

## PRIORS FOR THE LONG RUN: ONLINE APPENDIX

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This document contains some additional results not included in the main body of the paper. In particular, we present (i) the MSFEs of a naive model in which all variables follow an independent random walk with drift, showing that they are very similar to those of the DIFF-VAR; (ii) the evolution over time of the hyperparameters selected in the recursive estimation of the PLR-BVAR; (iii) the MSFEs obtained with various five-variable VARs; (iv) a comparison with the dummy-initial-observation (or single-unit-root) prior of [Sims and Zha \(1998\)](#). This supplement is not entirely self-contained, so readers might need to refer to the main paper.

### 1. A COMPARISON BETWEEN THE DIFF-VAR AND A NAIVE MODEL

In the main body of the paper, we have mentioned that the forecasting performance of a VAR specified in first differences is similar to that of a naive model in which all variables follow separate random walks with drifts. The former corresponds to imposing an infinitely tight PLR or sum-of-coefficients prior, while the latter coincides with an infinitely tight Minnesota prior. Figures [1.1](#) and [1.2](#) substantiate the previous claim, by showing that the MSFEs at various forecasting horizons of the DIFF-VAR are very similar to those produced by the naive model. In the 7-variable case, we present the MSFEs for only a representative subset of the variables and their linear combinations, to save space.

### 2. HYPERPARAMETER VALUES

In our recursive estimation and forecasting exercise, we choose the value of the hyperparameters for the PLR-BVAR by maximizing their posterior, according to a hierarchical interpretation of the model. Figure [2.1](#) and [2.2](#) report the selected hyperparameter values for the models with three and seven variables, as a function of the end point of the estimation sample.

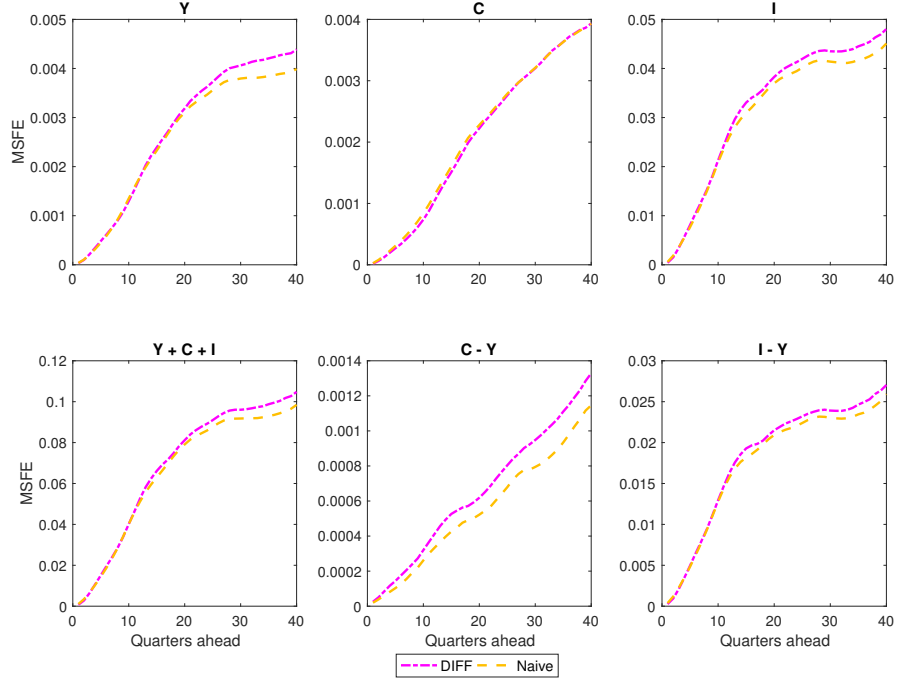


FIGURE 1.1. Mean squared forecast errors in models with three variables. DIFF: VAR with variables in first differences; Naive: random walk with drift for each variable.

### 3. 5-VARIABLE VAR

In the main text, we have stated that the PLR-BVAR dominates the alternatives also in the case of models with five variables. These models augment the small-scale ones (log-GDP, log-consumption and log-investment) with two labor-market variables, i.e. log-total compensation and log-hours worked. The PLR for this 5-variable model is set up using the  $5 \times 5$  upper-left block of the matrix  $H$  in section 5. Figure 3.1 plots the MSFEs at various forecasting horizons for the level of all the variables included in the VAR and for the linear combinations of the variables, obtained by multiplying the matrix  $H$  by the vector  $y$  (i.e. the common real trend, the great ratios and hours). Notice that the prediction accuracy of the SZ-BVAR deteriorates for GDP, consumption and investment, relative to the 3-variable case. The PLR-BVAR, instead, continues to forecast well, outperforming the MN-BVAR, SZ-BVAR and the DIFF-VAR model uniformly over variables and horizons. The only exceptions are consumption and the labor share, for which the forecasting accuracy of the PLR-BVAR is comparable to the DIFF-VAR and the MN-BVAR, respectively.

