_{(-\ell)}(t, \boldsymbol{O}_i) - \nu(t, \boldsymbol{O}_i) \right| \\
&= \left| \frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i - 1} \binom{n_i - 1}{s} \left\{ \widehat{\psi}_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) \right\} \sum_{\ell=0}^{n_i - a - s} \binom{n_i - a - s}{\ell} \frac{(-1)^\ell t^{\ell + a + s + 1}}{\ell + a + s + 1} \right| \\
&\le \frac{1}{n_i} \sum_{j=1}^{n_i} \sum_{a=0}^{1} \sum_{s=0}^{n_i - 1} \binom{n_i - 1}{s} \left| \widehat{\psi}_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) \right| \sum_{\ell=0}^{n_i - a - s} \binom{n_i - a - s}{\ell} \frac{t^{\ell + a + s + 1}}{\ell + a + s + 1} \\
&\le C' \max_{(a,s) \in \{0,1\} \otimes \{0,\ldots,M\}} \left| \widehat{\psi}_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) \right|
\end{aligned}
\tag{9}
$$

for some generic constant C'. The last inequality is from $t \in [0, 1]$ and bounded n_i. Also, we find the following result for $t \in (-\infty, 0)$.

$$
\left| \widehat{L}_{(-\ell)}(t, \boldsymbol{O}_i) - L(t, \boldsymbol{O}_i) \right| = \left| \widehat{\nu}_{(-\ell)}(0, \boldsymbol{O}_i) - \nu(0, \boldsymbol{O}_i) \right| = 0 \ .
$$

Finally, we find the following result for $t \in (1, \infty)$ for all $s = 0, 1, \ldots, n_i$.

$$\begin{aligned}\left|\widehat{L}_{(-\ell)}(t, \boldsymbol{O}_i) - L(t, \boldsymbol{O}_i)\right| &= \left|\widehat{\nu}_{(-\ell)}(1, \boldsymbol{O}_i) - \nu(1, \boldsymbol{O}_i)\right| \\ &\leq C' \max_{(a,s)\in\{0,1\}\otimes\{0,\ldots,M\}} \left|\widehat{\psi}_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})\right|\end{aligned}$$

where constant C' is given in (9). Therefore, for any θ, we find

$$\begin{aligned}\left|R(\theta) - \widehat{R}_{(-\ell)}(\theta)\right| &\leq \mathrm{E}\left[\left|L\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big) - \widehat{L}_{(-\ell)}\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\right| \,\middle|\, \mathcal{D}_\ell^c\right] \\ &\leq \mathrm{E}\left[\left|L\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big) - \widehat{L}_{(-\ell)}\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big)\right|^2 \,\middle|\, \mathcal{D}_\ell^c\right]^{1/2} \\ &\leq C'\mathrm{E}\left[\max_{(a,s)\in\{0,1\}\otimes\{0,\ldots,M\}} \left|\widehat{\psi}_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})\right|^2 \,\middle|\, \mathcal{D}_\ell^c\right]^{1/2} \\ &\leq C' \max_{(a,s)\in\{0,1\}\otimes\{0,\ldots,M\}} \left\|\widehat{\psi}_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})\right\|_{P,2}\end{aligned}$$

The first inequality is from the definition of R. The second inequality is from the Jensen's inequality. The third inequality is from the above results. The last inequality is from the definition of $\|\cdot\|_{P,2}$. Therefore, it suffices to bound $\|\widehat{\psi}_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_k(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})\|_{P,2}$ for all three types of ψ_k where $k \in \{\mathrm{IPW}, \mathrm{OR}, \mathrm{DR}\}$.

First, the difference between $\widehat{\psi}_{\mathrm{IPW}}$ and $\psi_{\mathrm{IPW}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})$ is

$$\begin{aligned}&\left|\widehat{\psi}_{\mathrm{IPW}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_{\mathrm{IPW}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})\right| \\ &= \left|\frac{Y_{ij}\mathbb{1}(A_{ij} = a, S_{i(-j)} = s)}{\widehat{e}_{(-\ell)}\big(a, s \,|\, \boldsymbol{X}_i\big)} - \frac{Y_{ij}\mathbb{1}(A_{ij} = a, S_{i(-j)} = s)}{e^*\big(a, s \,|\, \boldsymbol{X}_i\big)}\right| \\ &\leq \left|Y_{ij}\mathbb{1}(A_{ij} = a, S_{i(-j)} = s)\right| \frac{\left|e^*\big(a, s \,|\, \boldsymbol{X}_i\big) - \widehat{e}_{(-\ell)}\big(a, s \,|\, \boldsymbol{X}_i\big)\right|}{\widehat{e}_{(-\ell)}\big(a, s \,|\, \boldsymbol{X}_i\big) e^*\big(a, s \,|\, \boldsymbol{X}_i\big)} \\ &\leq \frac{\left|e^*\big(a, s \,|\, \boldsymbol{X}_i\big) - \widehat{e}_{(-\ell)}\big(a, s \,|\, \boldsymbol{X}_i\big)\right|}{cc'} \ .\end{aligned}$$

