Supplementary Material

Algorithm 1 Compute Betti number values and locations from barcodes

Input *sp* (smoothing parameter vector) and *barcodes* (sets of barcodes)
kholes ← NULL (List of Betti number values and locations, for each smoothing parameter value)
for $i = 1 \rightarrow$ length(*sp*) **do**
  kbarcode ← *barcodes*[[*i*]]{ dim = k, birth, death } (Select k-dimensional barcode obtained from ith smoothing parameter value)
  $m \leftarrow$ column length of kbarcode (Number of k-dimensional bars)
  if $m = 0$ (No k-dimensional bars) **then**
    track ← { filtration=0, numk= 0}
  else
    filtration ← { birth values in *kbarcode*; death values in *kbarcode* }
    kBetti ← { $\mathbf{1_m}$;$\mathbf{-1_m}$ }
    track ← { filtration, kBetti } (Bind two vectors into $(2m \times 2)$ matrix, thereby assigning 1 and -1 to the birth and death points, respectively)
    Sort *track* matrix by 'filtration' in ascending order
    track$kBetti ← cumsum(*track*$kBetti) (Replace the kBetti column by its cumulative sum)
  end if
  kholes[[*i*]] ← *track*
end for
return *kholes*

Algorithm 2 Compute persistence terrace matrix from Betti number values/locations

Input sp (smoothing parameter vector) and $kholes$ = { filtration, kth Betti number }
$n \leftarrow$ length(sp)
$xvec \leftarrow$ sp (x values vector: smoothing parameters)
$yvec \leftarrow$ NULL (y values vector: filtration values when the kth Betti number change)
for $i = 1 \rightarrow n$ **do**
  $yvec \leftarrow$ { $yvec$, $kholes[[i]]$\$filtration } (Stack all filtration values)
end for
$yvec \leftarrow$ sort($yvec$) (Sort $yvec$ in descending order)
$zmat \leftarrow \mathbf{0}_{\mathbf{length(xvec)} \times \mathbf{length(yvec)}}$ (z values matrix: kth Betti number)
for $p = 1 \rightarrow n$ **do**
  $filtration \leftarrow kholes[[p]]\$filtration$
  $kBetti \leftarrow kholes[[p]]\$kBetti$
  $zvec \leftarrow \mathbf{0}_{\mathbf{length(yvec)}}$
  for $q = 1 \rightarrow$ length($kBetti$) **do**
    $zvec = zvec + (filtration[q+1] < yvec) * (yvec \leq filtration[q]) * kBetti[q]$ (Fill out kth Betti numbers for all filtration values)
  end for
  $zmat[, p] \leftarrow zvec$ (Save $zvec$ to pth column of $zmat$)
end for
return $[xvec, yvec, zmat]$

