

This file serves as supplemental material for the article “Robust methods for moderation analysis with a two-level regression model” in the journal Multivariate Behavioral Research.

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The first part (pages 1-3) contains simulation results by normal-distribution-based maximum likelihood (NML), robust method using Student’s t distribution with 5 degrees of freedom (t_5), and robust method using Huber-type weight with $\alpha = .05$ ($H_{.05}$) under the condition that x and z are correlated. The second part (pages 4-6) contains simulation results comparing the performance of Theil-Sen (TS) and least trimmed squares (LTS) under the moderated multiple regression (MMR) model against that of t_5 and $H_{.05}$ under the two-level regression model. The third part (pages 7-8) introduces an R program to perform robust moderation analysis with the two-level regression model.

Part I

This part contains the simulation results by NML, t_5 , and $H_{.05}$ under the condition that x and z are correlated. Results include empirical bias, empirical root mean square error (RMSE), relative difference (RD) between the average of sandwich-type standard errors and empirical standard errors, type I error and power for testing the moderation effect. The results are arranged in the following order:

Table 1: bias

Table 2: RMSE

Table 3: RD

Table 4: type I error and power

Table 1: *Empirical biases for the estimates of regression coefficients using the methods NML, t_5 and $H_{.05}$.*

n, γ	Normal errors			Heavy-tailed errors			Skewed errors			Contaminated errors		
	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$
100												
γ_{00}	.002	.001	.001	-.001	.000	-.001	-.017	-.235	-.105	-.011	.001	.000
γ_{01}	-.002	.001	.000	.005	.008	.006	.000	-.004	.000	-.006	.001	-.001
γ_{10}	.008	.008	.009	.006	.002	.003	-.005	-.106	-.047	.012	.009	.011
γ_{11}	.001	.003	.001	.005	.005	.005	.009	.034	.023	.001	.003	.001
200												
γ_{00}	.002	.002	.002	.000	.000	.000	-.004	-.232	-.098	.020	.003	.002
γ_{01}	.000	.001	.000	.004	.003	.003	.001	-.004	-.001	-.012	.001	-.001
γ_{10}	.013	.013	.014	-.006	-.007	-.006	-.009	-.116	-.055	.023	.014	.017
γ_{11}	.001	.002	.001	.002	.003	.003	.001	.019	.010	-.013	.001	-.001
500												
γ_{00}	-.001	.000	-.001	.000	.001	.000	-.003	-.236	-.099	.003	.000	-.001
γ_{01}	.001	.002	.001	.000	.000	.001	.000	-.002	-.002	.005	.001	.002
γ_{10}	-.004	-.005	-.004	-.004	-.003	-.003	-.002	-.110	-.048	-.006	-.006	-.005
γ_{11}	-.003	-.002	-.003	.001	.001	.001	.003	.018	.011	-.006	-.002	-.002
1000												
γ_{00}	.000	.000	.000	-.001	.000	.000	-.002	-.236	-.100	.002	.000	.000
γ_{01}	-.002	-.003	-.003	.002	.002	.002	.001	.000	.000	-.001	-.003	-.002
γ_{10}	-.003	-.003	-.002	.000	.000	.000	-.001	-.109	-.047	-.002	-.003	-.003
γ_{11}	.000	.001	.000	.001	.000	.001	-.002	.010	.004	.000	.001	.001

Table 2: *Empirical root mean square errors for the estimates of regression coefficients using the methods NML, t_5 and $H_{.05}$.*

