A Cross-Entropy Approach to the Estimation of Generalised Linear Multilevel Models

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Supplementary Material

The present document includes the following tables and figures:

- **Tables SM.1-SM.6** – RMSE and computing times about simulations on the 2-level logistic model (1) with $\Omega = \sigma^2 I$. – Parameters N and ρ (Section 3.4.1 of the paper).
- **Tables SM.7-SM.12** – RMSE and computing times about simulations on the 2-level Poisson model (3) with $\Omega = \sigma^2 I$. – Parameters N and ρ (Section 3.4.1 of the paper).
- **Tables SM.13-SM.29** – RMSE, bias and computing times about simulations on the 2-level logistic model (1) with $\Omega = \Xi \otimes I_G$. Only RMSE for $\hat{\beta}_j$ $(j = 1, \ldots, k_1)$ and $\hat{\sigma}_j$ $(j = 1, \ldots, k_2)$ are reported. – Parameters N and ρ (Section 3.4.1 of the paper).
- **Tables SM.30-SM.35** – RMSE and computing times about simulations on the 2-level logistic model (1) with $\Omega = \sigma^2 I$. – Parameters β and q (Section 3.4.3 of the paper).
- **Tables SM.36-SM.51** – RMSE, bias and computing times about the simulations for the comparison between the cross-entropy and the AGQ estimates – For $G = 8, 12, 25, 50$ and $n = 250, 1000$ (Section 4 of the paper).
- **Figures SM.1-SM.15** – Boxplots of the point estimates about the simulations for the comparison between the cross-entropy and the AGQ estimates – For $G = 50$ and $n = 250$ (Section 4 of the paper).

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N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
100	27.56	32.48	25.54	28.68	31.28	26.67	31.64	39.55	42.27	41.50
180	24.30	23.56	25.16	24.20	25.72	28.15	28.35	28.54	30.51	33.30
320	25.83	23.52	23.57	24.12	23.50	23.57	24.24	27.60	28.62	30.15
560	23.98	24.01	23.94	24.77	23.49	24.27	24.73	25.79	27.19	29.07
1000	24.61	23.87	24.06	23.20	24.00	24.63	25.26	25.65	27.29	29.29
1800	23.30	23.41	23.62	24.20	24.03	24.01	24.62	25.62	26.43	26.60
3200	24.46	23.05	23.44	23.67	23.50	24.03	24.30	25.67	25.75	27.07
5600	23.62	23.56	23.76	23.35	23.65	23.83	24.60	25.07	26.31	27.75
10 000	23.92	23.50	23.50	23.55	23.84	24.05	24.32	25.00	25.88	27.00
18 000	23.92	23.36	23.47	23.49	23.64	23.82	24.28	24.97	25.64	26.90
32 000	23.85	23.51	23.50	23.48	23.53	23.86	24.24	25.06	25.70	26.72
56 000	23.74	23.57	23.39	23.46	23.61	23.88	24.25	24.94	25.69	26.79

Table SM.1: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_1$ of model (1) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
100	58.51	62.49	55.70	58.54	63.89	75.20	63.82	108.58	107.14	99.54
180	46.26	43.97	42.54	52.76	51.00	57.64	66.51	67.30	83.30	90.65
320	46.51	44.58	46.83	45.75	47.24	52.86	56.41	57.15	68.21	70.25
560	45.11	44.90	44.16	48.07	50.54	50.53	55.61	57.09	63.42	72.45
1000	44.69	43.84	44.21	45.10	47.43	50.86	52.70	57.27	66.70	72.42
1800	43.97	44.97	45.15	46.34	46.59	48.38	51.59	56.17	62.16	64.08
3200	45.92	44.36	44.87	44.92	46.55	48.24	51.13	54.77	60.25	63.08
5600	45.64	44.99	44.80	45.18	46.41	47.99	50.56	53.46	58.25	63.54
10 000	44.71	45.20	44.89	45.25	46.31	48.17	50.58	53.57	58.35	62.03
18 000	45.17	44.69	44.61	45.13	46.03	48.05	50.33	53.63	57.17	62.57
32 000	45.47	44.78	44.64	44.95	45.94	47.85	49.83	53.04	57.81	61.88
56 000	45.38	44.88	44.81	45.18	45.95	47.71	49.70	52.71	57.08	61.76

Table SM.2: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_2$ of model (1) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
100	80.57	76.55	76.87	86.71	95.30	98.70	86.66	141.60	140.92	132.80
180	63.44	63.32	58.26	67.61	66.87	73.45	89.89	87.30	108.71	126.67
320	63.81	58.49	61.62	62.62	64.72	68.50	77.05	78.00	87.95	93.92
560	58.27	59.00	60.44	61.77	66.83	66.31	72.94	75.02	84.31	96.93
1000	60.08	59.34	60.36	60.39	63.28	66.17	69.34	75.60	86.05	95.90
1800	59.55	59.36	60.35	61.69	62.59	64.71	68.63	73.41	82.87	86.54
3200	60.55	60.35	60.04	60.58	62.15	63.85	67.89	72.54	80.09	85.55
5600	60.77	60.48	60.46	60.44	61.60	64.00	66.54	70.29	77.20	83.95
10 000	60.78	60.66	59.98	60.19	62.13	64.17	67.40	71.50	76.65	82.86
18 000	61.09	60.22	60.11	60.63	61.52	64.18	66.82	71.44	75.72	83.31
32 000	61.18	60.53	60.27	60.33	61.55	63.91	66.19	70.35	76.34	82.25
56 000	61.46	60.38	60.24	60.50	61.48	63.72	66.25	70.09	75.39	82.07

Table SM.3: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_3$ of model (1) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
100	90.49	121.48	88.54	103.11	117.56	117.07	117.33	183.02	191.32	174.68
180	71.31	79.03	70.44	84.57	86.78	103.49	107.34	112.85	134.61	157.21
320	79.29	71.14	76.65	76.76	77.77	82.97	92.09	95.93	115.09	123.38
560	72.51	72.45	73.64	79.66	78.54	85.67	89.83	95.37	102.99	122.73
1000	75.86	72.12	73.10	73.31	79.26	84.73	88.04	95.94	108.78	119.13
1800	72.57	73.39	72.62	75.79	76.52	80.53	83.01	92.64	102.37	105.58
3200	75.40	72.41	73.29	74.15	76.38	78.30	83.63	90.87	97.92	105.70
5600	74.14	74.52	73.69	73.69	75.43	78.38	82.68	87.80	97.80	106.70
10 000	74.51	74.02	73.22	73.89	76.23	78.70	83.10	87.83	95.82	102.48
18 000	76.00	73.41	73.39	73.87	75.56	78.27	82.20	88.05	93.48	103.25
32 000	75.52	73.64	73.52	73.90	75.29	78.20	81.56	87.18	94.76	102.55
56 000	75.36	73.78	73.43	73.91	75.26	77.99	81.51	86.49	93.90	102.20

Table SM.4: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_4$ of model (1) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
100	11.59	14.73	11.74	8.50	11.34	8.80	9.94	10.06	10.17	8.12
180	12.83	14.92	11.76	11.73	9.96	9.44	9.65	8.15	8.90	8.92
320	13.28	13.13	12.8	12.32	10.16	11.22	9.09	8.75	8.56	8.10
560	16.70	14.15	13.33	11.88	10.92	10.01	9.49	8.86	8.19	9.01
1000	17.12	14.51	13.34	11.45	10.77	10.77	9.46	9.25	9.03	8.65
1800	17.37	15.16	13.22	12.26	10.74	10.27	9.69	8.92	8.65	8.18
3200	17.80	14.92	13.32	12.01	11.15	10.34	9.47	9.26	8.78	8.45
5600	17.42	14.91	13.48	12.16	11.06	10.24	9.73	9.12	8.46	8.38
10 000	17.75	14.91	13.38	12.13	11.14	10.18	9.61	8.95	8.53	8.27
18 000	17.55	14.91	13.37	12.13	10.96	10.27	9.51	9.02	8.56	8.24
32 000	17.54	15.11	13.34	12.06	11.08	10.25	9.59	9.05	8.59	8.18
56 000	17.42	14.97	13.39	12.07	11.05	10.24	9.55	9.01	8.56	8.19

Table SM.5: Sample Root Mean Squared Error (RMSE) for $\hat{\sigma}$ of model (1) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
100	0.05	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.07	0.09
180	0.08	0.08	0.07	0.09	0.08	0.09	0.09	0.10	0.13	0.17
320	0.15	0.13	0.12	0.15	0.15	0.15	0.17	0.18	0.20	0.28
560	0.28	0.25	0.23	0.27	0.28	0.25	0.28	0.33	0.49	0.54
1000	0.49	0.51	0.39	0.49	0.45	0.51	0.51	0.80	0.91	1.52
1800	0.93	0.89	0.80	0.97	1.02	0.92	1.31	1.70	2.29	2.71
3200	1.78	1.56	1.57	1.77	1.95	1.84	2.37	3.85	4.57	8.11
5600	3.10	2.98	3.17	3.36	3.73	3.29	4.57	6.33	8.84	15.20
10 000	7.37	5.80	6.45	7.22	7.41	7.18	11.56	15.01	23.15	27.02
18 000	13.90	11.57	12.06	12.60	13.78	14.58	25.71	32.73	45.27	57.71
32 000	26.27	23.69	24.93	25.54	27.72	28.12	52.18	68.06	84.42	100.77
56 000	56.43	46.61	47.47	61.65	60.47	68.46	102.44	136.53	151.21	179.59

Table SM.6: Computing times of model (1) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. The table reports the average computing times for one model, expressed in minutes.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	9.16	9.46	10.14	9.47	9.98	10.16	9.98	10.35	11.55	13.37
1000	9.72	9.45	9.55	9.43	9.82	9.98	10.78	10.75	11.37	13.13
1800	9.49	9.32	9.49	9.58	9.60	9.90	10.19	10.47	11.07	12.49
3200	9.38	9.38	9.45	9.48	9.61	9.83	10.26	10.39	10.99	12.62
5600	9.52	9.40	9.43	9.46	9.56	9.79	10.08	10.56	11.12	12.53
10 000	9.54	9.43	9.42	9.40	9.53	9.76	9.98	10.37	11.25	12.54

Table SM.7: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_1$ of model (3) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	7.52	7.48	7.04	7.18	7.16	6.81	7.18	7.14	7.04	8.16
1000	7.31	7.03	6.99	7.13	7.07	7.22	7.24	7.31	7.51	7.47
1800	6.90	7.07	7.02	7.05	7.04	7.29	7.04	7.26	7.24	7.44
3200	7.15	7.16	7.04	7.12	7.10	7.31	7.22	7.21	7.14	7.11
5600	6.94	6.95	6.90	7.03	7.09	7.08	7.19	7.12	7.17	7.27
10 000	7.07	7.06	6.95	7.03	7.02	7.11	7.16	7.05	7.20	7.30

Table SM.8: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_2$ of model (3) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	9.23	9.51	8.86	9.13	9.29	9.43	9.86	9.63	9.76	9.84
1000	9.04	9.16	9.06	9.44	9.23	9.33	9.50	9.40	9.69	9.34
1800	9.02	8.83	9.20	9.04	9.23	9.59	9.44	9.46	9.67	9.71
3200	9.13	9.06	8.93	9.27	9.23	9.33	9.22	9.32	9.34	9.72
5600	9.19	8.96	9.08	9.15	9.14	9.33	9.43	9.48	9.37	9.62
10 000	8.89	8.90	9.06	9.11	9.13	9.31	9.34	9.47	9.57	9.82