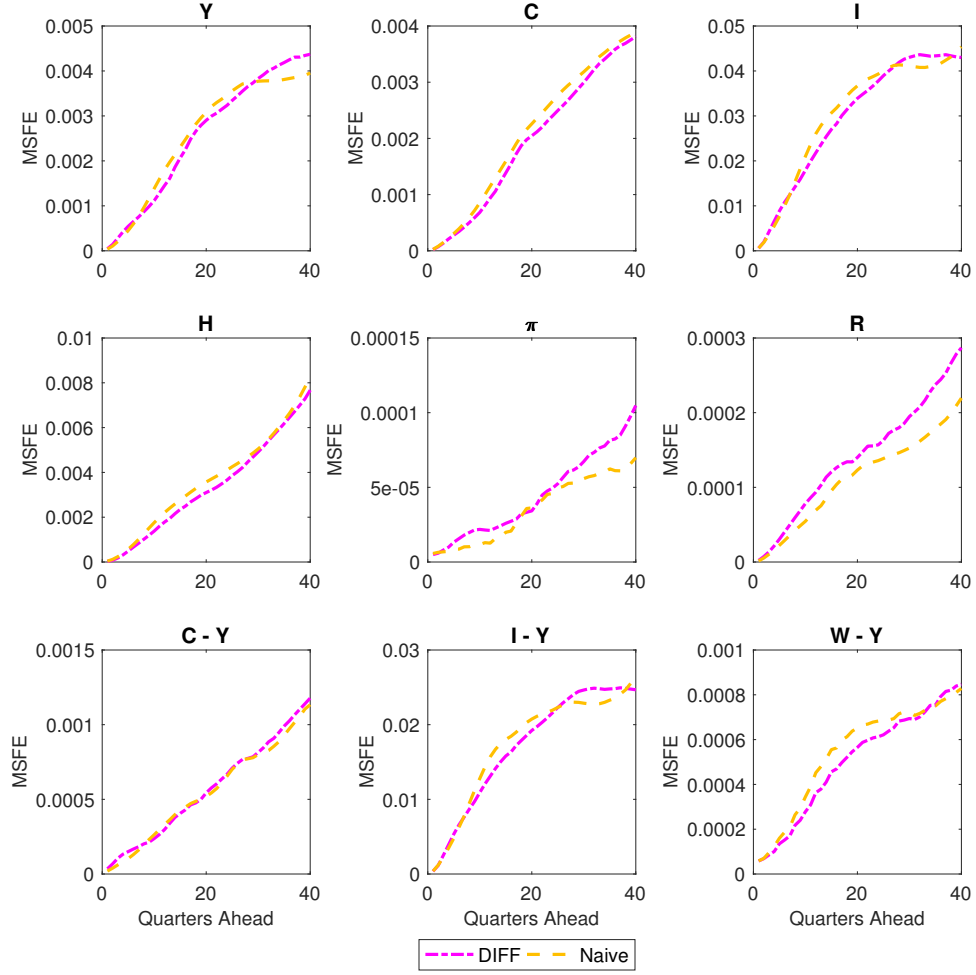


FIGURE 1.2. Mean squared forecast errors in models with seven variables. DIFF: VAR with variables in first differences; Naive: random walk with drift for each variable.

#### 4. A COMPARISON WITH THE DUMMY-INITIAL-OBSERVATION PRIOR

In this section, we evaluate the accuracy of the forecasts obtained when we include a dummy-initial-observation prior in the 3- and 7-variable VARs. This prior was designed to avoid the bias against cointegration of the sum-of-coefficients prior, while still reducing the explanatory power of the deterministic component of the model (see [Sims and Zha, 1998](#) for the details of its implementation). In the existing literature, it is often combined with the Minnesota and sum-of-coefficients priors (see, for example, [Sims and Zha, 1998](#) or [Giannone et al., 2015](#)).