The upper bound is from the bounded outcome and Assumptions (A3) and (E1) of the main paper. Thus, we find the following result with probability greater than $1 - \Delta_N$:

$$\begin{aligned}\left\|\widehat{\psi}_{\mathrm{IPW}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_{\mathrm{IPW}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})\right\|_{P,2} &\leq \frac{1}{cc'}\left\|e^*(a, s \,|\, \boldsymbol{X}_i) - \widehat{e}_{(-\ell)}(a, s \,|\, \boldsymbol{X}_i)\right\|_{P,2} \\ &\leq C_{\mathrm{IPW}} \cdot r_{e,N}(a, s)\end{aligned}$$

where $C_{\mathrm{IPW}} = 1/(cc')$.

Second, we study the difference between $\widehat{\psi}_{\mathrm{OR}}$ and ψ_{OR} which is

$$\begin{aligned}&\left|\widehat{\psi}_{\mathrm{OR}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)}) - \psi_{\mathrm{OR}}(a, s, \boldsymbol{O}_{ij}, \boldsymbol{O}_{i(-j)})\right| \\ &= \left|\widehat{\mu}_{(-\ell)}(a, s/(n_i - 1), \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)}) - \mu^*(a, s/(n_i - 1), \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)})\right| \ .\end{aligned}$$

Therefore, we find the following result with probability greater than $1-\Delta_N$.

$$
\begin{aligned}
&\big\|\widehat{\psi}_{\mathrm{OR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})-\psi_{\mathrm{OR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})\big\|_{P,2} \\
&=\left\|\widehat{\mu}_{(-\ell)}\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)-\mu^*\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)\right\|_{P,2}=C_{\mathrm{OR}}\cdot r_{\mu,N}
\end{aligned}
$$

where $C_{\mathrm{OR}}=1$.

Lastly, we prove the result when ψ_{DR} is chosen. From (9), we have

$$
\begin{aligned}
&-C'\Big[\widehat{\psi}_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})-\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})\Big] \\
&\le\widehat{L}_{(-\ell)}(t,\boldsymbol{O}_i)-L(t,\boldsymbol{O}_i)\le C'\Big[\widehat{\psi}_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})-\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})\Big]
\end{aligned}
$$

where the sign of C' is chosen to satisfy the inequality above. The expectation of $\widehat{\psi}_{\mathrm{DR}}-\psi_{\mathrm{DR}}$ is

$$
\begin{aligned}
&\mathrm{E}\Big\{\widehat{\psi}_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})-\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})\,\Big|\,\mathcal{D}_\ell^c\Big\} \\
&=\mathrm{E}\Bigg[\Bigg\{\frac{Y_{ij}-\widehat{\mu}_{(-\ell)}(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})}{\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)}-\frac{Y_{ij}-\mu(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})}{e^*(a,s\,|\,\boldsymbol{X}_i)}\Bigg\}\mathbb{1}(A_{ij}=a,S_{i(-j)}=s) \\
&\qquad+\widehat{\mu}_{(-\ell)}\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)-\mu^*\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)\,\Bigg|\,\mathcal{D}_\ell^c\Bigg] \\
&=\mathrm{E}\Bigg[\frac{\{\mu^*(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})-\widehat{\mu}_{(-\ell)}(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})\}e^*(a,s\,|\,\boldsymbol{X}_i)}{\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)} \\
&\qquad+\widehat{\mu}_{(-\ell)}\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)-\mu^*\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)\,\Bigg|\,\mathcal{D}_\ell^c\Bigg] \\
&=\mathrm{E}\Bigg[\frac{\{\mu^*(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})-\widehat{\mu}_{(-\ell)}(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})\}\{e^*(a,s\,|\,\boldsymbol{X}_i)-\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)\}}{\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)}\,\Bigg|\,\mathcal{D}_\ell^c\Bigg]\ .
\end{aligned}
$$

