Complex-valued wavelet lifting and applications

A Supplementary Material: comparison with competitor methods

This document contains additional coherence and phase analysis comparisons to Fourier-based techniques, in order to provide the reader with a clear balance of the advantages and disadvantages of our proposed methodology. The investigated examples range from traditional bivariate stationary signals sampled over regular grids to a combination of process nonstationarity and sampling irregularity.

A.1 Coherence analysis

This section provides a simple visual comparison of the coherence estimates obtained for two *regularly sampled*, *stationary* bivariate vector autoregressive (VAR) processes of length 200.

The data were simulated using parameters $\Phi_1 = \begin{pmatrix} 0.4 & 0.1 \\ 0.2 & -0.4 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$ for Model (1), and $\Phi_1 = \begin{pmatrix} 0.9 & 0.2 \\ 0.5 & -0.9 \end{pmatrix}$, $\Phi_2 = \begin{pmatrix} -0.286 & 0.006 \\ -0.034 & 0.040 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$ for Model (2).

The coherence estimates computed through classical Fourier techniques (recommended for stationary data with regular sampling) and through our proposed $\mathbb{C}$NLT are presented in Figures A.1 and A.2 below.

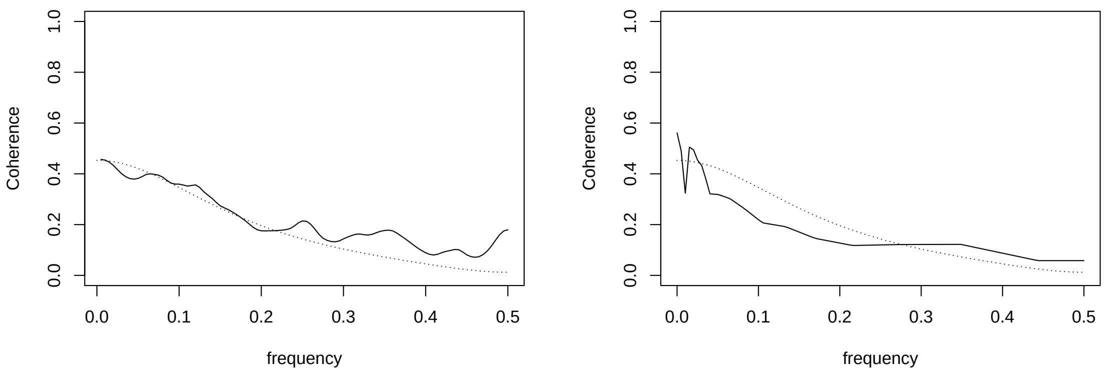

Figure A.1: Coherence between the bivariate VAR processes described by parameters of Model (1). Left: traditional Fourier-based estimate. Right: proposed lifting-based estimate. The dotted line shows true coherence.

Note that while the profiles of the two estimates (Fourier and lifting-based) are in overall agreement, it is clear that the flexibility of our method results in a small bias and variance price to pay by the lifting-based estimates. However, we reinforce the message that we propose our method not as a substitute of classical methods, but rather as a method designed to provide a solution when sampling irregularity and nonstationarity are present, thus it is pleasing to see its reasonable performance in a classical setting.

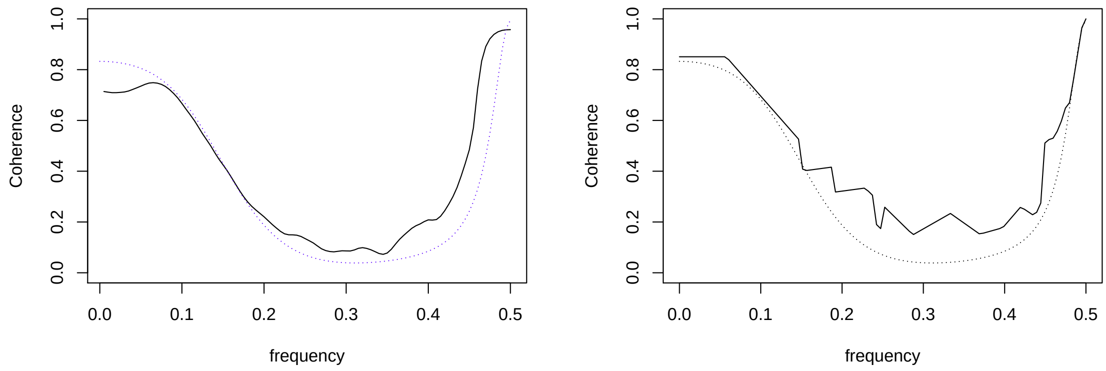

Figure A.2: Coherence between the bivariate VAR processes described by parameters of Model (2). Left: traditional Fourier-based estimate. Right: proposed lifting-based estimate. The dotted line shows true coherence.