Table SM.9: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_3$ of model (3) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	9.22	8.82	8.75	8.94	9.15	9.05	9.18	9.06	10.07	9.97
1000	8.77	9.05	8.71	8.78	9.10	9.08	9.43	9.40	9.50	10.32
1800	8.74	8.66	8.81	8.74	8.97	9.11	9.08	9.28	9.53	9.72
3200	8.81	8.71	8.58	8.68	8.89	8.95	9.02	9.24	9.34	9.82
5600	8.61	8.69	8.79	8.81	8.96	9.07	8.99	9.33	9.53	9.71
10 000	8.80	8.79	8.69	8.75	8.91	9.06	9.01	9.15	9.52	9.84

Table SM.10: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_4$ of model (3) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	3.96	4.00	4.27	5.48	7.46	8.42	8.52	8.44	8.31	8.14
1000	3.29	3.87	4.32	5.07	7.64	8.77	8.65	8.49	8.40	8.20
1800	3.29	3.87	4.23	4.83	7.97	8.79	8.64	8.52	8.35	8.19
3200	3.28	3.86	4.21	4.54	7.69	8.75	8.60	8.52	8.38	8.19
5600	3.28	3.84	4.19	4.52	7.73	8.81	8.66	8.55	8.39	8.24
10 000	3.25	3.78	4.18	4.51	7.91	8.76	8.69	8.51	8.36	8.25

Table SM.11: Sample Root Mean Squared Error (RMSE) for $\hat{\sigma}$ of model (3) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	10.50	10.73	10.78	10.40	8.59	6.89	6.23	6.35	6.30	6.22
1000	18.37	18.89	19.54	19.06	14.58	11.81	11.52	11.42	11.34	11.47
1800	33.33	34.11	35.20	34.93	25.00	20.49	20.02	20.50	20.43	20.32
3200	58.61	60.56	62.68	63.81	45.61	36.25	36.36	36.31	36.46	36.32
5600	102.70	105.50	108.88	110.84	80.55	64.35	64.04	63.56	63.58	63.35
10 000	182.85	191.19	194.84	199.70	137.64	114.04	114.48	113.98	114.32	114.35

Table SM.12: Computing times of model (3) obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 50. The table reports the average computing times for one model, expressed in minutes.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	25.57	23.63	23.75	23.31	23.66	23.46	24.37	24.05	23.21	24.00
1000	25.06	25.18	24.42	24.13	23.47	23.91	23.94	23.96	24.09	23.63
1800	24.69	24.75	23.85	23.27	23.43	23.69	23.76	23.18	23.74	23.56
3200	25.44	24.62	24.12	23.88	23.77	24.12	23.41	23.50	23.40	23.39
5600	24.79	24.25	24.12	24.15	23.72	23.60	23.69	23.55	23.67	23.67
10 000	25.01	24.25	24.06	24.10	23.99	23.73	23.54	23.51	23.37	23.52

Table SM.13: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_1$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	26.69	26.09	26.22	26.11	25.27	24.81	25.05	25.53	25.16	25.18
1000	25.33	25.46	25.38	25.26	25.38	24.87	24.88	24.60	25.60	25.36
1800	26.32	25.65	25.20	24.99	25.38	25.25	25.04	25.33	25.03	25.25
3200	25.47	25.56	24.89	25.27	25.13	24.77	24.68	24.73	24.89	25.05
5600	25.49	25.34	24.94	25.17	24.88	25.06	25.10	25.21	25.19	24.88
10 000	25.61	25.47	25.16	25.16	24.88	25.04	24.95	25.00	24.99	25.05

Table SM.14: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_2$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	37.38	36.45	35.97	35.29	34.61	34.42	34.78	34.16	33.45	33.71
1000	36.48	37.21	35.46	35.66	33.76	34.35	33.95	33.48	33.85	32.79
1800	37.08	37.33	36.21	35.38	34.45	34.39	34.21	34.23	33.62	33.45
3200	37.81	36.50	36.41	35.62	35.46	35.36	34.31	34.18	33.50	33.49
5600	37.46	36.26	36.28	35.57	34.43	34.92	34.75	34.05	33.76	33.41
10 000	38.14	36.55	36.00	35.90	35.04	35.14	34.42	34.26	33.66	34.01

Table SM.15: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_3$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	67.60	64.71	63.33	64.23	64.20	62.53	63.99	63.78	63.11	63.83
1000	66.72	65.18	64.51	65.52	63.81	64.43	63.53	62.78	63.15	63.06
1800	67.07	66.12	65.37	64.60	63.87	64.15	63.65	63.58	63.64	63.06
3200	67.04	65.41	63.80	65.00	63.86	63.68	63.08	63.47	63.35	63.17
5600	66.88	65.28	65.09	64.54	63.90	63.72	63.73	63.53	63.72	63.21
10 000	66.23	65.42	64.84	64.58	64.30	63.82	63.80	63.28	63.32	63.04

Table SM.16: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_4$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	12.14	11.12	12.47	12.77	11.86	11.31	12.42	9.85	10.82	11.12
1000	12.25	11.67	12.07	12.22	10.22	11.37	11.14	11.58	10.78	11.30
1800	12.65	12.74	11.98	11.78	11.80	12.29	11.23	10.78	10.76	10.63
3200	13.06	12.59	11.53	12.17	11.57	11.52	11.85	10.96	11.10	10.94
5600	11.75	12.08	12.20	12.01	11.50	11.79	11.60	11.15	11.23	10.87
10 000	12.72	12.17	12.34	12.00	11.84	11.50	11.48	11.25	11.44	10.78

Table SM.17: Sample Root Mean Squared Error (RMSE) for $\hat{\sigma}_1$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	17.57	17.78	18.38	18.37	17.81	18.14	18.07	17.84	17.84	17.76
1000	18.36	18.63	19.12	18.51	18.70	18.26	18.46	18.54	18.02	18.16
1800	18.86	18.83	18.92	18.77	18.75	18.28	18.62	18.18	18.44	18.26
3200	18.55	18.61	18.88	18.90	18.93	18.52	18.64	18.50	18.27	18.19
5600	19.07	18.92	19.10	18.96	18.81	18.66	18.63	18.48	18.40	18.46
10 000	19.02	19.07	19.08	18.95	18.83	18.69	18.53	18.45	18.36	18.32

Table SM.18: Sample Root Mean Squared Error (RMSE) for $\hat{\sigma}_2$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	42.28	4184.69	42.57	42.55	43.30	42.75	42.63	42.73	42.79	42.95
1000	41.70	42.51	42.77	43.53	43.17	43.08	42.97	43.12	43.71	42.91
1800	40.47	42.89	43.12	44.11	43.93	43.28	43.52	43.38	43.55	43.65
3200	39.81	42.56	42.86	43.64	43.71	43.89	43.45	43.82	43.76	43.69
5600	39.85	42.00	42.71	43.43	44.16	44.21	43.83	43.82	43.46	43.97
10 000	39.71	41.91	42.97	43.68	43.96	44.03	43.81	43.68	43.68	43.54

Table SM.19: Sample Root Mean Squared Error (RMSE) for $\hat{\sigma}_3$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	63.30	62.63	63.13	64.48	64.01	66.00	66.58	65.03	66.65	67.22
1000	58.32	60.95	62.56	62.69	62.52	64.39	65.15	66.12	64.63	66.95
1800	59.62	60.23	62.24	62.70	62.59	64.32	63.99	65.61	64.96	65.59
3200	57.43	60.57	60.90	62.53	63.14	63.64	65.06	64.51	65.80	66.41
5600	57.49	59.91	61.47	62.97	62.97	64.55	64.56	65.15	65.49	65.66
10 000	57.28	59.51	62.37	63.04	63.30	63.85	64.56	64.85	66.06	66.11

Table SM.20: Sample Root Mean Squared Error (RMSE) for $\hat{\sigma}_4$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	0.03	0.01	0.01	0.01	−0.01	0.00	−0.01	−0.01	−0.01	−0.02
1000	0.04	0.02	0.02	0.01	0.00	0.00	−0.01	−0.01	−0.01	−0.01
1800	0.04	0.03	0.01	0.01	0.00	0.00	0.00	−0.01	−0.01	−0.01
3200	0.03	0.02	0.02	0.01	0.01	0.00	0.00	−0.01	−0.01	−0.01
5600	0.03	0.03	0.02	0.01	0.01	0.00	0.00	0.00	−0.01	−0.01
10 000	0.04	0.02	0.02	0.01	0.01	0.00	0.00	−0.01	−0.01	−0.01

Table SM.21: Sample bias for $\hat{\beta}_1$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	0.02	0.00	0.00	0.00	−0.01	−0.02	−0.02	−0.03	−0.03	−0.03
1000	0.02	0.01	0.00	−0.01	−0.02	−0.01	−0.02	−0.02	−0.03	−0.03
1800	0.02	0.01	0.00	−0.01	−0.01	−0.02	−0.02	−0.03	−0.03	−0.03
3200	0.02	0.01	0.00	0.00	−0.01	−0.02	−0.02	−0.02	−0.03	−0.03
5600	0.02	0.01	0.00	0.00	−0.01	−0.01	−0.02	−0.02	−0.03	−0.03
10 000	0.02	0.01	0.00	0.00	−0.01	−0.01	−0.02	−0.02	−0.02	−0.03

Table SM.22: Sample bias for $\hat{\beta}_2$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	0.09	0.08	0.08	0.08	0.07	0.06	0.06	0.05	0.05	0.04
1000	0.10	0.09	0.08	0.07	0.06	0.07	0.06	0.05	0.05	0.04
1800	0.10	0.09	0.08	0.07	0.07	0.06	0.06	0.05	0.05	0.05
3200	0.10	0.09	0.08	0.08	0.07	0.07	0.06	0.05	0.05	0.05
5600	0.11	0.09	0.08	0.08	0.07	0.07	0.06	0.05	0.05	0.05
10 000	0.11	0.10	0.08	0.08	0.07	0.07	0.06	0.06	0.05	0.05

Table SM.23: Sample bias for $\hat{\beta}_3$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	0.01	−0.01	−0.02	−0.01	−0.02	−0.03	−0.02	−0.03	−0.03	−0.04
1000	0.01	0.02	0.00	−0.01	−0.01	−0.02	−0.03	−0.03	−0.04	−0.04
1800	0.03	0.01	0.00	0.00	−0.01	−0.02	−0.02	−0.03	−0.03	−0.04
3200	0.03	0.01	0.01	0.00	−0.01	−0.02	−0.02	−0.03	−0.03	−0.04
5600	0.02	0.01	0.01	0.00	−0.01	−0.01	−0.02	−0.02	−0.03	−0.03
10 000	0.04	0.02	0.01	0.00	0.00	−0.01	−0.02	−0.02	−0.03	−0.03

Table SM.24: Sample bias for $\hat{\beta}_4$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	−0.04	−0.06	−0.05	−0.05	−0.05	−0.06	−0.05	−0.07	−0.06	−0.06
1000	−0.04	−0.05	−0.05	−0.06	−0.07	−0.06	−0.06	−0.06	−0.06	−0.06
1800	−0.04	−0.05	−0.06	−0.06	−0.06	−0.06	−0.06	−0.07	−0.07	−0.07
3200	−0.04	−0.05	−0.06	−0.06	−0.06	−0.06	−0.06	−0.07	−0.06	−0.06
5600	−0.05	−0.06	−0.06	−0.06	−0.07	−0.06	−0.06	−0.07	−0.06	−0.06
10 000	−0.05	−0.06	−0.06	−0.06	−0.06	−0.07	−0.06	−0.07	−0.06	−0.07