n, γ	Normal errors			Heavy-tailed errors			Skewed errors			Contaminated errors		
	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$
100												
γ_{00}	.169	.177	.170	.169	.161	.161	.178	.284	.198	.346	.183	.188
γ_{01}	.175	.182	.176	.172	.158	.161	.173	.146	.156	.192	.181	.176
γ_{10}	.206	.214	.207	.199	.182	.186	.204	.200	.189	.475	.224	.237
γ_{11}	.191	.196	.191	.187	.179	.179	.199	.173	.184	.258	.196	.195
200												
γ_{00}	.118	.121	.118	.120	.113	.114	.115	.253	.146	.265	.129	.137
γ_{01}	.118	.122	.119	.119	.110	.112	.115	.095	.100	.220	.126	.132
γ_{10}	.154	.160	.155	.142	.132	.134	.150	.169	.144	.241	.162	.162
γ_{11}	.131	.134	.132	.127	.117	.119	.127	.103	.110	.211	.135	.134
500												
γ_{00}	.074	.077	.075	.073	.067	.069	.071	.244	.121	.165	.080	.082
γ_{01}	.075	.077	.075	.075	.070	.071	.075	.059	.064	.156	.079	.082
γ_{10}	.099	.101	.098	.098	.090	.092	.095	.135	.098	.248	.106	.113
γ_{11}	.087	.090	.087	.088	.083	.084	.088	.073	.078	.223	.094	.100
1000												
γ_{00}	.053	.055	.053	.052	.047	.048	.053	.241	.111	.131	.057	.060
γ_{01}	.054	.056	.054	.055	.050	.052	.052	.043	.045	.118	.058	.060
γ_{10}	.067	.071	.067	.065	.060	.061	.064	.121	.074	.154	.074	.076
γ_{11}	.064	.067	.064	.064	.060	.061	.065	.052	.055	.131	.068	.069

Table 3: *Relative differences between the averaged sandwich-type SEs and the empirical SEs for the estimates of the regression coefficients using the methods NML, t_5 and $H_{.05}$.*

n, γ	Normal errors			Heavy-tailed errors			Skewed errors			Contaminated errors		
	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$
100												
γ_{00}	.014	.036	.029	-.011	.011	.015	-.055	-.013	-.010	-.061	.037	.029
γ_{01}	-.044	-.002	-.029	-.053	.031	-.001	-.072	-.013	-.036	.053	.002	-.024
γ_{10}	-.032	.007	-.012	-.020	.047	.024	-.044	.037	.019	-.072	.007	-.020
γ_{11}	-.056	.015	-.031	-.059	-.013	-.026	-.116	-.049	-.083	.039	.011	-.027
200												
γ_{00}	-.003	.026	.007	-.043	-.030	-.035	.024	.054	.043	.004	.025	.006
γ_{01}	-.031	.001	-.019	-.053	-.015	-.033	-.011	.012	.013	-.003	.015	-.019
γ_{10}	-.064	-.044	-.057	-.005	.018	.009	-.051	.002	-.014	.017	-.022	-.041
γ_{11}	-.057	-.003	-.045	-.048	.007	-.011	-.041	.029	.016	-.031	.010	-.039
500												
γ_{00}	-.008	.004	-.002	-.002	.020	.007	.034	.025	.025	-.007	.005	.006
γ_{01}	-.010	.011	-.004	-.017	-.009	-.010	-.012	.031	.011	-.039	.016	.004
γ_{10}	-.045	-.022	-.035	-.041	-.033	-.037	-.011	-.002	-.006	-.081	-.017	-.030
γ_{11}	-.014	-.002	-.011	-.044	-.039	-.040	-.036	-.016	-.018	-.081	.006	-.006
1000												
γ_{00}	-.015	-.013	-.010	.005	.022	.020	-.006	.016	.015	-.047	-.011	-.007
γ_{01}	-.020	-.022	-.019	-.060	-.036	-.050	.001	-.014	.000	-.033	-.025	-.026
γ_{10}	-.022	-.032	-.024	-.006	-.001	.000	.015	.035	.036	-.006	-.037	-.030
γ_{11}	-.002	.004	.001	-.015	-.005	-.010	-.017	.017	.009	-.002	.001	.005

Table 4: *Empirical type I errors and empirical power for testing the moderation effect $H_0 : \gamma_{11} = 0$ using the methods NML, t_5 and $H_{.05}$.*