The equalities are straightforward from the definition of ψ_{DR} and the law of total expectation. Since $c'\le\widehat{e}_{(-\ell)}$, we find

$$
\begin{aligned}
&\mathrm{E}\Bigg[\frac{\{\mu^*(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})-\widehat{\mu}_{(-\ell)}(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)})\}\{e^*(a,s\,|\,\boldsymbol{X}_i)-\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)\}}{\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)}\,\Bigg|\,\mathcal{D}_\ell^c\Bigg] \\
&\le\frac{1}{c'}\mathrm{E}\Bigg[\left|\mu\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)-\widehat{\mu}_{(-\ell)}\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)\right|\left|e(a,s\,|\,\boldsymbol{X}_i)-\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)\right|\,\Bigg|\,\mathcal{D}_\ell^c\Bigg] \\
&\le\frac{1}{c'}\left\|\widehat{\mu}_{(-\ell)}\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)-\mu^*\left(a,\frac{s}{n_i-1},\boldsymbol{X}_{ij},\boldsymbol{X}_{i(-j)}\right)\right\|_{P,2}\left\|e^*(a,s\,|\,\boldsymbol{X}_i)-\widehat{e}_{(-\ell)}(a,s\,|\,\boldsymbol{X}_i)\right\|_{P,2}\ .
\end{aligned}
$$

The first inequality is straightforward. The second inquality is from the Hölder's inequality. From the last line, we find $\mathrm{E}\big\{\widehat{\psi}_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})-\psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)})\,\big|\,\mathcal{D}_\ell^c\big\}=O_P(r_{e,N}r_{\mu,N})$. As a result, we have the following result with probability greater than $1-\Delta_N$.

$$
\begin{aligned}
& \Big| R(\theta) - \widehat{R}_{(-\ell)}(\theta) \Big| \\
& = \Big| \mathrm{E}\Big\{ L\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big) - \widehat{L}_{(-\ell)}\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big) \,\Big|\, \mathcal{D}_\ell^c \Big\} \Big| \\
& \leq C' \max_{(a,s)\in\{0,1\}\otimes\{0,\ldots,M\}} \Big| \mathrm{E}\Big\{ \widehat{\psi}_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) - \psi_{\mathrm{DR}}(a,s,\boldsymbol{O}_{ij},\boldsymbol{O}_{i(-j)}) \,\Big|\, \mathcal{D}_\ell^c \Big\} \Big| \\
& \leq \frac{C}{c'} \max_{(a,s)\in\{0,1\}\otimes\{0,\ldots,M\}} \left[\begin{array}{l} \Big\| \mu^*(a, \tfrac{s}{n_i-1}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)}) - \widehat{\mu}_{(-\ell)}(a, \tfrac{s}{n_i-1}, \boldsymbol{X}_{ij}, \boldsymbol{X}_{i(-j)}) \Big\|_{P,2} \\ \quad \times \Big\| e^*(a, s \,|\, \boldsymbol{X}_i) - \widehat{e}_{(-\ell)}(a, s \,|\, \boldsymbol{X}_i) \Big\|_{P,2} \end{array} \right] \\
& \leq C_{\mathrm{DR}} \cdot r_{e,N} r_{\mu,N}
\end{aligned}
$$

where $C_{\mathrm{DR}} = C/c'$.

Combining the established results, we have the following results with probability greater than $1-\Delta_N$.

$$
\Big| R(\theta) - \widehat{R}_{(-\ell)}(\theta) \Big| \leq \begin{cases} C_{\mathrm{IPW}} \cdot r_{e,N} & \text{if } \psi_{\mathrm{IPW}} \text{ is chosen} \\ C_{\mathrm{OR}} \cdot r_{\mu,N} & \text{if } \psi_{\mathrm{OR}} \text{ is chosen} \\ C_{\mathrm{DR}} \cdot r_{e,N} r_{\mu,N} & \text{if } \psi_{\mathrm{DR}} \text{ is chosen} \end{cases} .
$$

This implies $\big| R(\theta) - \widehat{R}_{(-\ell)}(\theta) \big| \leq 0.5 c_6 r_N$ with probability greater than $1-\Delta_N$ where $c_6 = 2 \cdot \max\{C_{\mathrm{IPW}}, C_{\mathrm{OR}}, C_{\mathrm{DR}}\}$, $r_N = r_{e,N}$ if the inverse probability-weighted loss function is used, $r_N = r_{\mu,N}$ if the outcome regression loss function is used, and $r_N = r_{e,N} r_{\mu,N}$ if the doubly robust loss function is used.

B.5 Proof of Theorem 3.1 of the Main Paper

We only show the result related to the overall outcome case because the result related to the spillover outcome case is obtained in a similar manner. We start with defining the risk function and the MRTP associated with the estimated loss function. Let $\widehat{R}_{(-\ell)}(\theta) = \mathrm{E}\big\{ \widehat{L}_{(-\ell)}\big(\theta(\boldsymbol{X}_i), \boldsymbol{O}_i\big) \,\big|\, \mathcal{D}_\ell^c \big\}$ be the estimated risk function where the expectation is taken with respect to $\boldsymbol{O}_i$ while $\widehat{L}_{(-\ell)}$ is considered as a fixed function which is clarified by denoting $\mathcal{D}_\ell^c$ in the conditioning statement. Accordingly, let $\theta^*_{(-\ell)}$ be the approximated MRTP which is the minimizer of $\widehat{R}_{(-\ell)}(\theta)$, i.e., $\widehat{R}_{(-\ell)}(\theta^*_{(-\ell)}) \leq \widehat{R}_{(-\ell)}(\theta)$ for all $\theta \in \Theta$. We remark that $\theta^*_{(-\ell)} \in \Theta^{[0,1]} = \big\{ f \,\big|\, f(\boldsymbol{x}_i) \in [0,1] \big\}$, the collection of policies ranging over the unit interval. Using $\theta^*_{(-\ell)}$ as the intermediate quantities, we can establish the excess risk of $\widehat{\theta}_{(-\ell)}$.