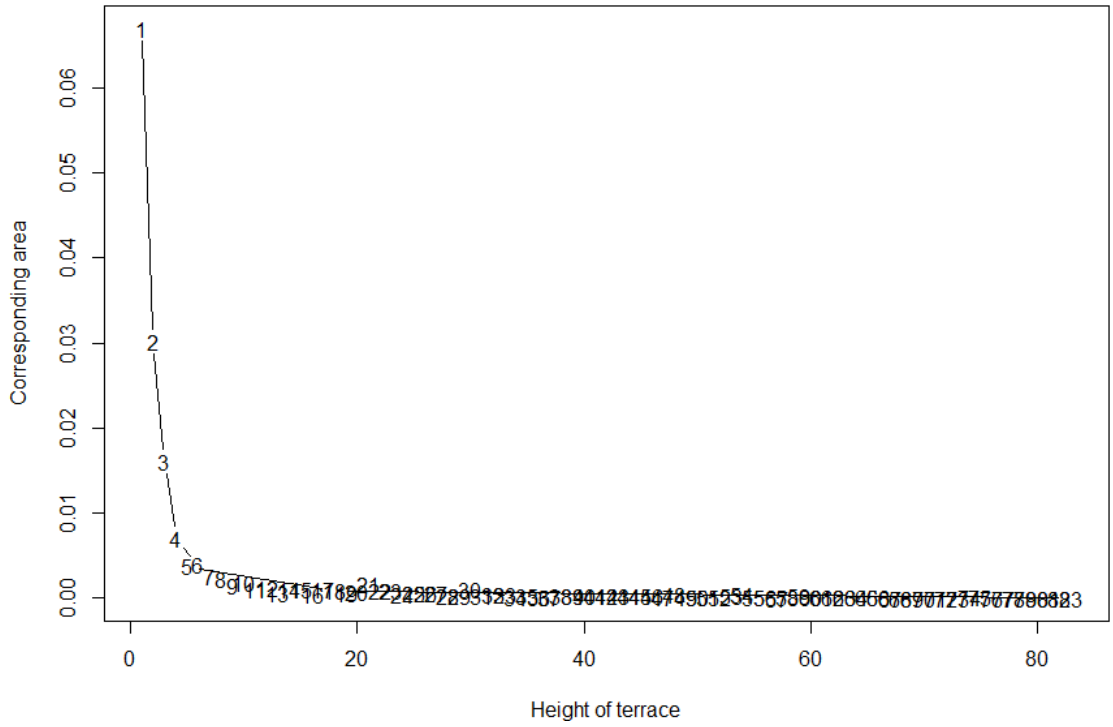


Figure 1: Terrace area plot for the simulated data set with four noisy features.