	Normal errors			Heavy-tailed errors			Skewed errors			Contaminated errors		
	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$	NML	t_5	$H_{.05}$
$n=100$												
type I error	.073	.063	.069	.071	.075	.071	.075	.070	.078	.048	.064	.073
power	.112	.102	.111	.119	.126	.126	.120	.133	.135	.062	.094	.103
$n=200$												
type I error	.067	.054	.065	.057	.054	.054	.06	.041	.044	.078	.051	.060
power	.165	.141	.153	.161	.149	.158	.139	.197	.156	.100	.141	.148
$n=500$												
type I error	.064	.057	.065	.058	.059	.055	.064	.067	.063	.06	.057	.059
power	.210	.198	.209	.231	.271	.258	.223	.394	.318	.093	.181	.171
$n=1000$												
type I error	.052	.054	.055	.049	.047	.054	.053	.048	.047	.040	.062	.048
power	.340	.315	.336	.362	.393	.390	.329	.575	.448	.121	.303	.291

Part II

This part contains the simulation results using the methods Theil-Sen (TS), least trimmed squares (LTS), NML, t_5 and $H_{.05}$. Results include empirical bias, empirical standard errors (SE_e) and empirical root mean square error (RMSE) for the regression coefficient estimates. Distributional conditions include four previously considered conditions (normal, heavy-tailed, skewed and contaminated) and data with high leverage points. Population values for the parameters are set to be $\gamma_{00} = \gamma_{01} = \gamma_{10} = \gamma_{11} = 1$, $\sigma_e^2 = \sigma_0^2 = \sigma_1^2 = 1$, $\sigma_{01} = 0$. Sample sizes are varied with $n = 200, 500, 1000$. The number of replication is 1000.

Data generation of high leverage points consists of two steps. First, n observations were generated from equation (6) with $x_i \stackrel{i.i.d.}{\sim} N(0,1)$, $z_i \stackrel{i.i.d.}{\sim} N(0,1)$. Then, x_i , z_i and y_i for the last 5% observations were replaced by their absolute values plus 3 times standard deviation of the corresponding variable. When applying the LTS method, we set the percentage of trimming to be 5%. The results are arranged in the following order:

Table 5: Condition with normally distributed errors

Table 6: Condition with heavy-tailed errors

Table 7: Condition with skewed errors

Table 8: Condition with contaminated errors

Table 9: Condition with high leverage points

Table 5: Condition with normally distributed errors.

n, γ	bias					SE_e					RMSE				
	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$
200															
γ_{00}	.285	-.029	.073	.079	.074	5.365	2.743	1.123	1.172	1.127	5.370	2.742	1.124	1.174	1.129
γ_{01}	-.047	-.001	-.008	-.009	-.008	.911	.443	.182	.191	.183	.912	.443	.182	.191	.183
γ_{10}	-.022	.013	-.003	.000	-.004	.642	.434	.223	.245	.225	.642	.434	.223	.245	.225
γ_{11}	.004	-.001	.000	.000	.000	.107	.069	.035	.038	.036	.107	.069	.035	.038	.036
500															
γ_{00}	.022	-.133	-.002	.008	-.001	2.967	1.705	.670	.705	.675	2.965	1.710	.670	.705	.674
γ_{01}	-.008	.015	.000	-.001	.000	.503	.276	.108	.113	.109	.503	.276	.108	.113	.109
γ_{10}	.003	.021	.001	.003	.002	.371	.275	.140	.146	.141	.371	.276	.140	.146	.141
γ_{11}	.000	-.003	.000	-.001	.000	.061	.044	.023	.024	.023	.061	.044	.023	.024	.023
1000															
γ_{00}	.083	.051	-.010	-.010	-.009	2.128	1.182	.452	.469	.452	2.129	1.182	.452	.469	.452
γ_{01}	-.012	-.004	.003	.003	.003	.365	.189	.073	.075	.073	.365	.189	.073	.075	.073
γ_{10}	-.010	-.008	.001	.000	.001	.263	.190	.100	.104	.101	.263	.190	.100	.104	.101
γ_{11}	.001	.000	.000	.000	.000	.044	.030	.015	.016	.015	.044	.030	.015	.016	.015

Table 6: Condition with heavy-tailed errors.