Table SM.25: Sample bias for $\hat{\sigma}_1$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	−0.12	−0.14	−0.13	−0.16	−0.15	−0.15	−0.15	−0.15	−0.15	−0.16
1000	−0.13	−0.14	−0.15	−0.15	−0.15	−0.16	−0.15	−0.15	−0.16	−0.16
1800	−0.13	−0.14	−0.15	−0.15	−0.16	−0.16	−0.16	−0.16	−0.16	−0.16
3200	−0.14	−0.15	−0.15	−0.15	−0.15	−0.16	−0.16	−0.16	−0.16	−0.16
5600	−0.14	−0.15	−0.15	−0.16	−0.16	−0.16	−0.16	−0.16	−0.16	−0.16
10 000	−0.14	−0.15	−0.15	−0.15	−0.15	−0.16	−0.16	−0.16	−0.16	−0.16

Table SM.26: Sample bias for $\hat{\sigma}_2$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	−0.30	3.89	−0.33	−0.33	−0.34	−0.34	−0.35	−0.35	−0.36	−0.37
1000	−0.27	−0.31	−0.32	−0.35	−0.35	−0.35	−0.36	−0.36	−0.38	−0.36
1800	−0.27	−0.32	−0.32	−0.35	−0.35	−0.35	−0.36	−0.35	−0.37	−0.37
3200	−0.26	−0.30	−0.32	−0.34	−0.35	−0.36	−0.36	−0.36	−0.37	−0.37
5600	−0.26	−0.30	−0.32	−0.34	−0.35	−0.35	−0.36	−0.36	−0.37	−0.38
10 000	−0.26	−0.30	−0.33	−0.34	−0.35	−0.35	−0.36	−0.36	−0.37	−0.37

Table SM.27: Sample bias for $\hat{\sigma}_3$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	−0.37	−0.43	−0.45	−0.50	−0.49	−0.51	−0.53	−0.53	−0.55	−0.57
1000	−0.38	−0.43	−0.44	−0.47	−0.47	−0.50	−0.52	−0.54	−0.54	−0.56
1800	−0.37	−0.41	−0.45	−0.47	−0.48	−0.50	−0.51	−0.53	−0.54	−0.55
3200	−0.38	−0.42	−0.44	−0.46	−0.48	−0.50	−0.52	−0.52	−0.54	−0.56
5600	−0.37	−0.41	−0.44	−0.47	−0.48	−0.50	−0.52	−0.53	−0.54	−0.55
10 000	−0.38	−0.41	−0.45	−0.47	−0.48	−0.49	−0.51	−0.52	−0.54	−0.55

Table SM.28: Sample bias for $\hat{\sigma}_4$ of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. All values of the RMSE are multiplied by 100.

N	ρ									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
560	0.68	0.68	0.69	0.78	0.74	0.78	0.84	0.83	0.85	0.89
1000	1.30	1.36	1.55	1.57	1.58	1.70	1.61	1.74	1.75	1.75
1800	2.44	2.70	2.91	3.37	3.32	3.43	3.63	3.59	3.49	3.82
3200	4.43	5.11	5.91	6.52	6.99	7.03	7.31	7.40	7.37	7.25
5600	9.11	10.81	12.84	13.59	13.47	13.27	15.00	14.33	14.45	14.34
10 000	18.52	22.93	25.16	27.15	27.59	29.11	31.65	32.84	30.58	29.81

Table SM.29: Computing times of model (1) with $\Omega = \Xi \otimes I_G$ obtained by means of Monte Carlo simulations for various values of the number of points (N) and the rarity parameter (ρ). For each pair (N, ρ), the number of replications is equal to 100. The table reports the average computing times for one model, expressed in minutes.

q	β									
	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
5	33.30	33.56	33.90	34.11	33.85	34.14	34.30	34.46	34.76	35.06
6	32.59	32.65	32.86	33.28	33.09	33.48	33.46	33.49	33.55	33.91
7	32.05	32.34	32.47	32.66	32.96	32.57	33.08	32.75	32.62	32.95
8	32.29	32.13	31.92	31.85	32.10	32.55	32.18	32.08	32.57	32.67
9	31.64	31.84	31.91	31.76	31.91	32.01	32.04	32.35	32.05	32.10
10	31.81	31.64	31.55	31.86	31.92	31.88	31.90	32.06	32.05	31.78

Table SM.30: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_1$ of model (1) obtained by means of Monte Carlo simulations for various values of the smoothing parameters q and β of Equation (15). For each pair (q, β), the number of replications is equal to 250. All values of the RMSE are multiplied by 100.

q	β									
	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
5	52.67	53.31	53.62	54.03	54.25	54.74	55.46	55.65	56.07	56.79
6	50.36	50.62	51.08	51.80	51.91	52.59	52.15	52.51	53.33	53.17
7	48.81	49.70	50.29	50.54	50.74	50.51	50.74	50.74	50.90	50.89
8	48.75	48.49	48.63	48.78	48.87	49.27	49.11	49.13	50.20	50.45
9	47.23	47.76	48.16	48.16	48.25	48.34	48.09	48.87	49.19	48.89
10	47.16	47.16	47.27	47.66	47.58	47.61	48.04	48.16	47.95	48.08

Table SM.31: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_2$ of model (1) obtained by means of Monte Carlo simulations for various values of the smoothing parameters q and β of Equation (15). For each pair (q, β), the number of replications is equal to 250. All values of the RMSE are multiplied by 100.

q	β									
	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
5	69.46	70.90	71.62	71.88	72.28	73.19	73.42	73.86	74.67	75.43
6	67.10	67.30	67.57	68.09	68.78	69.84	69.43	69.86	71.37	70.40
7	65.55	66.10	66.62	67.06	67.36	67.02	67.85	67.48	67.83	67.70
8	64.57	64.58	64.89	65.00	65.57	65.55	65.27	65.13	66.30	66.97
9	62.92	64.09	64.13	64.10	64.18	64.07	63.95	64.83	65.26	64.52
10	62.81	63.09	62.71	63.00	63.57	63.16	64.15	63.79	63.81	64.31

Table SM.32: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_3$ of model (1) obtained by means of Monte Carlo simulations for various values of the smoothing parameters q and β of Equation (15). For each pair (q, β), the number of replications is equal to 250. All values of the RMSE are multiplied by 100.

q	β									
	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
5	92.02	94.05	94.18	95.28	95.89	96.67	97.24	98.14	100.30	101.00
6	88.36	89.00	89.67	90.62	90.78	92.51	92.34	92.65	94.23	93.50
7	86.30	87.30	87.87	88.97	89.41	88.90	90.24	89.26	89.22	89.79
8	85.69	84.93	85.07	85.58	86.28	86.75	86.15	85.96	87.41	88.56
9	82.89	83.93	84.37	84.35	84.39	84.45	84.95	85.40	86.23	85.45
10	83.04	82.79	83.00	83.24	83.67	82.90	84.50	84.04	84.14	84.67

Table SM.33: Sample Root Mean Squared Error (RMSE) for $\hat{\beta}_4$ of model (1) obtained by means of Monte Carlo simulations for various values of the smoothing parameters q and β of Equation (15). For each pair (q, β), the number of replications is equal to 250. All values of the RMSE are multiplied by 100.

q	β									
	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
5	12.38	12.38	12.32	12.50	12.42	12.51	12.52	12.53	12.57	12.66
6	12.25	12.41	12.31	12.36	12.29	12.37	12.43	12.35	12.32	12.31
7	12.16	12.27	12.37	12.18	12.36	12.27	12.24	12.32	12.38	12.16
8	12.21	12.31	12.27	12.11	12.11	12.24	12.22	12.11	12.44	12.19
9	12.19	12.30	12.21	12.14	12.17	12.15	12.15	12.19	12.22	12.19
10	12.20	12.16	12.13	12.08	12.10	12.17	12.28	12.32	12.20	12.13

Table SM.34: Sample Root Mean Squared Error (RMSE) for $\hat{\sigma}$ of model (1) obtained by means of Monte Carlo simulations for various values of the smoothing parameters q and β of Equation (15). For each pair (q, β), the number of replications is equal to 250. All values of the RMSE are multiplied by 100.

q	β									
	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99
5	19.57	18.79	18.48	18.25	17.99	17.82	17.60	17.09	17.36	16.91
6	18.74	18.21	17.96	18.15	17.24	17.56	17.66	17.45	16.88	16.86
7	19.18	18.87	18.90	17.97	17.96	17.78	17.84	17.39	17.53	17.60
8	20.26	19.84	20.05	19.58	19.90	19.79	19.08	18.16	18.69	19.11
9	22.16	23.48	22.78	22.74	22.01	21.63	21.43	20.89	21.32	20.88
10	25.23	24.72	23.79	24.35	23.12	22.94	22.68	22.85	22.70	22.76

Table SM.35: Computing times of model (1) obtained by means of Monte Carlo simulations for various values of the smoothing parameters q and β of Equation (15). For each pair (q, β), the number of replications is equal to 250. The table reports the average computing times for one model, expressed in seconds.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	30.85	32.19	27.82	28.59	29.31
AG quadrature ($m = 3$)	30.85	32.18	27.82	28.58	29.31
AG quadrature ($m = 5$)	30.85	32.19	27.82	28.58	29.31
Cross-entropy ($N = 1000$)	30.28	32.17	27.47	28.74	29.44
Cross-entropy ($N = 2000$)	30.60	32.31	27.55	28.71	29.60
Cross-entropy ($N = 5000$)	30.38	32.23	27.47	28.73	29.47
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	35.81	45.97	40.33	38.72	41.78
AG quadrature ($m = 3$)	35.80	45.96	40.32	38.71	41.78
AG quadrature ($m = 5$)	35.81	45.97	40.33	38.72	41.78
Cross-entropy ($N = 1000$)	35.90	45.65	39.93	39.17	42.13
Cross-entropy ($N = 2000$)	35.22	45.25	40.03	38.81	42.16
Cross-entropy ($N = 5000$)	35.42	45.52	40.16	38.94	41.94
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	53.39	57.88	52.47	57.48	57.77
AG quadrature ($m = 3$)	53.38	57.86	52.46	57.46	57.77
AG quadrature ($m = 5$)	53.39	57.87	52.47	57.47	57.77
Cross-entropy ($N = 1000$)	52.61	58.28	52.48	58.24	57.90
Cross-entropy ($N = 2000$)	52.22	57.93	52.71	56.81	58.09
Cross-entropy ($N = 5000$)	52.02	58.06	52.62	57.52	57.92
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	69.37	84.50	72.04	72.93	77.63
AG quadrature ($m = 3$)	69.37	84.49	72.03	72.92	77.62
AG quadrature ($m = 5$)	69.38	84.51	72.04	72.93	77.63
Cross-entropy ($N = 1000$)	67.95	85.13	71.55	73.92	78.84
Cross-entropy ($N = 2000$)	67.36	84.24	71.58	73.14	78.55
Cross-entropy ($N = 5000$)	67.42	84.99	71.73	73.23	78.57
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	28.08	26.17	25.44	43.18	90.11
AG quadrature ($m = 3$)	28.10	26.20	25.45	43.18	90.10
AG quadrature ($m = 5$)	28.13	26.23	25.47	43.18	90.09
Cross-entropy ($N = 1000$)	40.02	30.85	22.20	15.83	63.34
Cross-entropy ($N = 2000$)	39.92	31.72	21.83	15.84	63.15
Cross-entropy ($N = 5000$)	40.01	31.36	22.00	15.76	63.13
Computing time					
AG quadrature ($m = 1$)	0.30	0.21	0.19	0.19	0.19
AG quadrature ($m = 3$)	0.26	0.24	0.22	0.22	0.22
AG quadrature ($m = 5$)	0.28	0.26	0.24	0.24	0.24
Cross-entropy ($N = 1000$)	23.65	21.79	20.47	19.83	20.34
Cross-entropy ($N = 2000$)	45.32	41.21	40.93	39.19	39.76
Cross-entropy ($N = 5000$)	110.79	98.41	99.85	98.27	97.90