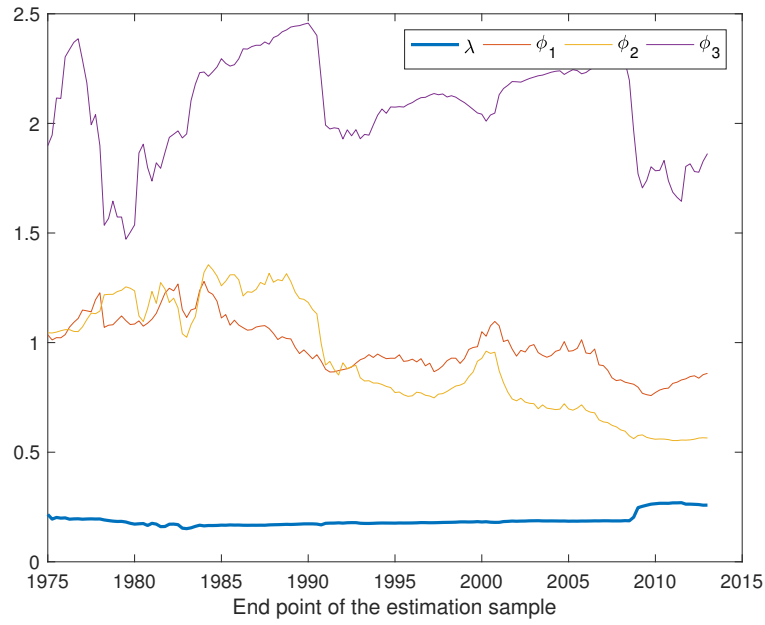


FIGURE 2.1. Posterior mode of the hyperparameters in the recursive estimation of the 3-variable PLR-BVAR.  $\lambda$  is the hyperparameter of the Minnesota prior; the  $\phi$ 's are the hyperparameters of the PLR.

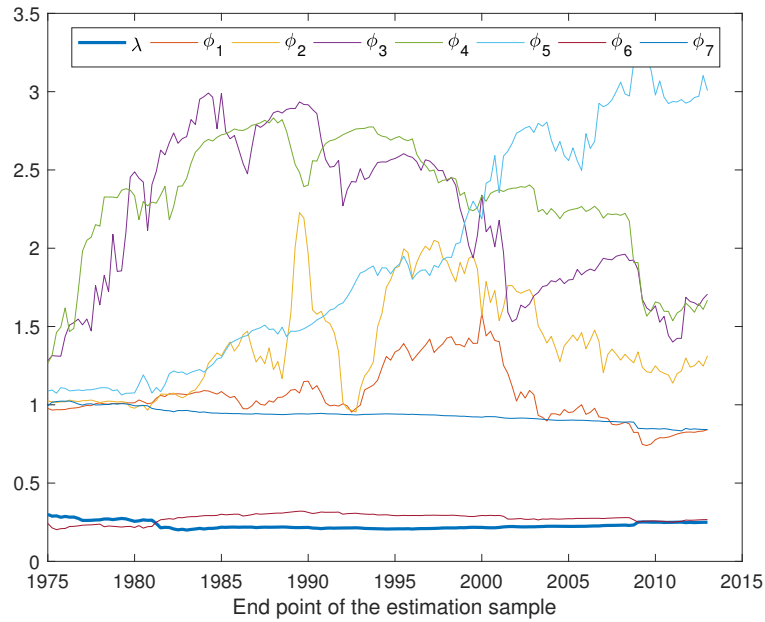


FIGURE 2.2. Posterior mode of the hyperparameters in the recursive estimation of the 7-variable PLR-BVAR.  $\lambda$  is the hyperparameter of the Minnesota prior; the  $\phi$ 's are the hyperparameters of the PLR.

