

June 17, 2016

Dear Reviewer,

I would like to thank you for your valuable comments and question. Please find attached the new version of the article that integrate the relevant given recommendations. In fact, the general presentation is changed and the several sections are reorganised. The problem statement is now separated from the main contribution of this paper. The transitions between section are smoother and the understanding is easier. In addition, More attention is given to the assumptions and the existence conditions for every defined quantity or equations. To summarise the proposed work an algorithm is added to explain the several steps in order to implement the order reduction method. Further comments are given in the last section and results are discussed more objectively. Finally, answers to your comments are give as follow (N.B. Equations references in the answers concern the previous version. Between brackets are given the equivalent equations references in the new version when if they differ)

1. In the formulated problem and the theoretical development, the  $D$  matrix of the closed-loop system is not equal to zero. However, this matrix is zero in (12), (34) and (44). This might be a mistake or typo. Moreover, the matrices  $B_2, C_2, D_{12}$  and  $D_{21}$  in (42) should be constant rather than parameter-dependent. In addition, the meaning of  $n_K$  in (25) should be provided just after it appears.  
Answer: Actually,  $D$  matrix is not equal to zero in (12), (34)[(44) in the new version] and (44)[(52) in the new version], it was a typo. The same comment holds for equation (12) where matrices  $B_2, C_2, D_{12}$  and  $D_{21}$  should be constant. The new version is corrected in this way.

2. It is unknown for what cases the conditions in (35) and (36) will be satisfied. Can you provide some comments for this? Straightforwardly it seems that the column space of  $B(\rho)$  must be a sub-space of that of  $B_\Omega(\rho)$ , but no further intuitive explanation can be obtained. By the way, please state whether they are satisfied in the considered example.

Answer: This idea is introduced first in [GJ90]. It means that the column space of  $B(\rho)$  must be a sub-space of that of  $B_\Omega(\rho)$  and the the row space of  $C(\rho)$  must be a sub-space of that of  $C_\Omega(\rho)$ , i.e  $\exists K, L$  with suitable dimension so that  $B(\rho) = B_\Omega(\rho)K$  and  $C(\rho) = LB_\Omega(\rho)$  with  $K$  and  $L$  have the suitable dimensions. Moreover, based in generic rank characterisation introduced in [AC81], it was shown in [WSL99] that this assumption hold for almost all the cases. Indeed, during our simulations, the assumption has always been satisfied. In the new version this comment is given in remark 1.

3. In the proof of Theorem 1, the integration range in (40) and (41) is limited but not infinite, so in general they cannot be derived from (38) and (39) by Parseval equivalence (where both integration ranges are infinite). Please explain more in detail.

Answer: For sick of place, this part was shortly introduced in the article. In the

new version, more details are given. Thus, the function  $f_\rho(\tau)$  contains term on  $H(-\tau)$ , the Heavside function that makes the equations (35)[(45) in the new version] and (36)[(46) in the new version] clear and allows the use of Parseval relationship from  $-\infty$  to  $+\infty$ . In addition, note that  $\mathcal{R}_\omega(\rho) \neq \mathcal{R}(\rho)$  and  $\mathcal{O}_\omega(\rho) \neq \mathcal{O}(\rho)$ . In fact,  $\mathcal{R}(\rho)$  and  $\mathcal{O}(\rho)$  are the exact reachability and observability Gramians in the frequency domain and  $\mathcal{R}_\omega(\rho)$  and  $\mathcal{O}_\omega(\rho)$  are a frequency dependent reachability and observability Gramians.

4. An adequate survey on frequency limited order reduction should be provided in the paper. A most recent development in this direction is finite frequency H-infinity model reduction (see doi: 10.1016/j.automatica.2014.07.001 and doi: 10.1109/TAC.2014.2352751). Note that the latter paper also explains why the frequency limited truncation method in [GA04] may be ineffective sometimes. This also should be pointed out in the paper.

Answer: In fact the state-of-the-art about the frequency limited techniques was short in the previous version. In the new one, a survey about those methods and their development is added in the introduction part. Moreover, the two stated references seem of great interest. The discussion given in the second one about the limits is useful. In fact, the FWBT does not guarantee the passivity of the model. in our work we focus just on the stability. however, these works give us some ideas to try to exploit them or extended them from a LTI PR model order reduction to LPV PR controller reduction. This discussion is recalled in the new version.

5. The important fact of the suspension system example is that the sensitive frequency range of human is limited, which inspires the frequency limited order reduction problem. However, as mentioned in Figure 1, a direct design paradigm is to directly design reduced-order or static output feedback controllers. Is it possible to extend the problem to the case? In this direction, some latest results (e.g., doi:10.1109/TAC.2013.2281472; doi: 10.1109/TCYB.2014.2364587) can be quoted.

Answer: Of course, the direct way to get a reduced order controller seems to be the best one to get a fixed order controller. Numerous work are given in this sense for the LTI case. However, almost all the developed techniques are suffered of the numerical implementation and the high cost in the computation resources in addition to optimisation constraints (convexification for instance ). They are based on the resolution of LMIs or BMIs. We made recently a review on these exiting methods([AG95][AHH11]...etc.) with a comparison on bechmarks examples. The work is not published yet. For LPV controllers, we focus in this paper the 'order reduction' way instead of the 'fixed order' way. The stated recent references seem of big interest and we are pleased to explore them because no methods using the 'frequency-limited' approach are developed in this direction.

6. Many typos, grammatical errors or unconventional math symbols can be found. Just to mention a few,

- In Definition 1, it is inappropriate to use only if in a definition statement.
- Line 40, page 3: and where should be and;
- Line 43, page 3: then should be Then;

- Line 27, page 4: used design should be used to design;
- Line 44, page 4: denoted (\*) should be denoted as (\*) or denoted by (\*);
- Lines 35-18, page 6: A dot should be added at the end of (19) and using fact should be Using the fact;
- Line 9, page 7: the semicolon in  $\Omega$  should be a comma;
- Lines 20 and 21, page 8: are and is should be being.

Answer: the paper is revised, Many typos and grammatical mistakes are rectified including the cited above.

Yours sincerely,