Table SM.36: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) and computing times obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 8$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All the values of the RMSE are multiplied by 100; computing times are in seconds.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	6.19	7.60	3.07	9.12	7.34
AG quadrature ($m = 3$)	6.19	7.59	3.06	9.12	7.34
AG quadrature ($m = 5$)	6.19	7.60	3.06	9.12	7.34
Cross-entropy ($N = 1000$)	6.15	7.57	3.21	9.48	7.24
Cross-entropy ($N = 2000$)	6.00	7.67	3.01	9.53	7.54
Cross-entropy ($N = 5000$)	6.14	7.70	3.14	9.54	7.55
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	11.05	16.27	10.10	12.69	10.19
AG quadrature ($m = 3$)	11.04	16.27	10.09	12.69	10.19
AG quadrature ($m = 5$)	11.05	16.28	10.09	12.69	10.19
Cross-entropy ($N = 1000$)	11.45	15.96	10.03	13.13	10.60
Cross-entropy ($N = 2000$)	10.88	16.08	10.15	12.72	10.66
Cross-entropy ($N = 5000$)	10.82	16.18	10.26	12.91	10.72
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	18.73	21.32	10.75	19.64	15.19
AG quadrature ($m = 3$)	18.73	21.31	10.74	19.64	15.19
AG quadrature ($m = 5$)	18.74	21.32	10.75	19.64	15.19
Cross-entropy ($N = 1000$)	18.99	21.24	11.43	20.33	15.33
Cross-entropy ($N = 2000$)	18.67	21.07	10.85	19.68	15.63
Cross-entropy ($N = 5000$)	18.53	21.30	11.27	20.05	15.83
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	23.76	32.95	19.61	28.74	26.11
AG quadrature ($m = 3$)	23.76	32.96	19.61	28.73	26.11
AG quadrature ($m = 5$)	23.77	32.97	19.61	28.74	26.12
Cross-entropy ($N = 1000$)	23.94	32.51	19.39	29.35	26.38
Cross-entropy ($N = 2000$)	23.79	32.83	19.35	29.21	26.63
Cross-entropy ($N = 5000$)	23.65	33.04	19.75	29.39	26.71
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	14.23	5.16	−5.54	−36.41	−87.19
AG quadrature ($m = 3$)	14.24	5.18	−5.53	−36.40	−87.18
AG quadrature ($m = 5$)	14.26	5.19	−5.52	−36.39	−87.16
Cross-entropy ($N = 1000$)	38.39	28.80	19.30	−11.40	−62.39
Cross-entropy ($N = 2000$)	38.41	29.51	19.09	−11.40	−62.19
Cross-entropy ($N = 5000$)	38.50	29.26	19.23	−11.10	−62.19

Table SM.37: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 8$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All the values of the bias are multiplied by 100.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	29.16	28.73	30.61	30.34	30.52
AG quadrature ($m = 3$)	29.13	28.71	30.59	30.33	30.50
AG quadrature ($m = 5$)	29.14	28.72	30.60	30.33	30.51
Cross-entropy ($N = 1000$)	28.73	27.90	30.03	30.11	30.38
Cross-entropy ($N = 2000$)	28.59	28.36	29.92	29.80	30.76
Cross-entropy ($N = 5000$)	28.69	28.20	29.76	29.86	30.54
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	48.02	43.05	48.24	45.08	49.16
AG quadrature ($m = 3$)	47.99	43.04	48.23	45.05	49.14
AG quadrature ($m = 5$)	48.02	43.08	48.24	45.07	49.15
Cross-entropy ($N = 1000$)	46.05	39.59	48.68	44.08	48.71
Cross-entropy ($N = 2000$)	45.63	39.67	48.61	44.49	49.29
Cross-entropy ($N = 5000$)	45.90	39.97	48.42	44.39	48.94
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	61.14	63.12	56.78	68.26	64.05
AG quadrature ($m = 3$)	61.09	63.17	56.76	68.19	64.00
AG quadrature ($m = 5$)	61.13	63.23	56.77	68.21	64.02
Cross-entropy ($N = 1000$)	58.60	56.62	56.67	67.54	63.38
Cross-entropy ($N = 2000$)	57.98	57.30	56.85	67.70	63.88
Cross-entropy ($N = 5000$)	58.44	57.57	56.56	67.43	63.87
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	79.63	83.91	80.30	80.85	89.07
AG quadrature ($m = 3$)	79.60	83.98	80.28	80.79	89.02
AG quadrature ($m = 5$)	79.64	84.06	80.31	80.82	89.06
Cross-entropy ($N = 1000$)	77.56	74.08	79.29	80.55	88.37
Cross-entropy ($N = 2000$)	77.03	75.18	79.35	81.03	88.66
Cross-entropy ($N = 5000$)	77.63	75.04	79.05	80.37	88.39
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	34.08	33.94	28.05	40.08	85.46
AG quadrature ($m = 3$)	34.13	34.04	28.07	40.09	85.45
AG quadrature ($m = 5$)	34.21	34.14	28.13	40.10	85.43
Cross-entropy ($N = 1000$)	40.45	31.83	22.08	13.30	60.38
Cross-entropy ($N = 2000$)	40.83	32.74	22.45	13.34	60.07
Cross-entropy ($N = 5000$)	41.02	32.50	22.37	13.13	60.03
Computing time					
AG quadrature ($m = 1$)	0.20	0.20	0.19	0.20	0.21
AG quadrature ($m = 3$)	0.23	0.23	0.23	0.23	0.24
AG quadrature ($m = 5$)	0.25	0.25	0.25	0.26	0.26
Cross-entropy ($N = 1000$)	20.28	19.68	20.37	20.25	21.02
Cross-entropy ($N = 2000$)	39.39	39.18	40.20	40.67	42.74
Cross-entropy ($N = 5000$)	99.93	98.18	99.43	101.17	105.74

Table SM.38: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) and computing times obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 12$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All the values of the RMSE are multiplied by 100; computing times are in seconds.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	6.03	9.90	9.48	4.00	11.49
AG quadrature ($m = 3$)	6.02	9.89	9.47	3.99	11.48
AG quadrature ($m = 5$)	6.03	9.90	9.48	4.00	11.48
Cross-entropy ($N = 1000$)	5.74	9.11	8.91	3.62	11.34
Cross-entropy ($N = 2000$)	5.64	9.54	8.82	3.49	11.50
Cross-entropy ($N = 5000$)	5.74	9.13	8.92	3.72	11.40
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	14.15	16.00	18.24	13.98	20.23
AG quadrature ($m = 3$)	14.14	15.99	18.23	13.97	20.22
AG quadrature ($m = 5$)	14.16	16.01	18.25	13.98	20.23
Cross-entropy ($N = 1000$)	12.75	13.90	17.36	12.81	19.51
Cross-entropy ($N = 2000$)	12.86	14.25	17.34	12.86	19.89
Cross-entropy ($N = 5000$)	13.06	14.15	17.27	13.22	19.67
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	21.36	21.37	23.86	22.08	26.88
AG quadrature ($m = 3$)	21.35	21.37	23.85	22.06	26.85
AG quadrature ($m = 5$)	21.37	21.41	23.87	22.08	26.87
Cross-entropy ($N = 1000$)	19.42	18.13	22.91	20.45	25.63
Cross-entropy ($N = 2000$)	19.18	18.50	22.74	20.83	26.25
Cross-entropy ($N = 5000$)	19.89	18.42	22.71	20.91	26.27
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	30.73	30.08	31.64	25.80	42.36
AG quadrature ($m = 3$)	30.73	30.09	31.64	25.79	42.35
AG quadrature ($m = 5$)	30.76	30.14	31.67	25.82	42.37
Cross-entropy ($N = 1000$)	28.61	25.41	29.87	23.19	41.26
Cross-entropy ($N = 2000$)	28.18	26.14	30.06	24.14	41.75
Cross-entropy ($N = 5000$)	29.01	25.67	29.74	24.34	41.45
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	18.54	11.62	−1.47	−29.10	−81.52
AG quadrature ($m = 3$)	18.57	11.66	−1.45	−29.08	−81.51
AG quadrature ($m = 5$)	18.61	11.70	−1.41	−29.04	−81.48
Cross-entropy ($N = 1000$)	39.38	30.41	19.96	−9.36	−59.66
Cross-entropy ($N = 2000$)	39.78	31.18	20.32	−9.27	−59.37
Cross-entropy ($N = 5000$)	40.04	31.06	20.36	−9.25	−59.35

Table SM.39: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 12$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All the values of the bias are multiplied by 100.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	30.26	37.22	37.24	30.79	33.94
AG quadrature ($m = 3$)	30.18	36.77	36.98	30.70	33.72
AG quadrature ($m = 5$)	30.22	37.04	37.11	30.75	33.85
Cross-entropy ($N = 1000$)	30.05	33.30	34.57	29.43	31.54
Cross-entropy ($N = 2000$)	30.26	33.20	34.51	29.51	31.52
Cross-entropy ($N = 5000$)	30.36	33.14	34.54	29.62	31.44
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	48.92	53.11	56.87	47.95	44.24
AG quadrature ($m = 3$)	48.75	52.43	56.49	47.82	43.95
AG quadrature ($m = 5$)	48.88	53.10	56.74	47.92	44.17
Cross-entropy ($N = 1000$)	47.57	43.75	52.12	46.77	40.38
Cross-entropy ($N = 2000$)	47.18	43.14	51.52	46.67	40.36
Cross-entropy ($N = 5000$)	47.60	43.12	52.08	46.77	40.43
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	58.91	80.12	74.15	70.76	59.14
AG quadrature ($m = 3$)	58.55	78.91	73.53	70.52	58.64
AG quadrature ($m = 5$)	58.80	79.89	73.98	70.71	59.01
Cross-entropy ($N = 1000$)	55.30	66.05	65.86	66.99	54.73
Cross-entropy ($N = 2000$)	54.24	64.89	65.21	67.29	54.51
Cross-entropy ($N = 5000$)	54.93	64.63	65.64	67.70	54.21
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	82.07	108.53	104.37	95.68	80.64
AG quadrature ($m = 3$)	81.75	106.90	103.56	95.51	80.14
AG quadrature ($m = 5$)	82.02	108.45	104.18	95.81	80.64
Cross-entropy ($N = 1000$)	79.08	88.06	92.33	89.50	74.04
Cross-entropy ($N = 2000$)	78.55	86.77	91.90	89.64	74.32
Cross-entropy ($N = 5000$)	78.94	86.17	92.03	89.79	73.75
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	38.16	43.79	39.55	43.05	86.16
AG quadrature ($m = 3$)	37.89	42.99	38.81	42.98	86.06
AG quadrature ($m = 5$)	38.28	43.79	39.42	43.14	86.09
Cross-entropy ($N = 1000$)	39.61	32.14	22.40	10.93	59.90
Cross-entropy ($N = 2000$)	40.30	32.01	22.60	10.79	59.73
Cross-entropy ($N = 5000$)	40.23	32.50	22.76	10.57	59.65
Computing time					
AG quadrature ($m = 1$)	0.34	0.24	0.22	0.22	0.21
AG quadrature ($m = 3$)	0.30	0.28	0.26	0.26	0.24
AG quadrature ($m = 5$)	0.33	0.31	0.28	0.28	0.27
Cross-entropy ($N = 1000$)	26.38	23.24	22.49	22.23	21.55
Cross-entropy ($N = 2000$)	49.39	43.91	43.95	42.75	43.42
Cross-entropy ($N = 5000$)	116.40	108.10	108.12	107.31	107.54