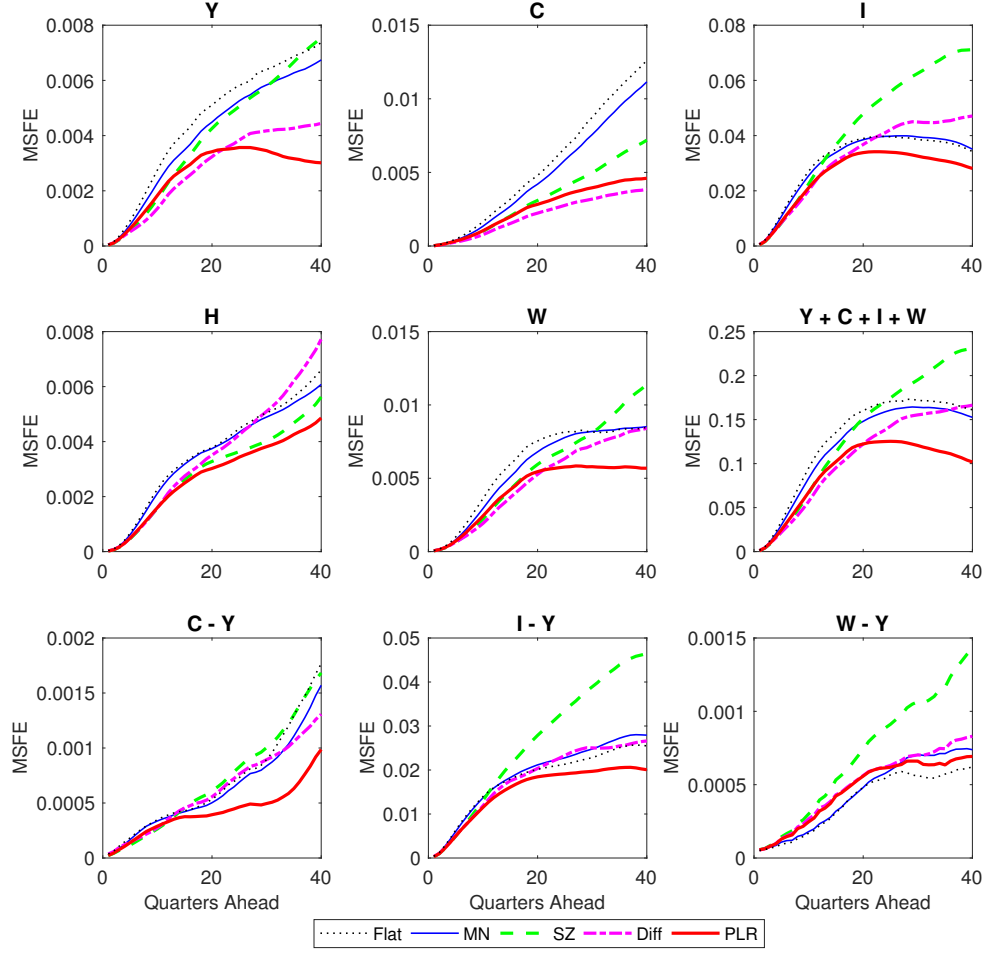


FIGURE 3.1. Mean squared forecast errors in models with five variables. Flat: BVAR with a flat prior; MN: BVAR with the Minnesota prior; SZ: BVAR with the Minnesota and sum-of-coefficients priors; DIFF: VAR with variables in first differences; PLR: BVAR with the Minnesota prior and the prior for the long run.

Figure 4.1 and 4.2 compare the forecasting performance (in terms of MSFEs) of the 3- and 7-variable MN- and SZ-BVARs *without* (as in the main text of the paper) and *with* the dummy-initial-observation prior. As usual, all hyperparameters are selected by maximizing their posterior. These figures make clear that the marginal contribution of the dummy-initial-observation prior is negligible, and the forecasting results of the MN+DIO- and SZ+DIO-BVARs are nearly identical to those of the MN- and SZ-BVARs reported in the main text of the paper.

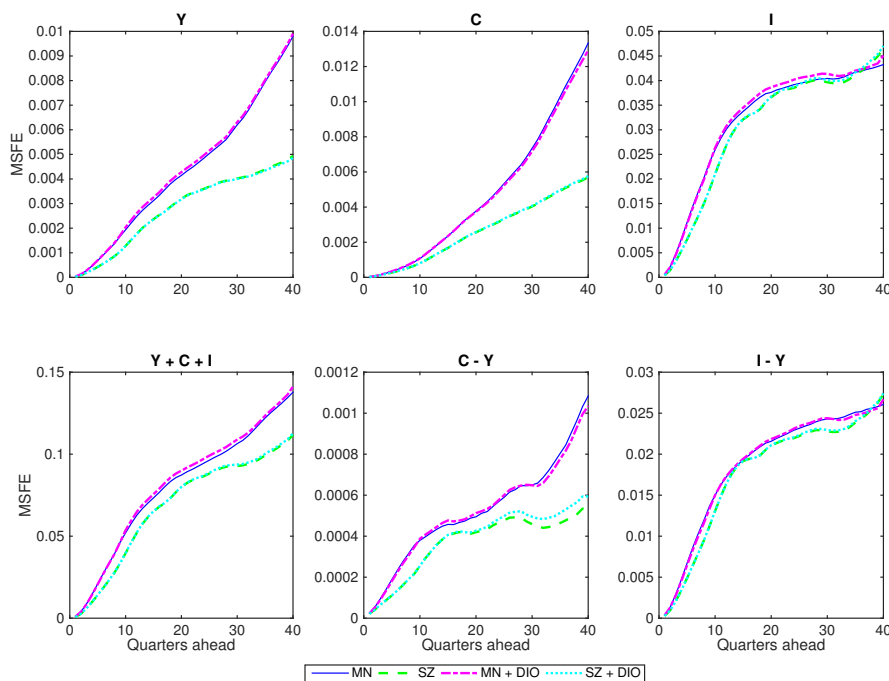


FIGURE 4.1. Mean squared forecast errors in models with three variables. MN: BVAR with the Minnesota prior; SZ: BVAR with the Minnesota and sum-of-coefficients priors; MN+DIO: BVAR with Minnesota and dummy-initial-observation priors; SZ+DIO: BVAR with the Minnesota, sum-of-coefficients and dummy-initial-observation priors.