We decompose the excess risk as follows.

$$\begin{aligned}
&\left|R\big(\widehat{\theta}_{(-\ell)}\big) - R\big(\theta^*\big)\right| \\
&= \underbrace{\left|R\big(\widehat{\theta}_{(-\ell)}\big) - \widehat{R}_{(-\ell)}\big(\widehat{\theta}_{(-\ell)}\big)\right|}_{(A)} + \underbrace{\left|\widehat{R}_{(-\ell)}\big(\widehat{\theta}_{(-\ell)}\big) - \widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big)\right|}_{(B)} + \underbrace{\left|\widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*\big)\right|}_{(C)} .
\end{aligned}$$

In the rest of the proof, we bound terms (A), (B), and (C).

From Lemma B.2 of the Supplementary Material, we find the upper bound of (A) with probability greater than $1-\Delta_N$:

$$\left|R\big(\widehat{\theta}_{(-\ell)}\big) - \widehat{R}_{(-\ell)}\big(\widehat{\theta}_{(-\ell)}\big)\right| \leq 0.5 c_6 r_N .$$

Next, we bound (B). Since $\widehat{L}_{(-\ell)}$ satisfies Assumptions (C1)-(C5) and $\theta^*_{(-\ell)}$ belongs to a Besov space $\mathcal{B}^{\beta}_{1,\infty}(\mathbb{R}^d)$, Theorem B.1 can be applied. Hence, the following result is satisfied with probability greater than $1-3e^{-\tau}$:

$$\begin{aligned}
&\left|\widehat{R}_{(-\ell)}\big(\widehat{\theta}_{(-\ell)}\big) - \widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big)\right| \\
&\leq c_1 \lambda_N \gamma_N^{-d} + c_2 \gamma_N^{\beta} + c_3 \Big\{ \gamma_N^{(1-p)(1+\epsilon)d} \lambda_N^p N \Big\}^{-\frac{1}{2-p}} + c_4 N^{-1/2} \tau^{1/2} + c_5 N^{-1} \tau .
\end{aligned}$$

Lastly, we bound (C). Since θ^* is the minimizer of $R(\theta)$, we find

$$\widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) = R\big(\theta^*_{(-\ell)}\big) + \widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*_{(-\ell)}\big) \geq R\big(\theta^*\big) + \widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*_{(-\ell)}\big) ,$$

and this implies $\widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*_{(-\ell)}\big) \leq \widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*\big)$. Similarly, since $\theta^*_{(-\ell)}$ is the minimizer of $\widehat{R}_{(-\ell)}(\theta)$, we find

$$R\big(\theta^*\big) = \widehat{R}_{(-\ell)}\big(\theta^*\big) + R\big(\theta^*\big) - \widehat{R}_{(-\ell)}\big(\theta^*\big) \geq \widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) + R\big(\theta^*\big) - \widehat{R}_{(-\ell)}\big(\theta^*\big) ,$$

and this implies $R\big(\theta^*\big) - \widehat{R}_{(-\ell)}\big(\theta^*\big) \leq R\big(\theta^*\big) - \widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big)$, i.e., $\widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*\big) \leq \widehat{R}_{(-\ell)}\big(\theta^*\big) - R\big(\theta^*\big)$. Combining two results, we have

$$\left|\widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*\big)\right| \leq \max\left\{ \left|\widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*_{(-\ell)}\big)\right|, \left|\widehat{R}_{(-\ell)}\big(\theta^*\big) - R\big(\theta^*\big)\right| \right\} .$$

From Lemma B.2 of the Supplementary Material, the right hand side of the above term is upper bounded by $0.5c_6 r_N$ with probability greater than $1-\Delta_N$ because $\theta^*_{(-\ell)}, \theta^* \in \Theta^{[0,1]}$. As a result, we find an upper bound of (C), which is $\big|\widehat{R}_{(-\ell)}\big(\theta^*_{(-\ell)}\big) - R\big(\theta^*\big)\big| = O_P(r_N)$. We remark that (A) and (C) are both $O_P(r_N)$ with probability greater than $1-\Delta_N$. This concludes the desired result.

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