Table SM.40: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) obtained by means of Monte Carlo simulations for various estimation techniques and various values of σ and for $G = 25$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All values of the RMSE are multiplied by 100.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	9.39	10.54	11.71	11.28	8.15
AG quadrature ($m = 3$)	9.32	10.35	11.56	11.22	8.05
AG quadrature ($m = 5$)	9.36	10.47	11.65	11.27	8.11
Cross-entropy ($N = 1000$)	8.55	7.79	9.23	9.83	7.00
Cross-entropy ($N = 2000$)	8.53	7.97	9.51	9.90	7.00
Cross-entropy ($N = 5000$)	8.66	7.92	9.48	10.00	7.03
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	17.25	19.22	25.19	18.12	15.83
AG quadrature ($m = 3$)	17.17	19.00	25.00	18.06	15.71
AG quadrature ($m = 5$)	17.25	19.22	25.15	18.14	15.82
Cross-entropy ($N = 1000$)	15.30	14.15	20.81	16.02	13.41
Cross-entropy ($N = 2000$)	15.32	14.02	20.72	15.74	13.08
Cross-entropy ($N = 5000$)	15.35	14.21	21.09	16.05	13.17
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	20.34	30.40	32.84	26.41	20.31
AG quadrature ($m = 3$)	20.21	30.01	32.58	26.30	20.12
AG quadrature ($m = 5$)	20.33	30.35	32.80	26.44	20.28
Cross-entropy ($N = 1000$)	17.23	23.03	26.29	22.48	16.45
Cross-entropy ($N = 2000$)	17.01	22.72	26.52	22.69	16.90
Cross-entropy ($N = 5000$)	17.11	22.89	27.08	22.80	16.61
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	34.85	41.02	50.75	40.21	31.64
AG quadrature ($m = 3$)	34.72	40.54	50.41	40.12	31.45
AG quadrature ($m = 5$)	34.88	41.02	50.72	40.30	31.66
Cross-entropy ($N = 1000$)	30.74	30.75	41.74	34.69	26.73
Cross-entropy ($N = 2000$)	30.79	30.29	41.69	35.15	27.18
Cross-entropy ($N = 5000$)	30.82	30.49	42.10	34.91	26.77
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	19.54	16.23	5.88	−27.35	−78.87
AG quadrature ($m = 3$)	19.49	15.94	5.67	−27.40	−79.02
AG quadrature ($m = 5$)	19.65	16.26	5.92	−27.23	−78.83
Cross-entropy ($N = 1000$)	39.14	31.12	21.30	−8.83	−59.51
Cross-entropy ($N = 2000$)	39.81	31.14	21.50	−8.65	−59.33
Cross-entropy ($N = 5000$)	39.78	31.59	21.68	−8.39	−59.30

Table SM.41: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 25$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All the values of the bias are multiplied by 100.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	269.64	137.11	45.04	34.29	187.36
AG quadrature ($m = 3$)	34.79	134.51	38.49	33.48	126.86
AG quadrature ($m = 5$)	35.84	113.69	39.38	33.79	258.89
Cross-entropy ($N = 1000$)	29.08	28.08	32.47	31.94	30.68
Cross-entropy ($N = 2000$)	29.42	28.18	32.36	31.82	30.76
Cross-entropy ($N = 5000$)	29.07	28.10	32.41	31.98	30.76
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	450.98	212.74	71.31	45.50	483.74
AG quadrature ($m = 3$)	50.70	225.85	53.12	44.40	294.71
AG quadrature ($m = 5$)	52.70	219.29	54.83	44.92	423.53
Cross-entropy ($N = 1000$)	41.91	39.34	43.67	41.86	51.45
Cross-entropy ($N = 2000$)	41.93	39.22	42.95	41.33	51.21
Cross-entropy ($N = 5000$)	41.88	39.58	43.29	41.63	51.61
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	523.82	363.11	87.71	68.97	658.52
AG quadrature ($m = 3$)	72.79	403.71	69.50	66.92	392.21
AG quadrature ($m = 5$)	75.80	392.26	71.83	67.84	645.28
Cross-entropy ($N = 1000$)	55.51	53.77	56.01	61.04	62.51
Cross-entropy ($N = 2000$)	55.61	53.81	55.72	60.59	61.95
Cross-entropy ($N = 5000$)	55.33	53.60	55.67	60.93	62.45
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	951.81	486.40	128.73	90.05	965.47
AG quadrature ($m = 3$)	103.17	524.54	98.23	87.73	516.14
AG quadrature ($m = 5$)	107.86	483.75	101.66	88.86	926.51
Cross-entropy ($N = 1000$)	76.99	73.74	78.09	81.96	81.37
Cross-entropy ($N = 2000$)	77.86	73.49	77.29	80.56	81.11
Cross-entropy ($N = 5000$)	77.61	73.81	78.02	81.16	81.62
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	561.45	393.75	73.21	47.91	801.28
AG quadrature ($m = 3$)	58.16	305.73	51.27	45.98	250.10
AG quadrature ($m = 5$)	61.32	237.42	54.14	46.99	286.78
Cross-entropy ($N = 1000$)	40.51	31.54	22.06	10.81	59.19
Cross-entropy ($N = 2000$)	41.24	31.77	22.74	10.32	58.69
Cross-entropy ($N = 5000$)	41.24	31.83	22.63	10.13	58.45
Computing time					
AG quadrature ($m = 1$)	0.27	0.25	0.25	0.24	0.26
AG quadrature ($m = 3$)	0.29	0.29	0.29	0.28	0.30
AG quadrature ($m = 5$)	0.32	0.32	0.31	0.30	0.32
Cross-entropy ($N = 1000$)	24.39	25.06	24.59	24.83	24.80
Cross-entropy ($N = 2000$)	48.07	48.42	49.11	48.78	48.45
Cross-entropy ($N = 5000$)	119.64	120.12	120.57	120.86	120.87

Table SM.42: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) obtained by means of Monte Carlo simulations for various estimation techniques and various values of σ and for $G = 50$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All values of the RMSE are multiplied by 100.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	30.67	20.75	12.78	9.97	30.72
AG quadrature ($m = 3$)	11.26	19.11	10.56	9.51	25.54
AG quadrature ($m = 5$)	11.78	15.16	11.02	9.73	33.17
Cross-entropy ($N = 1000$)	7.10	3.02	6.25	7.31	9.96
Cross-entropy ($N = 2000$)	7.41	2.91	6.15	7.10	9.79
Cross-entropy ($N = 5000$)	7.25	3.02	6.40	7.06	9.94
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	54.32	39.76	29.89	21.16	69.91
AG quadrature ($m = 3$)	22.05	39.40	25.84	20.44	56.04
AG quadrature ($m = 5$)	23.04	34.15	26.68	20.86	60.04
Cross-entropy ($N = 1000$)	13.83	10.41	17.50	15.44	20.49
Cross-entropy ($N = 2000$)	14.00	10.73	17.71	15.32	20.01
Cross-entropy ($N = 5000$)	13.92	10.90	17.60	15.57	20.22
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	69.82	64.62	40.90	27.08	87.97
AG quadrature ($m = 3$)	31.76	65.68	35.98	25.97	68.50
AG quadrature ($m = 5$)	33.19	56.20	37.18	26.60	79.86
Cross-entropy ($N = 1000$)	19.31	16.70	24.55	18.31	21.26
Cross-entropy ($N = 2000$)	19.43	16.81	24.04	18.21	20.67
Cross-entropy ($N = 5000$)	19.36	16.82	24.27	18.61	21.10
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	114.41	90.78	59.69	38.23	120.01
AG quadrature ($m = 3$)	46.30	91.51	52.34	36.87	87.84
AG quadrature ($m = 5$)	48.33	76.99	54.01	37.70	109.19
Cross-entropy ($N = 1000$)	28.20	26.64	36.20	27.36	26.13
Cross-entropy ($N = 2000$)	29.27	26.94	35.63	26.64	25.91
Cross-entropy ($N = 5000$)	29.10	27.04	36.03	27.04	26.28
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	76.27	62.46	19.31	−20.02	0.48
AG quadrature ($m = 3$)	36.01	51.43	14.62	−21.09	−41.89
AG quadrature ($m = 5$)	37.42	40.25	15.84	−20.35	−43.03
Cross-entropy ($N = 1000$)	40.12	30.98	21.35	−9.52	−58.81
Cross-entropy ($N = 2000$)	40.86	31.19	22.11	−8.94	−58.34
Cross-entropy ($N = 5000$)	40.90	31.31	22.01	−8.75	−58.11

Table SM.43: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulations for various estimation techniques and various values of σ and for $G = 50$ and $n = 250$. For each value of σ, the number of replications is equal to 200. All values of the bias are multiplied by 100.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	14.44	13.54	13.72	15.39	14.62
AG quadrature ($m = 3$)	14.44	13.55	13.72	15.39	14.62
AG quadrature ($m = 5$)	14.44	13.55	13.72	15.39	14.62
Cross-entropy ($N = 1000$)	14.35	13.54	13.71	15.40	14.74
Cross-entropy ($N = 2000$)	14.41	13.46	13.80	15.44	14.66
Cross-entropy ($N = 5000$)	14.39	13.52	13.71	15.42	14.58
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	19.30	17.91	18.26	19.88	19.28
AG quadrature ($m = 3$)	19.30	17.91	18.26	19.88	19.28
AG quadrature ($m = 5$)	19.30	17.91	18.26	19.88	19.28
Cross-entropy ($N = 1000$)	19.25	17.89	18.46	19.88	19.34
Cross-entropy ($N = 2000$)	19.24	17.73	18.41	19.80	19.24
Cross-entropy ($N = 5000$)	19.37	17.81	18.29	19.77	19.17
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	27.70	24.24	24.26	25.28	26.25
AG quadrature ($m = 3$)	27.70	24.24	24.26	25.28	26.25
AG quadrature ($m = 5$)	27.70	24.24	24.26	25.28	26.25
Cross-entropy ($N = 1000$)	27.65	24.05	24.29	25.56	26.40
Cross-entropy ($N = 2000$)	27.62	24.14	24.49	25.42	26.29
Cross-entropy ($N = 5000$)	27.70	24.21	24.31	25.20	26.08
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	33.34	31.11	27.87	30.47	35.05
AG quadrature ($m = 3$)	33.34	31.11	27.87	30.47	35.05
AG quadrature ($m = 5$)	33.34	31.11	27.87	30.47	35.05
Cross-entropy ($N = 1000$)	33.25	31.04	28.22	30.46	35.13
Cross-entropy ($N = 2000$)	33.33	31.00	27.99	30.37	35.15
Cross-entropy ($N = 5000$)	33.40	30.98	27.97	30.17	34.82
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	12.31	12.76	17.72	44.85	94.54
AG quadrature ($m = 3$)	12.32	12.77	17.72	44.85	94.54
AG quadrature ($m = 5$)	12.32	12.77	17.72	44.85	94.54
Cross-entropy ($N = 1000$)	18.49	11.34	5.52	31.61	81.69
Cross-entropy ($N = 2000$)	18.59	11.36	5.48	31.52	81.29
Cross-entropy ($N = 5000$)	18.75	11.32	5.40	31.45	81.35
Computing time					
AG quadrature ($m = 1$)	0.73	0.62	0.61	0.62	0.61
AG quadrature ($m = 3$)	0.84	0.81	0.79	0.79	0.79
AG quadrature ($m = 5$)	0.93	0.91	0.88	0.89	0.89
Cross-entropy ($N = 1000$)	31.42	30.93	29.69	29.94	30.28
Cross-entropy ($N = 2000$)	60.48	59.52	59.11	59.44	59.52
Cross-entropy ($N = 5000$)	147.59	146.34	148.34	147.93	146.35

