**SUPPLEMENTARY MATERIAL**

# A Appendix

This appendix verifies equation (16): for any past and future screening number $K_1 \geq 1$ and $K \geq 1$,

$$\begin{aligned}
&\sum_{i=1}^{4} P(\text{Case } i, H_{K_1}|T = t_{K_1+K}) = P(H_{K_1}|T \geq t_{K_1}) \\
&= 1 - \int_0^{t_{K_1}} w(x)dx + \int_{t_{K_1-1}}^{t_{K_1}} w(x)Q(t_{K_1} - x)dx \\
&\quad + \sum_{j=0}^{K_1-1} (1-\beta_j)\cdots(1-\beta_{K_1-1}) \int_{t_{j-1}}^{t_j} w(x)Q(t_{K_1} - x)dx.
\end{aligned}$$

First, we combine the probabilities of case 3 and 4, since they are similar. Denote by $I$ their sum:

$$\begin{aligned}
I &= P(\text{Case } 3, H_{K_1}|T = t_{K_1+K}) + P(\text{Case } 4, H_{K_1}|T = t_{K_1+K}) \\
&= \sum_{j=K_1}^{K_1+K-1} \beta_j \left\{ \sum_{i=0}^{j-1} (1-\beta_i)\cdots(1-\beta_{j-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_j - x)dx \right. \\
&\quad \left. + \int_{t_{j-1}}^{t_j} w(x)Q(t_j - x)dx \right\}.
\end{aligned}$$

We split $P\{\text{Case } 2, H_{K_1}|T = t_{K_1+K}\}$ into 4 items, denoted II, III, IV, V, as follows:

$$\begin{aligned}
&P(\text{Case } 2, H_{K_1}|T = t_{K_1+K}) \qquad (1) \\
&= \sum_{j=K_1+1}^{K_1+K} \left\{ \sum_{i=0}^{j-1} (1-\beta_i)\cdots(1-\beta_{j-1}) \int_{t_{i-1}}^{t_i} w(x)[Q(t_{j-1} - x) - Q(t_j - x)]dx \right. \\
&\quad \left. + \int_{t_{j-1}}^{t_j} w(x)[1 - Q(t_j - x)]dx \right\} \\
&= \sum_{j=K_1+1}^{K_1+K} \sum_{i=0}^{j-1} (1-\beta_i)\cdots(1-\beta_{j-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_{j-1} - x)dx \\
&\quad - \sum_{j=K_1+1}^{K_1+K} \sum_{i=0}^{j-1} (1-\beta_i)\cdots(1-\beta_{j-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_j - x)dx \\
&\quad + \sum_{j=K_1+1}^{K_1+K} \int_{t_{j-1}}^{t_j} w(x)dx - \sum_{j=K_1+1}^{K_1+K} \int_{t_{j-1}}^{t_j} w(x)Q(t_j - x)dx = II - III + IV - V.
\end{aligned}$$

We combines items I and II, and cancel redundant items, first by changing the index $l = j - 1$ in item II, then changing $l$ back to $j$:

$$
\begin{aligned}
II = & \sum_{l=K_1}^{K_1+K-1} \sum_{i=0}^{l} (1-\beta_i)\cdots(1-\beta_l) \int_{t_{i-1}}^{t_i} w(x)Q(t_l - x)dx \qquad (2) \\
= & \sum_{j=K_1}^{K_1+K-1} \sum_{i=0}^{j} (1-\beta_i)\cdots(1-\beta_j) \int_{t_{i-1}}^{t_i} w(x)Q(t_j - x)dx \\
= & \sum_{j=K_1}^{K_1+K-1} (1-\beta_j) \left\{ \sum_{i=0}^{j-1} (1-\beta_i)\cdots(1-\beta_{j-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_j - x)dx \right. \\
& \left. + \int_{t_{j-1}}^{t_j} w(x)Q(t_j - x)dx \right\}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
I + II \quad = & \sum_{j=K_1}^{K_1+K-1} \sum_{i=0}^{j-1} (1-\beta_i)\cdots(1-\beta_{j-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_j - x)dx \\
& + \sum_{j=K_1}^{K_1+K-1} \int_{t_{j-1}}^{t_j} w(x)Q(t_j - x)dx.
\end{aligned}
$$

Notice that the first term in $I + II$ is similar to that of III, the second term in $I + II$ is similar to that of V, and $IV = \int_{t_{K_1}}^{t_{K_1+K}} w(x)dx$. This leads to

$$
\begin{aligned}
& \sum_{i=2}^{4} P(\text{Case } i, H_{K_1} | T = t_{K_1+K}) = I + II - III + IV - V \\
= & \sum_{i=0}^{K_1-1} (1-\beta_i)\cdots(1-\beta_{K_1-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_{K_1} - x)dx \\
& - \sum_{i=0}^{K_1+K-1} (1-\beta_i)\cdots(1-\beta_{K_1+K-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_{K_1+K} - x)dx \\
& + \int_{t_{K_1-1}}^{t_{K_1}} w(x)Q(t_{K_1} - x)dx - \int_{t_{K_1+K-1}}^{t_{K_1+K}} w(x)Q(t_{K_1+K} - x)dx + \int_{t_{K_1}}^{t_{K_1+K}} w(x)dx.
\end{aligned}
$$

Compare with:

$$
\begin{aligned}
& P(\text{Case } 1, H_{K_1} | T = t_{K_1+K}) \\
= & \ 1 - \int_0^{t_{K_1+K}} w(x)dx + \int_{t_{K_1+K-1}}^{t_{K_1+K}} w(x)Q(t_{K_1+K} - x)dx \\
& + \sum_{j=0}^{K_1+K-1} (1-\beta_j)\cdots(1-\beta_{K_1+K-1}) \int_{t_{j-1}}^{t_j} w(x)Q(t_{K_1+K} - x)dx.
\end{aligned}
$$

Many terms will cancel, leaving

$$
\begin{aligned}
& \sum_{i=1}^{4} P(\text{Case } i, H_{K_1} | T = t_{K_1+K}) \\
= \ & 1 - \int_0^{t_{K_1}} w(x)dx + \sum_{i=0}^{K_1-1} (1-\beta_i)\cdots(1-\beta_{K_1-1}) \int_{t_{i-1}}^{t_i} w(x)Q(t_{K_1}-x)dx \\
& + \int_{t_{K_1-1}}^{t_{K_1}} w(x)Q(t_{K_1}-x)dx \\
= \ & P(H_{K_1} | T \geq t_{K_1}).
\end{aligned}
$$

This completes the proof.