n, γ	bias					SE _e					RMSE				
	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$
200															
γ_{00}	.116	.056	.071	.047	.054	4.484	2.249	1.139	1.061	1.075	4.484	2.249	1.140	1.061	1.076
γ_{01}	-.021	-.011	-.008	-.005	-.006	.763	.360	.185	.173	.176	.763	.360	.185	.173	.176
γ_{10}	-.024	-.017	-.019	-.015	-.017	.537	.327	.219	.195	.202	.537	.328	.220	.195	.202
γ_{11}	.003	.002	.002	.001	.002	.090	.052	.036	.031	.033	.090	.052	.036	.031	.033
500															
γ_{00}	.116	-.013	.019	-.001	.006	2.513	1.298	.664	.598	.607	2.514	1.297	.664	.598	.607
γ_{01}	-.024	-.001	-.003	-.001	-.002	.428	.211	.106	.096	.098	.428	.211	.106	.096	.098
γ_{10}	-.011	.004	-.003	.002	.000	.310	.187	.137	.118	.122	.310	.187	.137	.118	.122
γ_{11}	.003	.000	.001	.000	.001	.052	.031	.023	.019	.020	.052	.031	.023	.019	.020
1000															
γ_{00}	.025	.015	-.004	.003	-.002	1.745	.922	.452	.416	.421	1.745	.922	.452	.416	.421
γ_{01}	-.006	-.005	.000	-.001	.000	.297	.143	.074	.068	.068	.296	.143	.074	.068	.068
γ_{10}	-.004	-.003	.000	-.002	-.001	.214	.136	.095	.083	.086	.214	.136	.095	.083	.086
γ_{11}	.001	.001	.000	.000	.000	.036	.021	.015	.013	.014	.036	.021	.015	.013	.014

Table 7: Condition with skewed errors.

n, γ	bias					SE _e					RMSE				
	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$
200															
γ_{00}	1.085	1.376	-.093	.036	.033	2.687	1.455	1.125	.948	.983	2.896	2.002	1.128	.948	.983
γ_{01}	-.111	-.001	.001	-.004	-.002	.457	.235	.179	.148	.153	.470	.234	.179	.148	.153
γ_{10}	-.470	-.399	.008	-.234	-.119	.296	.180	.223	.157	.176	.555	.438	.223	.281	.212
γ_{11}	.011	.000	-.001	.000	.000	.050	.029	.035	.024	.026	.051	.029	.035	.024	.026
500															
γ_{00}	1.255	1.431	-.021	.003	.037	1.413	.890	.658	.547	.577	1.890	1.685	.658	.547	.578
γ_{01}	-.140	-.006	-.003	.000	-.002	.239	.144	.103	.084	.088	.277	.144	.103	.084	.088
γ_{10}	-.506	-.415	-.005	-.239	-.127	.154	.115	.136	.098	.110	.529	.431	.136	.258	.168
γ_{11}	.015	.001	.001	.000	.001	.026	.018	.021	.014	.016	.031	.018	.021	.014	.016
1000															
γ_{00}	1.276	1.451	-.024	.021	.038	.985	.645	.472	.391	.412	1.612	1.587	.473	.391	.413
γ_{01}	-.142	-.007	-.001	-.002	-.002	.167	.101	.076	.061	.064	.219	.102	.076	.061	.064
γ_{10}	-.510	-.418	.003	-.239	-.123	.107	.079	.102	.069	.080	.521	.426	.102	.249	.147
γ_{11}	.016	.001	.000	.000	.000	.019	.013	.016	.011	.012	.024	.013	.016	.011	.012

Table 8: Condition with contaminated errors.