Table SM.44: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) and computing times obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 8$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All the values of the RMSE are multiplied by 100; computing times are in seconds.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	0.78	0.50	0.50	0.36	0.12
AG quadrature ($m = 3$)	0.78	0.50	0.50	0.36	0.12
AG quadrature ($m = 5$)	0.78	0.50	0.50	0.36	0.12
Cross-entropy ($N = 1000$)	0.84	0.47	0.63	0.30	0.11
Cross-entropy ($N = 2000$)	0.68	0.40	0.54	0.31	0.32
Cross-entropy ($N = 5000$)	0.74	0.43	0.52	0.27	0.19
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	3.92	3.17	1.36	2.57	3.13
AG quadrature ($m = 3$)	3.92	3.17	1.36	2.57	3.13
AG quadrature ($m = 5$)	3.92	3.17	1.36	2.57	3.13
Cross-entropy ($N = 1000$)	3.92	2.97	1.38	2.61	3.29
Cross-entropy ($N = 2000$)	3.90	2.80	1.46	2.51	3.28
Cross-entropy ($N = 5000$)	4.04	3.01	1.41	2.48	3.27
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	5.15	5.21	2.81	3.06	4.46
AG quadrature ($m = 3$)	5.15	5.21	2.81	3.06	4.46
AG quadrature ($m = 5$)	5.15	5.21	2.81	3.06	4.46
Cross-entropy ($N = 1000$)	5.15	4.90	3.01	3.26	4.75
Cross-entropy ($N = 2000$)	5.26	4.95	3.04	3.12	4.74
Cross-entropy ($N = 5000$)	5.35	5.08	2.97	3.11	4.75
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	4.97	7.35	3.47	2.09	8.12
AG quadrature ($m = 3$)	4.97	7.35	3.47	2.09	8.12
AG quadrature ($m = 5$)	4.97	7.35	3.47	2.09	8.12
Cross-entropy ($N = 1000$)	4.96	7.29	3.85	2.30	8.47
Cross-entropy ($N = 2000$)	5.12	7.02	3.81	2.22	8.53
Cross-entropy ($N = 5000$)	5.27	7.22	3.71	2.21	8.46
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	5.57	−1.54	−14.19	−43.51	−93.96
AG quadrature ($m = 3$)	5.58	−1.54	−14.19	−43.51	−93.96
AG quadrature ($m = 5$)	5.58	−1.54	−14.19	−43.51	−93.96
Cross-entropy ($N = 1000$)	17.71	9.82	−1.48	−31.17	−81.53
Cross-entropy ($N = 2000$)	17.79	9.81	−1.41	−31.10	−81.11
Cross-entropy ($N = 5000$)	17.96	9.88	−1.26	−31.02	−81.19

Table SM.45: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 8$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All the values of the bias are multiplied by 100.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	14.47	15.07	13.29	13.96	15.00
AG quadrature ($m = 3$)	14.47	15.07	13.29	13.96	15.00
AG quadrature ($m = 5$)	14.47	15.07	13.29	13.96	15.00
Cross-entropy ($N = 1000$)	14.57	15.13	13.48	13.94	15.05
Cross-entropy ($N = 2000$)	14.38	15.07	13.23	13.91	14.89
Cross-entropy ($N = 5000$)	14.35	15.06	13.21	14.01	15.00
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	19.46	18.79	19.16	20.47	19.40
AG quadrature ($m = 3$)	19.46	18.79	19.16	20.47	19.40
AG quadrature ($m = 5$)	19.46	18.79	19.16	20.47	19.40
Cross-entropy ($N = 1000$)	19.46	18.94	19.33	20.54	19.51
Cross-entropy ($N = 2000$)	19.43	18.97	19.19	20.55	19.42
Cross-entropy ($N = 5000$)	19.34	18.73	19.15	20.55	19.39
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	25.81	25.52	24.21	23.90	24.29
AG quadrature ($m = 3$)	25.82	25.52	24.21	23.90	24.29
AG quadrature ($m = 5$)	25.82	25.52	24.21	23.90	24.29
Cross-entropy ($N = 1000$)	25.51	25.63	24.27	24.13	24.19
Cross-entropy ($N = 2000$)	25.43	25.45	23.93	24.07	24.28
Cross-entropy ($N = 5000$)	25.50	25.29	24.00	24.02	24.26
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	32.97	31.79	30.92	33.60	30.37
AG quadrature ($m = 3$)	32.97	31.79	30.92	33.60	30.37
AG quadrature ($m = 5$)	32.97	31.79	30.92	33.60	30.37
Cross-entropy ($N = 1000$)	32.63	31.81	30.98	33.91	30.72
Cross-entropy ($N = 2000$)	32.45	31.75	30.68	33.64	30.63
Cross-entropy ($N = 5000$)	32.48	31.56	30.83	33.66	30.52
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	14.74	12.59	17.58	44.57	93.00
AG quadrature ($m = 3$)	14.76	12.60	17.58	44.57	93.00
AG quadrature ($m = 5$)	14.76	12.60	17.58	44.57	93.00
Cross-entropy ($N = 1000$)	19.29	10.67	4.41	31.54	80.91
Cross-entropy ($N = 2000$)	19.34	10.56	4.30	31.54	80.81
Cross-entropy ($N = 5000$)	19.26	10.73	4.24	31.43	80.72
Computing time					
AG quadrature ($m = 1$)	0.64	0.63	0.64	0.63	0.64
AG quadrature ($m = 3$)	0.81	0.80	0.81	0.81	0.81
AG quadrature ($m = 5$)	0.92	0.91	0.92	0.90	0.91
Cross-entropy ($N = 1000$)	31.70	31.00	31.45	31.19	32.14
Cross-entropy ($N = 2000$)	61.13	61.23	60.50	62.37	62.01
Cross-entropy ($N = 5000$)	153.70	153.65	152.07	153.65	155.82

Table SM.46: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) and computing times obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 12$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All the values of the RMSE are multiplied by 100; computing times are in seconds.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	1.97	0.86	0.75	1.40	0.78
AG quadrature ($m = 3$)	1.97	0.86	0.75	1.40	0.78
AG quadrature ($m = 5$)	1.97	0.86	0.75	1.40	0.78
Cross-entropy ($N = 1000$)	1.86	0.85	0.93	1.40	0.66
Cross-entropy ($N = 2000$)	1.82	0.81	0.73	1.36	0.75
Cross-entropy ($N = 5000$)	1.85	0.80	0.66	1.36	0.67
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	1.72	2.55	2.31	2.54	2.76
AG quadrature ($m = 3$)	1.72	2.55	2.31	2.54	2.76
AG quadrature ($m = 5$)	1.72	2.55	2.31	2.54	2.76
Cross-entropy ($N = 1000$)	1.63	2.50	2.48	2.64	2.63
Cross-entropy ($N = 2000$)	1.44	2.50	2.37	2.48	2.72
Cross-entropy ($N = 5000$)	1.46	2.44	2.29	2.59	2.61
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	4.23	2.28	3.38	4.92	4.15
AG quadrature ($m = 3$)	4.23	2.28	3.38	4.92	4.15
AG quadrature ($m = 5$)	4.23	2.28	3.38	4.92	4.15
Cross-entropy ($N = 1000$)	4.20	2.32	3.60	5.13	4.14
Cross-entropy ($N = 2000$)	3.99	2.19	3.53	4.86	4.20
Cross-entropy ($N = 5000$)	4.00	2.15	3.43	4.98	3.98
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	4.55	3.77	6.20	7.40	5.40
AG quadrature ($m = 3$)	4.56	3.77	6.20	7.40	5.40
AG quadrature ($m = 5$)	4.56	3.77	6.20	7.40	5.40
Cross-entropy ($N = 1000$)	4.51	3.89	6.55	7.68	5.43
Cross-entropy ($N = 2000$)	4.28	3.77	6.33	7.36	5.59
Cross-entropy ($N = 5000$)	4.26	3.63	6.30	7.54	5.23
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	7.29	−1.56	−13.28	−43.21	−92.20
AG quadrature ($m = 3$)	7.30	−1.55	−13.27	−43.20	−92.19
AG quadrature ($m = 5$)	7.30	−1.55	−13.27	−43.20	−92.19
Cross-entropy ($N = 1000$)	18.77	9.72	−1.14	−31.24	−80.79
Cross-entropy ($N = 2000$)	18.80	9.54	−0.97	−31.23	−80.68
Cross-entropy ($N = 5000$)	18.77	9.76	−0.99	−31.12	−80.60

Table SM.47: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 12$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All the values of the bias are multiplied by 100.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	13.63	14.98	14.96	14.61	14.06
AG quadrature ($m = 3$)	13.63	14.98	14.96	14.61	14.06
AG quadrature ($m = 5$)	13.63	14.98	14.96	14.61	14.06
Cross-entropy ($N = 1000$)	13.65	15.05	14.88	14.58	14.11
Cross-entropy ($N = 2000$)	13.52	14.90	14.79	14.64	14.00
Cross-entropy ($N = 5000$)	13.58	15.02	14.86	14.57	14.02
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	18.31	22.00	19.77	21.44	18.56
AG quadrature ($m = 3$)	18.31	22.01	19.77	21.45	18.56
AG quadrature ($m = 5$)	18.31	22.01	19.77	21.45	18.56
Cross-entropy ($N = 1000$)	18.21	21.69	19.81	21.19	18.57
Cross-entropy ($N = 2000$)	18.20	21.62	19.61	20.95	18.38
Cross-entropy ($N = 5000$)	18.28	21.65	19.65	20.86	18.39
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	23.64	25.12	28.53	27.92	25.83
AG quadrature ($m = 3$)	23.65	25.13	28.53	27.93	25.83
AG quadrature ($m = 5$)	23.65	25.13	28.54	27.93	25.83
Cross-entropy ($N = 1000$)	23.42	24.90	28.16	27.67	25.49
Cross-entropy ($N = 2000$)	23.52	24.89	28.14	27.43	25.25
Cross-entropy ($N = 5000$)	23.43	24.82	28.23	27.32	25.33
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	29.56	32.52	34.94	34.80	34.00
AG quadrature ($m = 3$)	29.56	32.52	34.94	34.80	34.00
AG quadrature ($m = 5$)	29.56	32.52	34.94	34.81	34.00
Cross-entropy ($N = 1000$)	29.59	32.14	34.70	34.01	33.53
Cross-entropy ($N = 2000$)	29.42	31.95	34.39	33.88	33.45
Cross-entropy ($N = 5000$)	29.57	32.06	34.59	33.86	33.37
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	17.33	14.20	18.71	41.63	89.21
AG quadrature ($m = 3$)	17.37	14.23	18.72	41.61	89.19
AG quadrature ($m = 5$)	17.37	14.23	18.72	41.61	89.19
Cross-entropy ($N = 1000$)	19.30	10.22	3.37	29.98	79.79
Cross-entropy ($N = 2000$)	19.20	10.11	3.24	29.83	79.66
Cross-entropy ($N = 5000$)	19.26	10.29	3.19	29.66	79.57
Computing time					
AG quadrature ($m = 1$)	0.67	0.66	0.67	0.67	0.68
AG quadrature ($m = 3$)	0.84	0.86	0.87	0.85	0.86
AG quadrature ($m = 5$)	0.96	0.96	0.97	0.97	0.98
Cross-entropy ($N = 1000$)	37.56	37.63	38.18	38.25	37.95
Cross-entropy ($N = 2000$)	73.76	74.02	74.62	75.40	74.79
Cross-entropy ($N = 5000$)	182.87	181.81	183.77	181.28	183.17

