To Referee no 2

June 17, 2016

Dear Reviewer,
I would like to thank you for your valuable comments and question. Please find attached the new version of the article that integrate the relevant given recommendations. In fact, the general presentation is changed and the several sections are reorganised. The problem statement is now separated from the main contribution of this paper. The transitions between section are smoother and the understanding is easier. In addition, More attention is given to the assumptions and the existence conditions for every defined quantity or equations. To summarise the proposed work an algorithm is added to explain the several steps in order to implement the order reduction method. Further comments are given in the last section and results are discussed more objectively. Finally, answers to your comments are give as follow (N.B. Equations references in the answers concern the previous version. Between brackets are given the equivalent equations references in the new version when if they differ)

1. Quadratic stability and robust stability are presented in Section 2. No assumption is clearly made about the stability of system (7).
   Answer: In fact, no assumption about the stability of the system (7) is given. The aim of the proposed method is to preserve the closed-loop stability. Then, the closed-loop system (12) is assumed 'quadratically stable' instead of the LPV-system (7). However, for control propose, the following assumptions about (7) must hold
   - The matrix function pair $[A(\rho), B_2(\rho)]$ is parameter-dependent stabilisable and detectable,
   - The matrix $[C_2(\rho), D_{21}(\rho)]$ has full row rank for all $\rho \in \mathbf{\Delta}_\rho$.

   These assumptions are recalled in the new version.
2. Minimality is completely omitted in the paper despite its importance in balanced truncation based methods. What authors have assumed in terms of controllability and observability for the system (7)? These properties must be defined for system (7) especially since it is well known that for systems with some nearly uncontrollable and/or nearly unobservable state variables, balanced transformation may be numerically poorly conditioned.
   Answer: This comment is important. We have omitted to remind the minimality condition. However, this assumption is to be considered to the closed-loop system (12) and not the system (7) as the FWBT is applied to the closed-loop system and not the system. Then, in the new version the assumption is recalled at many places in the paper.
3. As the paper proposes an adaptation of the FWBT method to deal with Hinf controller order reduction, it would be clearer if the FWBT method initially based on the controllability and observability gramians within a frequency range, can be recalled before the extended version based on the generalized gramians. In

fact, the method presented in subsection 4.3 is a modified version of the FWBT which was introduced in [GA04] in order to treat the truncation errors and the stability issues that are inherent to balanced truncation methods. Authors propose this modified version with the generalized gramians without any prior discussion and/or motivation. Only a brief comment is provided later in the paper. This doesnt help for easy lecture of the paper.
Answer: As suggested, the new version is modified in order to introduce the classical Gramians before extending them to LPV systems. Then, the subsection 4.2 is reorganised. In addition, discussion and motivation about their use are given in more details.

4. No algorithm is given in the paper; neither for the proposed FWTB with the generalized gramians nor for the method used for Hinf - control synthesis. It would be more rigorous and more helpful if authors can formalize the proposed methods with algorithms.
Answer: An algorithm that summarises the proposed method is given in the new version at the end of Section 3. It describes the necessary steps to get a an LPV-reduced-order controller from a LPV-full-order model using the modified FWBT method.

5. Theorem 1 is largely inspired from Theorem 11 in [GA04] where the asymptotic stability and an error bound are defined for the FWTB of LTI systems. Theorem 11 in [GA04] is based on the minimality assumption (reachability and observability). The demonstrated asymptotic stability results from reachability and observability properties. However, in the presented paper, no assumption is made about these properties for the used LPV system. Also, since theorem 1 shows exponential stability for the closed-loop (12), it would be clearer and more consistent if the exponential stability notion can be defined in the LPV framework especially as the quadratic and robust stability have been initially presented, unless authors believe that robust stability is equivalent to the exponential stability.
Answer: The minimality assumption in theorem 1 has been totally omitted. In the new version, this assumption is checked and recalled. However, minimality in this case should be assumed for the closed-loop system (necessarily stable) and not the for open-loop LPV-model. The other important assumption is about the stability of the system in theorem 1: In the new version we use the 'quadratic stable' instead of the 'exponential stability'. In fact, definition of LPV quadratic stability is reduced to a necessary and sufficient condition for exponential stability when the parameter trajectories are assumed to be constant (i.e. LTI case).

6. The LPV-model is not suitably presented for readers who are not familiar with the used application. It is not sufficiently clearly described for a good understanding although authors have referenced it. The way considered by author to present their application is suitable for a conference paper.
Answer: Regarding the big size of the article, we shortened the LPV model details. Finally this part became more detailed and easier to understand in this new version.

7. Results need to be discussed more objectively. In fact, the plotted results are

briefly commented without any quantitative indicators.
Answer: Results discussion is expanded in the new version and more attention is given to quantify observations.