n, γ	bias					SE _e					RMSE				
	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$
200															
γ_{00}	.248	-.001	-1.676	.064	.082	5.871	2.840	56.429	1.267	1.303	5.873	2.839	56.426	1.268	1.305
γ_{01}	-.039	-.003	.223	-.007	-.009	1.002	.466	7.404	.207	.215	1.003	.466	7.404	.207	.215
γ_{10}	-.017	.011	.091	-.003	-.005	.699	.408	3.184	.251	.258	.699	.407	3.184	.251	.258
γ_{11}	.003	-.001	-.013	.000	.000	.116	.065	.440	.040	.041	.116	.065	.440	.039	.041
500															
γ_{00}	-.012	-.094	.001	.005	-.011	3.272	1.745	1.504	.742	.773	3.270	1.747	1.504	.742	.773
γ_{01}	-.003	.005	-.004	.000	.001	.554	.280	.237	.119	.124	.554	.280	.237	.119	.124
γ_{10}	.008	.018	.004	.003	.003	.406	.255	.331	.152	.160	.405	.255	.331	.152	.160
γ_{11}	-.001	-.002	-.001	-.001	-.001	.067	.041	.052	.025	.026	.067	.041	.052	.025	.026
1000															
γ_{00}	.056	.015	-.028	-.013	-.015	2.319	1.226	1.015	.492	.509	2.319	1.225	1.015	.491	.509
γ_{01}	-.007	.000	.006	.003	.003	.397	.199	.162	.080	.082	.397	.199	.162	.080	.082
γ_{10}	-.007	-.004	.000	.000	.001	.284	.180	.236	.107	.113	.284	.179	.236	.107	.113
γ_{11}	.001	.000	.000	.000	.000	.047	.029	.038	.017	.017	.047	.029	.038	.017	.017

Table 9: Condition with high leverage points.

n, γ	bias					SE _e					RMSE				
	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$	TS	LTS	NML	t_5	$H_{.05}$
200															
γ_{00}	.108	.022	.035	.034	.035	.158	.131	.124	.131	.126	.191	.133	.129	.135	.131
γ_{01}	.269	.078	.129	.125	.127	.135	.161	.142	.146	.142	.301	.179	.192	.192	.191
γ_{10}	.292	.095	.196	.189	.194	.165	.220	.163	.170	.164	.335	.239	.255	.255	.254
γ_{11}	.329	.363	.704	.688	.700	.162	.425	.117	.125	.118	.367	.559	.714	.699	.710
500															
γ_{00}	.108	.017	.034	.031	.033	.103	.088	.082	.086	.082	.149	.089	.088	.091	.088
γ_{01}	.269	.076	.135	.128	.132	.086	.116	.091	.094	.091	.282	.139	.162	.159	.161
γ_{10}	.291	.095	.193	.186	.190	.098	.148	.098	.102	.098	.307	.176	.216	.212	.214
γ_{11}	.332	.364	.705	.688	.701	.103	.359	.077	.083	.078	.347	.511	.709	.693	.705
1000															
γ_{00}	.113	.019	.036	.034	.035	.072	.064	.057	.060	.057	.134	.067	.067	.069	.067
γ_{01}	.269	.074	.131	.125	.129	.059	.098	.063	.065	.064	.275	.122	.145	.141	.144
γ_{10}	.296	.100	.197	.191	.196	.068	.116	.069	.070	.069	.303	.153	.209	.203	.207
γ_{11}	.334	.361	.703	.687	.699	.068	.328	.050	.052	.050	.340	.488	.705	.689	.701

Part III

This part introduces an R program to perform robust moderation analysis with the two-level regression model. Robustness is achieved using Student's t distribution and Huber-type weights. In order that users can easily compare the results of robust analyses with those of NML analysis, we also include the code for NML estimation in our program. A simulated data set with three variables (a dependent variable named **DV**, an independent variable names **IV**, and a moderator variable named **Moderator**) and 500 cases is used to illustrate the application of the program. The R program with data generation can be downloaded at [\[Insert a link here for supplemental file 2\]](#). In this illustration, we save the data file in the folder **c:/moderation** and name it **simdata.txt**. Part of the code that needs users' modification in a typical application is copied below.

```
setwd("c:/moderation")
sim <- read.table('simdata.txt', header=T)
NML(data=sim, y=DV, x=IV, z=Moderator)
TML(data=sim, y=DV, x=IV, z=Moderator, m=5)
Huber(data=sim, y=DV, x=IV, z=Moderator, alpha=.05)
```