Table SM.48: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) obtained by means of Monte Carlo simulations for various estimation techniques and various values of σ and for $G = 25$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All values of the RMSE are multiplied by 100.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	0.79	1.43	2.89	3.68	1.70
AG quadrature ($m = 3$)	0.79	1.43	2.89	3.68	1.71
AG quadrature ($m = 5$)	0.79	1.43	2.89	3.68	1.71
Cross-entropy ($N = 1000$)	0.43	1.12	2.73	3.32	1.51
Cross-entropy ($N = 2000$)	0.60	1.23	2.57	3.41	1.37
Cross-entropy ($N = 5000$)	0.57	1.27	2.63	3.45	1.35
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	2.09	4.09	4.16	6.17	3.68
AG quadrature ($m = 3$)	2.09	4.09	4.16	6.18	3.69
AG quadrature ($m = 5$)	2.09	4.09	4.16	6.18	3.69
Cross-entropy ($N = 1000$)	1.53	3.71	3.85	5.75	2.96
Cross-entropy ($N = 2000$)	1.57	3.81	3.70	5.50	3.10
Cross-entropy ($N = 5000$)	1.74	3.72	3.74	5.54	2.97
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	2.61	5.10	6.59	9.62	6.71
AG quadrature ($m = 3$)	2.62	5.10	6.59	9.63	6.71
AG quadrature ($m = 5$)	2.62	5.10	6.59	9.63	6.72
Cross-entropy ($N = 1000$)	2.21	4.57	6.02	8.64	5.68
Cross-entropy ($N = 2000$)	2.13	4.68	5.96	8.64	5.80
Cross-entropy ($N = 5000$)	2.17	4.68	6.05	8.63	5.81
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	3.80	7.50	9.97	12.24	9.19
AG quadrature ($m = 3$)	3.80	7.51	9.97	12.25	9.20
AG quadrature ($m = 5$)	3.80	7.51	9.98	12.25	9.20
Cross-entropy ($N = 1000$)	3.20	6.88	9.23	11.00	8.01
Cross-entropy ($N = 2000$)	3.15	6.85	9.18	11.00	8.13
Cross-entropy ($N = 5000$)	3.31	6.99	9.38	11.02	7.92
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	9.22	−1.02	−10.15	−38.31	−87.83
AG quadrature ($m = 3$)	9.24	−1.00	−10.13	−38.29	−87.80
AG quadrature ($m = 5$)	9.24	−1.00	−10.13	−38.28	−87.80
Cross-entropy ($N = 1000$)	19.05	9.68	0.00	−29.80	−79.73
Cross-entropy ($N = 2000$)	18.98	9.65	0.08	−29.67	−79.61
Cross-entropy ($N = 5000$)	19.07	9.87	0.17	−29.49	−79.51

Table SM.49: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulation for various estimation techniques and various values of σ and for $G = 25$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All the values of the bias are multiplied by 100.

	σ				
RMSE of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	13.84	14.26	13.83	13.20	15.69
AG quadrature ($m = 3$)	13.85	14.27	13.83	13.21	15.70
AG quadrature ($m = 5$)	13.85	14.27	13.83	13.21	15.70
Cross-entropy ($N = 1000$)	13.60	13.95	13.75	13.16	15.37
Cross-entropy ($N = 2000$)	13.52	13.77	13.64	13.05	15.42
Cross-entropy ($N = 5000$)	13.62	13.84	13.56	13.00	15.39
RMSE of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	20.56	21.40	19.85	20.23	20.30
AG quadrature ($m = 3$)	20.57	21.41	19.86	20.23	20.31
AG quadrature ($m = 5$)	20.57	21.42	19.86	20.24	20.31
Cross-entropy ($N = 1000$)	19.61	20.68	19.02	19.49	19.88
Cross-entropy ($N = 2000$)	19.78	20.60	19.01	19.23	19.85
Cross-entropy ($N = 5000$)	19.72	20.49	19.07	19.24	19.85
RMSE of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	25.09	26.65	24.69	25.18	27.61
AG quadrature ($m = 3$)	25.10	26.67	24.71	25.19	27.62
AG quadrature ($m = 5$)	25.10	26.67	24.71	25.19	27.62
Cross-entropy ($N = 1000$)	23.19	25.27	23.72	24.48	27.39
Cross-entropy ($N = 2000$)	23.63	25.39	23.70	24.41	27.01
Cross-entropy ($N = 5000$)	23.55	25.51	23.75	24.41	26.98
RMSE of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	32.33	35.02	32.60	31.61	32.65
AG quadrature ($m = 3$)	32.35	35.05	32.62	31.63	32.67
AG quadrature ($m = 5$)	32.35	35.06	32.62	31.64	32.67
Cross-entropy ($N = 1000$)	30.59	33.18	31.08	29.85	32.05
Cross-entropy ($N = 2000$)	30.63	33.37	31.23	29.78	31.91
Cross-entropy ($N = 5000$)	30.75	33.43	31.33	29.88	31.91
RMSE of $\hat{\sigma}$					
AG quadrature ($m = 1$)	21.85	18.72	18.68	42.38	89.51
AG quadrature ($m = 3$)	21.94	18.81	18.73	42.36	89.47
AG quadrature ($m = 5$)	21.95	18.83	18.74	42.36	89.47
Cross-entropy ($N = 1000$)	19.03	10.43	2.40	30.21	79.75
Cross-entropy ($N = 2000$)	19.33	10.55	2.32	30.10	79.84
Cross-entropy ($N = 5000$)	19.43	10.54	2.24	29.99	79.65
Computing time					
AG quadrature ($m = 1$)	0.71	0.72	0.71	0.71	0.71
AG quadrature ($m = 3$)	0.91	0.90	0.90	0.90	0.89
AG quadrature ($m = 5$)	1.02	1.01	1.03	1.01	1.02
Cross-entropy ($N = 1000$)	49.01	45.23	45.13	45.38	44.43
Cross-entropy ($N = 2000$)	97.30	88.74	88.18	88.49	88.56
Cross-entropy ($N = 5000$)	224.44	221.45	222.62	219.23	222.36

Table SM.50: Sample Root Mean Squared Error (RMSE) for all the parameters of model (1) obtained by means of Monte Carlo simulations for various estimation techniques and various values of σ and for $G = 50$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All values of the RMSE are multiplied by 100.

	σ				
Bias of $\hat{\beta}_0$	0.01	0.10	0.20	0.50	1.00
AG quadrature ($m = 1$)	2.07	2.64	2.48	1.97	1.37
AG quadrature ($m = 3$)	2.08	2.65	2.49	1.98	1.37
AG quadrature ($m = 5$)	2.08	2.65	2.49	1.98	1.37
Cross-entropy ($N = 1000$)	1.44	2.08	1.74	1.51	0.89
Cross-entropy ($N = 2000$)	1.53	2.00	1.90	1.37	0.86
Cross-entropy ($N = 5000$)	1.59	2.12	1.94	1.41	0.90
Bias of $\hat{\beta}_1$					
AG quadrature ($m = 1$)	1.71	5.38	5.42	3.92	4.82
AG quadrature ($m = 3$)	1.72	5.39	5.43	3.93	4.83
AG quadrature ($m = 5$)	1.72	5.39	5.43	3.93	4.83
Cross-entropy ($N = 1000$)	0.65	4.35	4.44	2.93	3.97
Cross-entropy ($N = 2000$)	0.83	4.39	4.25	3.04	3.77
Cross-entropy ($N = 5000$)	0.81	4.32	4.33	2.82	3.73
Bias of $\hat{\beta}_2$					
AG quadrature ($m = 1$)	4.88	7.28	4.67	5.14	7.12
AG quadrature ($m = 3$)	4.89	7.30	4.69	5.15	7.13
AG quadrature ($m = 5$)	4.90	7.30	4.69	5.16	7.14
Cross-entropy ($N = 1000$)	3.13	5.62	3.10	3.61	6.00
Cross-entropy ($N = 2000$)	3.39	5.60	3.15	3.67	5.88
Cross-entropy ($N = 5000$)	3.47	5.85	3.18	3.63	5.68
Bias of $\hat{\beta}_3$					
AG quadrature ($m = 1$)	6.74	10.00	8.80	5.57	8.17
AG quadrature ($m = 3$)	6.76	10.03	8.83	5.59	8.19
AG quadrature ($m = 5$)	6.77	10.04	8.83	5.60	8.19
Cross-entropy ($N = 1000$)	4.68	7.99	6.81	3.65	6.60
Cross-entropy ($N = 2000$)	4.81	8.17	6.79	3.73	6.50
Cross-entropy ($N = 5000$)	5.04	8.07	6.96	3.70	6.42
Bias of $\hat{\sigma}$					
AG quadrature ($m = 1$)	11.60	2.95	−6.02	−38.15	−87.70
AG quadrature ($m = 3$)	11.65	3.02	−5.95	−38.10	−87.64
AG quadrature ($m = 5$)	11.66	3.03	−5.94	−38.09	−87.63
Cross-entropy ($N = 1000$)	18.87	10.10	0.21	−30.10	−79.71
Cross-entropy ($N = 2000$)	19.16	10.28	0.41	−30.00	−79.81
Cross-entropy ($N = 5000$)	19.29	10.28	0.47	−29.89	−79.63

Table SM.51: Bias for all the parameters of model (1) obtained by means of Monte Carlo simulations for various estimation techniques and various values of σ and for $G = 50$ and $n = 1000$. For each value of σ, the number of replications is equal to 200. All values of the bias are multiplied by 100.

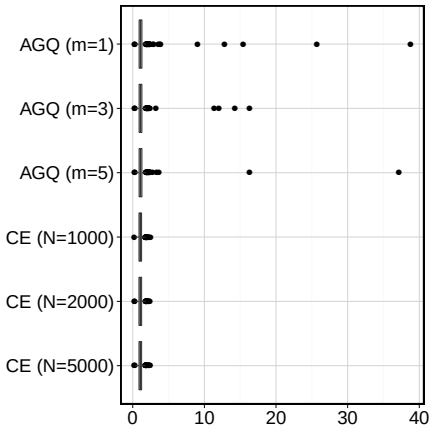

Figure SM.1: Boxplots of $\hat{\beta}_1$ for models with $G = 50$ and $\sigma = 1$ according to the estimation method.