8. 25 frozen values of ρ_1 and ρ_2 are considered by authors. It is said that the associated bode diagrams are plotted for the three transfer functions in Figures 5, 6 and 7. The dispersion, induced by ρ_1 and ρ_2, of the bode diagram of the open-loop system in figure (5) diagrams seems (at least visually) to be not so important. The closed-loop with K_{full} have modified this dispersion in particular for relatively low frequencies. This dispersion is much more important for a larger frequency bandwidth when using the closed-loop with K-FWBT. In my opinion and from the shown plots, the used reduced controller has increased the sensitivity of the system to parameter variations. This sensitivity is also affected by the used reduced controller within relatively high frequency in figure (7). These points need to be commented, discussed and explained.
Answer: In fact the open-loop plot has no importance in this case and has been removed in the new version. For the sensitivity regarding the reduction step, this degradation is expected. Indeed, the given application is a quasi-LPV sytem where ρ_1 and ρ_2 are depending on the state vector. Then, reducing the states number will affect ρ_1 and ρ_2. However, the expected degradation when reducing is under control for two reasons: the first one, the stability of reduced order closed-loop is preserved and the error is guaranteed limited. The second one is that this dispersion is weak in the required frequency range (sensitivity more important outside the desired frequency range): then, performance are not affected. This discussion is reported in the new version.

9. Is the previous sensitivity increased in the temporal space? It would be more interesting if the chassis position and the wheel position can be plotted for the 25 frozen values of ρ_1 and ρ_2 to observe the influence of these parameters on the closed-loop performance when both the reduced controller and full controller are used respectively.
Answer: In this contribution, time analysis is shown: the chassis position and the wheel position are plotted supposing the car travelling a bump of 0.01m x2m at 30 km/h. However, sensitivity or parameters dispersion could not be visible here (as expected): the temporal test draw two output signals (chassis and wheel positions) regarding the input (the road profile) and by the way there will be just one plot of each transfer. Parameters ρ_1 and ρ_2 are internal (Figure1). Note that the several frequency transfers shown in Fig. 5, 6 and 7 are plotted in function of ρ_1 and ρ_2 due to the the impossibility to define a 'function transfer' for the LPV-systems. A pseudo-Bode plot is proposed in the literature ([PV08]) but we preferred plotting the several transfers at frozen values which is more reliable.

10. No quantitative measure is used to assess the performance of the used reduced controller. It would be more objective if the authors can quantify the performance loss induced by the use of the reduced controller in comparison with the performance of the full order controller. It would be more objective and rigorous if error bound given by (37) can be estimated for the frozen values of ρ_1 and ρ_1. Then, the mean value and the associated standard deviation can be used.
Answer: In fact, in the previous version, results discussion was not enough com-

mented. In the new version, performance in term of comfort and road holding are more commented. In addition errors between the full and the reduced order closed systems are shown with theirs upper bounds for several frozen values of ρ_1 and ρ_2 at the reduced order 5.

11. The truncation order is fixed to 5. How this order has been chosen? This point needs to be discussed

    Answer: To chose the right order when reducing, no reliable method exists and it remains an open research subject for the model-order-reduction community. Some suggestions given in the literature consist on the analysis of the Hankel singular values of the original model and the order is chosen when these values decrease brutally. For our application, an heuristic test is made, and all the orders are tested. The order 5 is appeared as the smallest one that gives an acceptable performance comparing to the original one .

12. The transmission matrix is sometimes put equal to zero as in expression (12) which is not correct. For the close loop system, the transmission matrix is given by $D_{11} + D_{12}D_K(\rho)D_{21}$. The same error is in expression (34).

    Answer: This error is fixed in the new version.

13. I am not very convinced by the originality of results in Lemma 1. (14) instead of (13) yields (19) . Otherwise, the transition from (19) to (20) from the derivative with respect to t_0 is not correct. The transition is possible by considering $\frac{\phi(t,t_0)}{dt} = A(\rho(t))\phi(t,t_0)$. Proof of Lemma 1 concerning the inequality (18) is required. In the next version, personally, I prefer the proof of Lemma 1 concerning (18) to be included in the next version of the paper

    Answer: These errors are now fixed. Then, in the new version, the (14) instead of (13) yields to (19)[20 in the new version]. In addition, the transition from (19) to (20) [(20) to (21) in the new version] is possible regarding the fact that $\frac{\partial}{\partial t}\Phi_\rho(t,t_0) = A(\rho(t))\Phi_\rho(t,t_0)$ (and not $\frac{\partial}{\partial t_0}\Phi_\rho(t,t_0) = -\Phi_\rho(t,t_0)A(\rho(t_0))$). Finally, the proof of (18)[19.a in the new version] is added in the new version.

Yours sincerely,