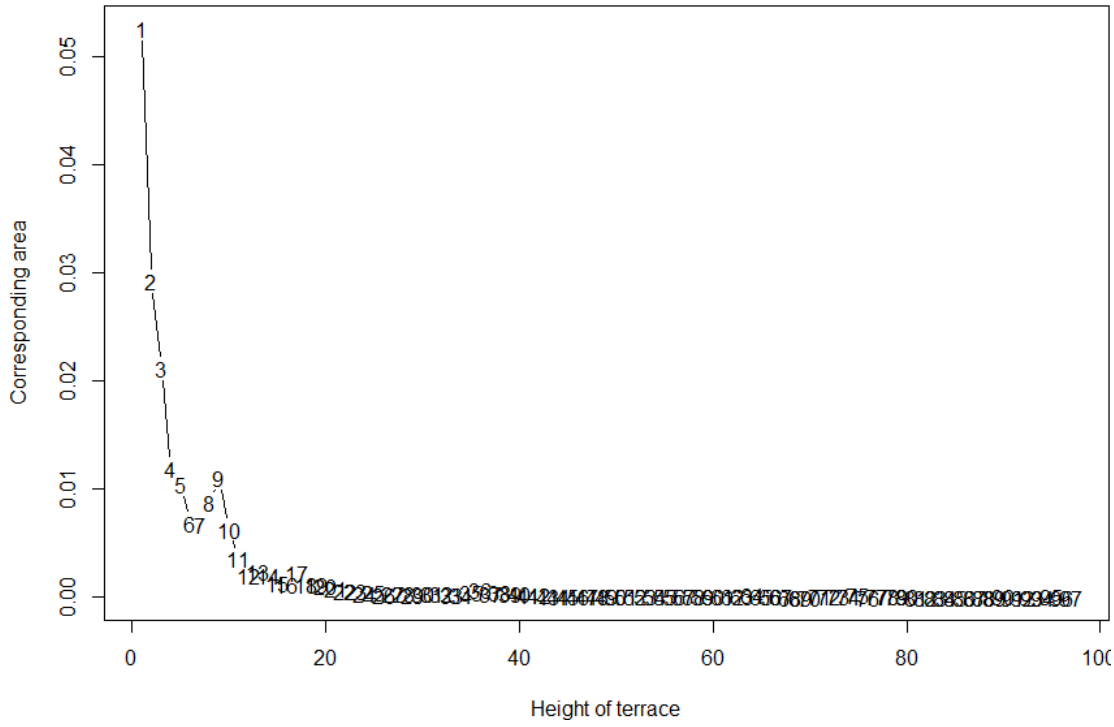
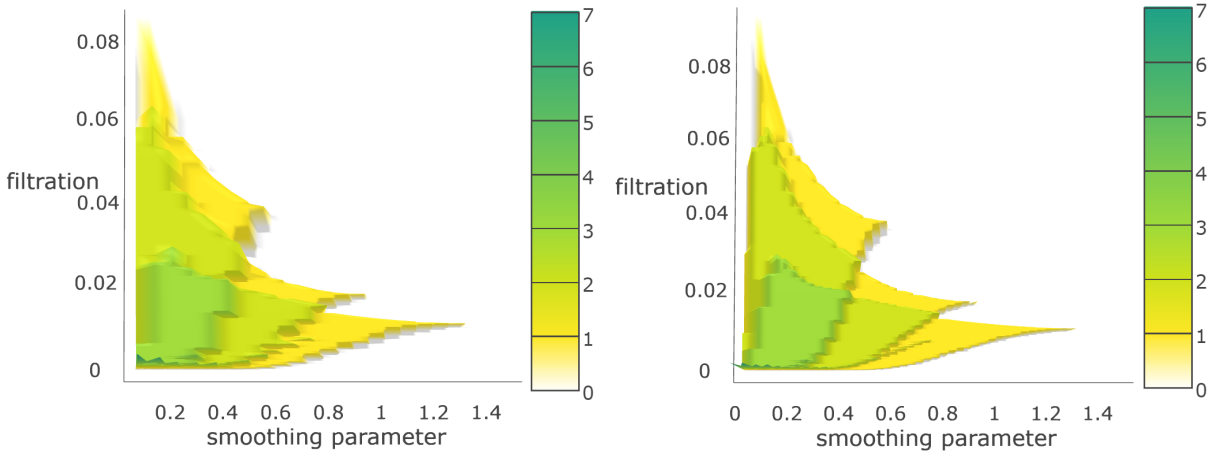
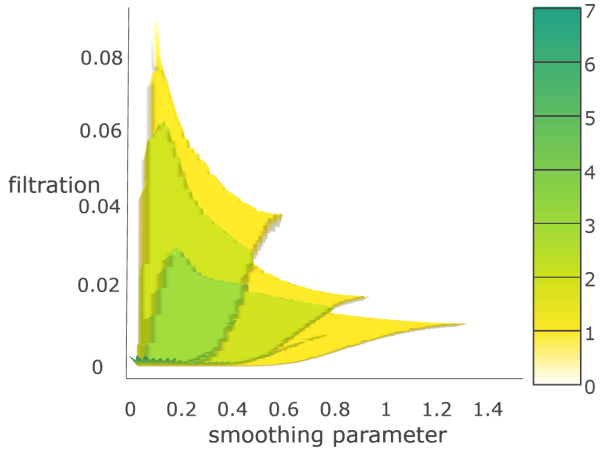


Figure 2: Terrace area plot for the muscle fiber data

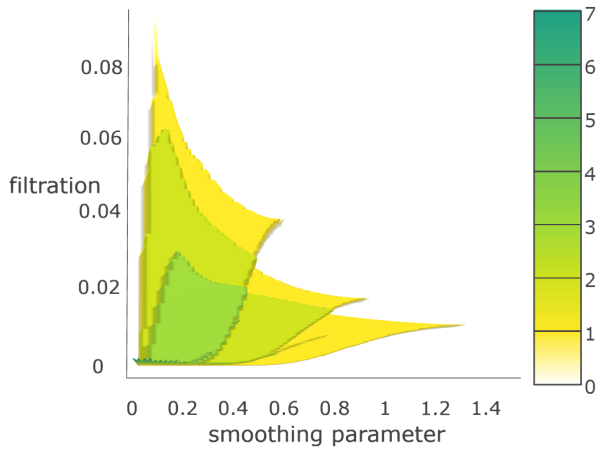


(a) 25 smoothing parameters

(b) 50 smoothing parameters



(c) 75 smoothing parameters



(d) 100 smoothing parameters

Figure 3: Resolution of the persistence terrace according to the number of smoothing parameters.

With an increase in the number of smoothing parameters, the resolution increases, although the general picture stays the same.