A.2 Phase analysis

Next we present additional phase illustrations for traditional methods of analysis for the simulated data in Section 3.3.1. In particular, we describe analyses from two Fourier-based techniques: a modified short-time Fourier transform (STFT) and a Lomb-Scargle analysis (see Section 3.3.1 for more details). The analysis for the two methods was also performed in *R* with the *e1071* package (Meyer et al., 2015) and the *lomb* package (Ruf, 2013) respectively.

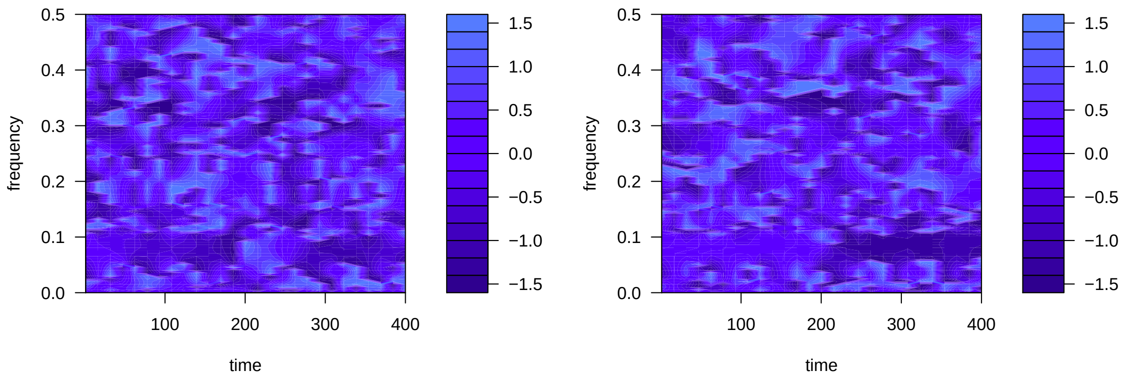

Figure A.3: Phase estimation using the STFT on an interpolated sampling grid: data observed on the same original irregular sampling grid (left); data observed on different original irregular sampling grids (right).

As traditional methods do not readily handle data that feature irregular sampling, STFT required further intervention– in order to obtain the desired phase analysis, we mapped the irregular data to a regular grid (by e.g. interpolation) and then used STFT in order to capture the nonstationary time-frequency content of the data. The resulting phase estimation plots of the STFT method for both same and different irregular sampling grids are shown in Figure A.3. The phase estimates show little resolution in time or frequency; additional blurring features in the plot for different irregular grids (Figure A.3 b)).

By contrast, the Lomb-Scargle method is able to deal naturally with the irregular sampling structure of the signals but fails to account for nonstationarity. Hence the results do not contain any time-phase

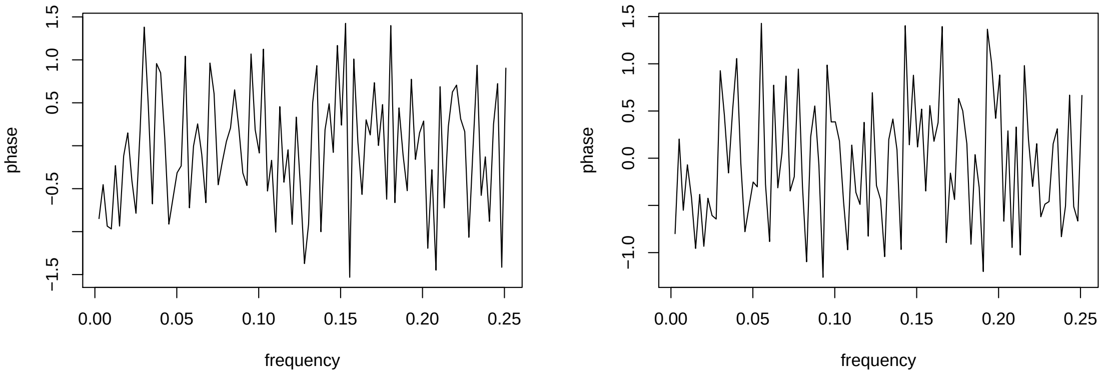

Figure A.4: Phase estimation using the Lomb-Scargle spectrum: data observed on the same irregular sampling grid (left); data observed on different irregular sampling grids (right).

information and do not show a strong differentiation in frequency at timepoints where the phase is large (Figure A.4). These features highlight the appeal of our complex-valued lifting technique for data that feature potential nonstationarities and/or irregular sampling.

References

Meyer, D., E. Dimitriadou, K. Hornik, A. Weingessel, F. Leisch, C.-C. Chang, and C.-C. Lin (2015). *e1071: Misc Functions of the Department of Statistics, Probability Theory Group (Formerly: E1071), TU Wien.*

Ruf, T. (2013). *lomb: Lomb-Scargle Periodogram.*