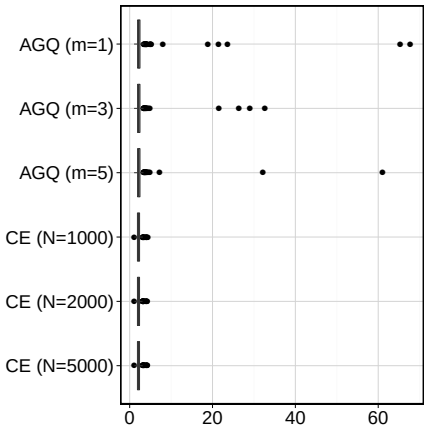

Figure SM.2: Boxplots of $\hat{\beta}_2$ for models with $G = 50$ and $\sigma = 1$ according to the estimation method.

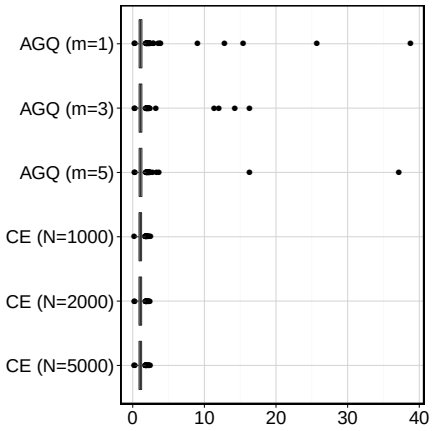

Figure SM.3: Boxplots of $\hat{\beta}_3$ for models with $G = 50$ and $\sigma = 1$ according to the estimation method.

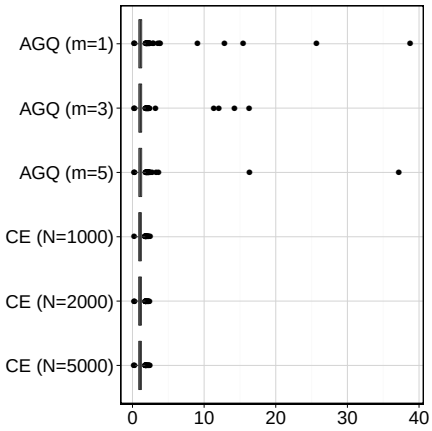

Figure SM.4: Boxplots of $\hat{\beta}_4$ for models with $G = 50$ and $\sigma = 1$ according to the estimation method.

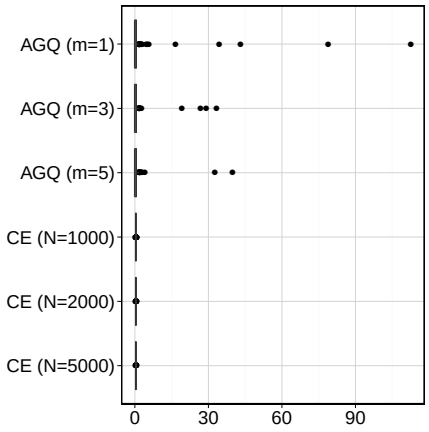

Figure SM.5: Boxplots of $\hat{\sigma}$ for models with $G = 50$ and $\sigma = 1$ according to the estimation method.

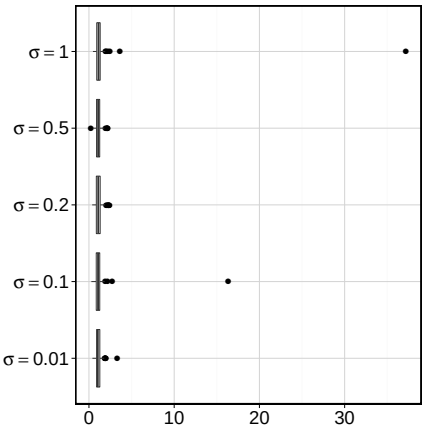

Figure SM.6: Boxplots of $\hat{\beta}_1$ for models with $G = 50$ and various values of σ, estimated by means of AGQ with $m = 5$.

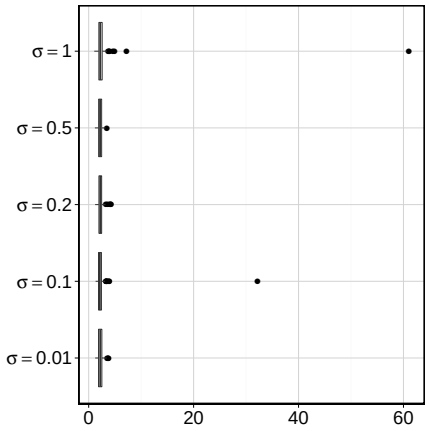

Figure SM.7: Boxplots of $\hat{\beta}_2$ for models with $G = 50$ and various values of σ, estimated by means of AGQ with $m = 5$.

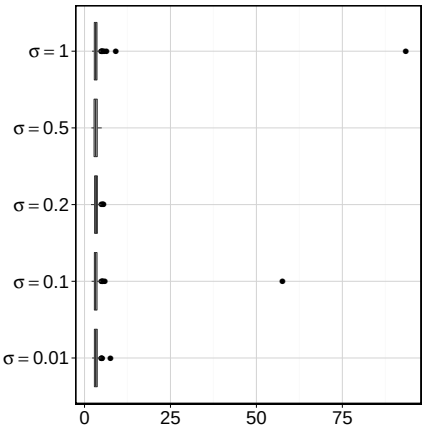

Figure SM.8: Boxplots of $\hat{\beta}_3$ for models with $G = 50$ and various values of σ, estimated by means of AGQ with $m = 5$.

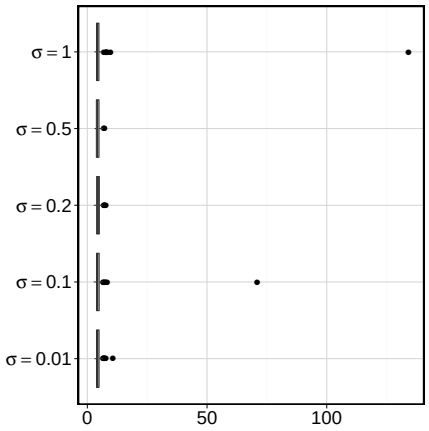

Figure SM.9: Boxplots of $\hat{\beta}_4$ for models with $G = 50$ and various values of σ, estimated by means of AGQ with $m = 5$.

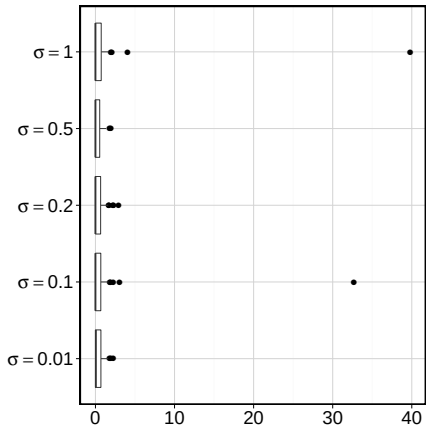

Figure SM.10: Boxplots of $\hat{\sigma}$ for models with $G = 50$ and various values of σ, estimated by means of AGQ with $m = 5$.

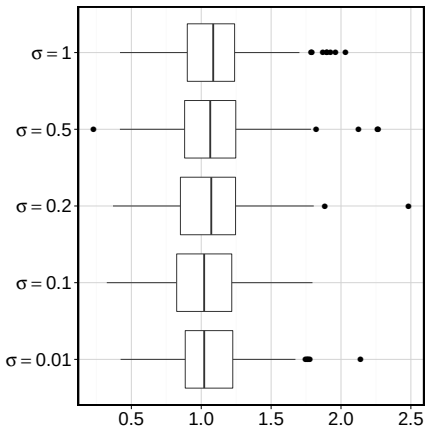

Figure SM.11: Boxplots of $\hat{\beta}_1$ for models with $G = 50$ and various values of σ, estimated by means of the cross-entropy algorithm with $N = 1000$.

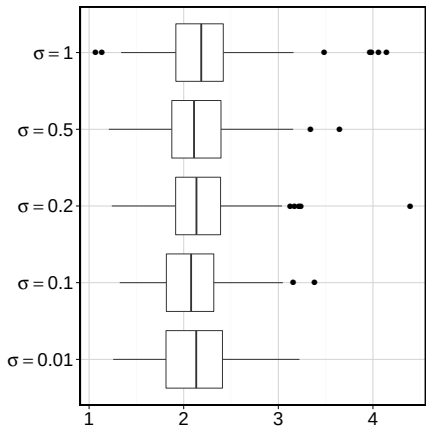

Figure SM.12: Boxplots of $\hat{\beta}_2$ for models with $G = 50$ and various values of σ, estimated by means of the cross-entropy algorithm with $N = 1000$.

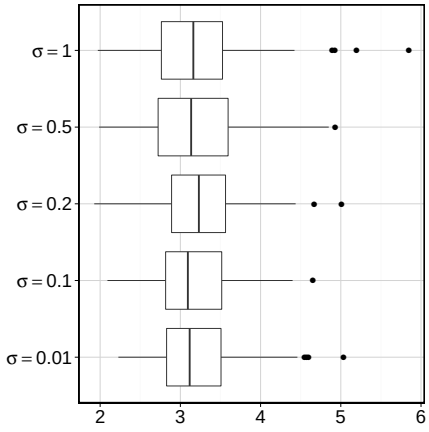

Figure SM.13: Boxplots of $\hat{\beta}_3$ for models with $G = 50$ and various values of σ, estimated by means of the cross-entropy algorithm with $N = 1000$.

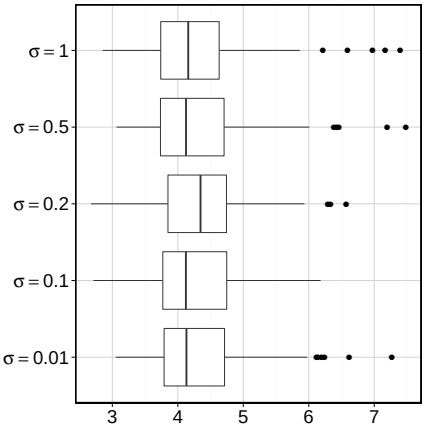

Figure SM.14: Boxplots of $\hat{\beta}_4$ for models with $G = 50$ and various values of σ, estimated by means of the cross-entropy algorithm with $N = 1000$.

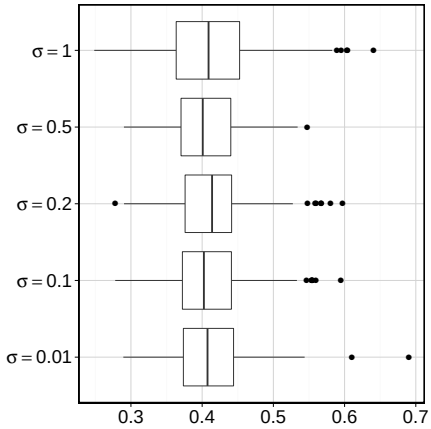

Figure SM.15: Boxplots of $\hat{\sigma}$ for models with $G = 50$ and various values of σ, estimated by means of the cross-entropy algorithm with $N = 1000$.