#### REFERENCES

- GIANNONE, D., M. LENZA, AND G. E. PRIMICERI (2015): “Prior Selection for Vector Autoregressions,” *The Review of Economics and Statistics*, 97, 436–451.
- SIMS, C. A. AND T. ZHA (1998): “Bayesian Methods for Dynamic Multivariate Models,” *International Economic Review*, 39, 949–68.

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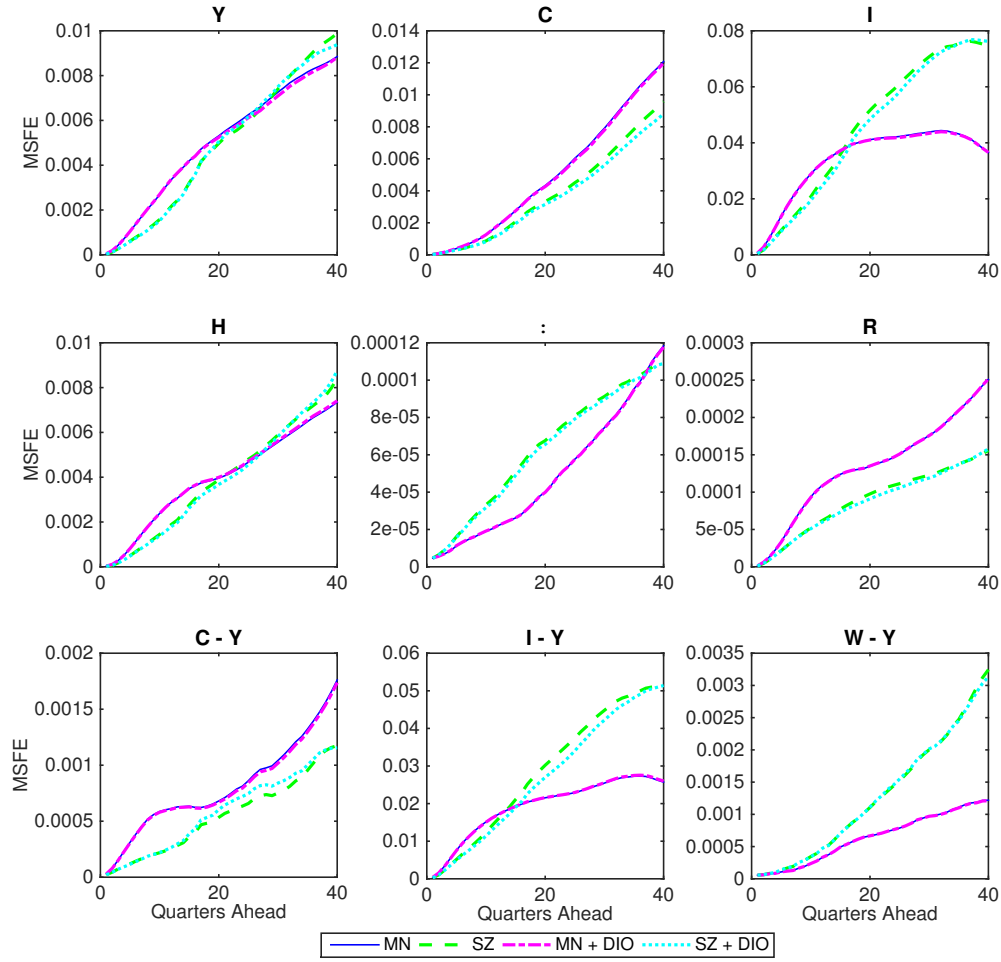


FIGURE 4.2. Mean squared forecast errors in models with seven variables. MN: BVAR with the Minnesota prior; SZ: BVAR with the Minnesota and sum-of-coefficients priors; MN+DIO: BVAR with Minnesota and dummy-initial-observation priors; SZ+DIO: BVAR with the Minnesota, sum-of-coefficients and dummy-initial-observation priors. To save space, the figure presents the MSFEs for only a subset of the variables and linear combinations.