The code `setwd("c:/moderation")` sets the working directory to the folder that contains the data file and the R program. The second line of the code uses the R function `read.table` to read data file **simdata.txt** into R and saves the data into an object called **sim**. The argument `header=T` tells R that the data file contains variable names. The **NML** function performs moderation analysis based on the NML estimation method. The first argument `data=sim` specifies the data to be used. If users read their data into R with a name other than **sim**, the name should be specified after `data=`. The subsequent arguments `y=DV`, `x=IV`, `z=Moderator` identify the dependent variable, the independent variable and the moderator in the **sim** data, respectively. Users need to specify their own variable names of the corresponding variables after each equal sign. The **TML** function performs robust moderation analysis using Student's t distribution, with the argument `m=5` specifying the degrees of freedom. As mentioned previously, the degrees of freedom is related with the extent to which each case is downweighted. We set the default value at 5. Users can specify a different value according to their needs. The **Huber** function performs robust moderation analysis using Huber-type weights. The argument `alpha=.05` specifies the proportion of cases to be downweighted. While the default value is .05, users can use a different value.

The output from running the R program with our simulated data set is given below. Lines 2 to 16 correspond to the results of NML analysis. The first part (lines 2 to 10) contains the parameter estimates and their SE_{sws} , z -scores and p -values, where **Intercept**, **Moderator**, **IV** and **IV*Moderator** correspond to the regression coefficients γ_{00} , γ_{01} , γ_{10} and γ_{11} , respectively; **Var1**, **Cov01** and **Var(0+e)** correspond to the variance parameters σ_1^2 , σ_{01} and σ_{0e} , respectively. The second part (lines 12 to 13) reports BIC and the third part (lines 15 to 16) reports R^2 . Lines 18 to 32 correspond to the results of robust analysis using Student's t distribution. Lines 34 to 48 correspond to the results of robust analysis using Huber-type weights. The formats and labels for the output of the robust analyses are the same as those for the output of NML except that the **Huber** function does not provide a BIC due to its inapplicability.

```

#-----Output from running NML-----#;
$Parameter
      Estimate      SE_sw      z      Pr(>|z|)
Intercept      0.124706468 0.12614248 0.9886159 3.228511e-01
Moderator     -0.021618610 0.01993419 -1.0844992 2.781435e-01
IV            -0.011912506 0.10951129 -0.1087788 9.133779e-01
IV*Moderator  -0.004166654 0.01868545 -0.2229892 8.235439e-01
Var1           0.012128246 0.05336834 0.2272555 8.202251e-01
Cov01         -0.086761221 0.03985895 -2.1767059 2.950252e-02
Var(0+e)       1.384580491 0.13668594 10.1296485 4.081140e-24

$BIC
[1] 1624.382

$R.square
[1] 0.0095741
#-----Output from running TML-----#;
$Parameter
      Estimate      SE_sw      z      Pr(>|z|)
Intercept      0.120963204 0.12220146 0.98986709 3.222391e-01
Moderator     -0.020171290 0.01951313 -1.03372930 3.012627e-01
IV            -0.008108447 0.12423129 -0.06526895 9.479599e-01
IV*Moderator  -0.006273076 0.02061655 -0.30427382 7.609193e-01
Var1           0.128591829 0.08479855 1.51643892 1.294084e-01
Cov01         -0.098669564 0.05576367 -1.76942376 7.682318e-02
Var(0+e)       1.437552541 0.15459572 9.29878595 1.420584e-20

$BIC
[1] 1619.345

$R.square
[1] 0.002062293
#-----Output from running Huber-----#;
$Parameter
      Estimate      SE_sw      z      Pr(>|z|)
Intercept      0.1253255754 0.12313089 1.017824014 3.087616e-01
Moderator     -0.0226876906 0.01949674 -1.163666002 2.445594e-01
IV            0.0008755584 0.11075120 0.007905634 9.936923e-01
IV*Moderator  -0.0068283692 0.01877173 -0.363758216 7.160386e-01
Var1           0.0983897392 0.06827898 1.440996082 1.495858e-01
Cov01         -0.0818461430 0.04676923 -1.749999799 8.011835e-02
Var(0+e)       1.1695725384 0.13011242 8.988938219 2.496303e-19

$BIC
[1] NA

$R.square
[1] 0.003190037

```