Continuous metabolic energy model

The model by Umberger et al. [1,2] calculates the energy as a function of the heat rate from the activation of muscles and its maintenance, $\dot{h}_{AM}$, the heat rate due to shortening and lengthing of muscles, $\dot{h}_{SL}$, and the mechanical work rate, w_{CE}:

$$\dot{E}(t) = \dot{h}_{AM} + \dot{h}_{SL} + w_{CE} \tag{1}$$

The following equation is used to find the activation/maintenance heat rate $\dot{h}_{AM}$:

$$\dot{h}_{AM} = (0.4 + 0.6 fl_{CE})\dot{\bar{h}}_{AM} A_{AM} S \tag{2}$$

Where S is a factor equal to 1.5 for aerobic conditions [3], $A_{AM} = A^{0.6}$ is related to the activation and stimulation as described in equation 6 of the paper (see below), and fl_{CE} and $\dot{\bar{h}}_{AM}$ are determined as follows:

$$fl_{CE} = \begin{cases} 1 & l_{CE} \leq l_{CE(OPT)} \\ f(l_{CE}) & l_{CE} > l_{CE(OPT)} \end{cases} \tag{3}$$

$$\dot{\bar{h}}_{AM} = \begin{cases} 25 & a \leq ST \\ 128FT + 25 & a > ST \end{cases} \tag{4}$$

Equation 6 in the paper:

$$A = u + \frac{1}{2}\left(\frac{a-u}{2} + \sqrt{\left(\frac{a-u}{2}\right)^2 + \varepsilon^2}\right)$$

where 128 W/kg and 25 W/kg are constants found using regression, $f(l_{CE})$ is the location on the force-length relationship of the muscle, and FT and ST are the ratios of fast-twitch and slow-twitch fibers in the muscle, respectively, $l_{CE(OPT)}$ is the optimal fiber length, and l_{CE} is the current fiber length.

When the fiber length is longer than optimal, $\dot{h}_{AM}$ is split up into two parts, where 40% represents the activation heat rate and 60% the activation heat rate, which is dependent on the location on the force-length relationship [4]. This does not create a discontinuity in the equation, since the derivative of $f(l_{CE})$ is zero when the fiber length is optimal. Also, equation 4 is continuous since $\dot{\bar{h}}_{AM}$ is a constant.

The shortening-lengthening heat rate is calculated as follows:

$$\dot{h}_{SL} = A_{SL}\dot{\bar{h}}_{SL} fl_{CE} S \tag{5}$$

Where A_{SL} is equal to A^2, and $\dot{\bar{h}}_{SL}$ is determined as follows:

$$\dot{\bar{h}}_{SL} = \alpha_L \bar{v}_{CE(l)} - \alpha_{FT}\bar{v}_{CE(S)}FT + \begin{cases} 100 & \alpha_{ST}\bar{v}_{CE(max)_{ST}} < -\alpha_{ST}\bar{v}_{CE(S)}ST \\ -\alpha_{ST}\bar{v}_{CE(S)}ST & \alpha_{ST}\bar{v}_{CE(max)_{ST}} > -\alpha_{ST}\bar{v}_{CE(S)}ST \end{cases} \tag{6}$$

Where $\bar{v}_{CE(l)}$ and $\bar{v}_{CE(s)}$ are the shortening and lengthening velocities normalized to optimal fiber length, respectively. $\bar{v}_{CE(max)_{ST}}$ is the normalized maximum shortening velocity for slow-twitch fibers, 4.8 fiber lengths per second. α_{ST}, α_{FT} and $\alpha_L$are the shortening heat coefficients for slow-twitch and fast-twitch fibers in J/kg, and the lengthening heat coefficient, respectively:

$$\alpha_{ST} = \frac{100}{\bar{v}_{CE(max)_{ST}}}, \quad \alpha_{FT} = \frac{153}{\bar{v}_{CE(max)_{FT}}}, \quad \alpha_L = 0.3\alpha_{ST}$$

Where $\bar{v}_{CE(max)_{FT}}$ is the maximum shortening velocity for fast-twitch fibers, assumed to be 12 fiber lengths per second.

The shortening and lengthening velocities are determined as described in the paper:

$$\bar{v}_{CE(l)} = \frac{1}{2}\left(\bar{v}_{CE} + \sqrt{\bar{v}_{CE}^2 + \varepsilon^2}\right) \tag{7}$$

$$\bar{v}_{CE(s)} = \frac{1}{2}\left(\bar{v}_{CE} - \sqrt{\bar{v}_{CE}^2 + \varepsilon^2}\right) \tag{8}$$

Note that the term $\alpha_{ST}\bar{v}_{CE(S)}ST$ for $\dot{\bar{h}}_{SL}$ cannot exceed 100 W/kg. However, this level is not reached during gait.

The work rate is determined as follows to ensure that it is never negative:

$$w_{CE} = \frac{1}{2}\left(\widetilde{w}_{CE} + \sqrt{\widetilde{w}_{CE}^2 + \varepsilon^2}\right) \tag{9}$$

where

$$\widetilde{w}_{CE} = -\frac{F_{CE}v_{CE}}{m_{mus}} \tag{10}$$

Where m_{mus} is the muscle mass, F_{CE} is the force in the contractile element, and v_{CE} is the fiber velocity, negative when shortening. ε is a small number, used to decrease the nonlinearity of the problem. For simplicity, the same value for ε is used for the shortening/lengthening velocity and the work rate.

The muscle mass is determined as follows:

$$m_{mus} = \frac{F_{max}\rho}{\sigma} l_{CE(OPT)} \tag{11}$$

Where F_{max} is the maximum isometric force, σ is the maximum muscle stress, 250 kPa, ρ is the muscle density, 1059.7 kg/m^3, and $l_{CE(OPT)}$ is the optimal fiber length.

Predictive gait simulation

The following objective was used with direct collocation using N collocation nodes:

$$J(x,u) = \frac{1}{vM}\sum_{j=1}^{16}\frac{1}{N}\sum_{i=1}^{N}\dot{E}(x(i),u(i),\varepsilon)m_{mus}(j) + \frac{W_{Reg}N}{N_{st}+N_{con}}\sum_{i=1}^{N}\left(\sum_{s=1}^{N_{st}}\big(x_s(i+1)-x_s(i)\big)^2 + \sum_{c=1}^{N_{con}}\big(u_c(i+1)-u_c(i)\big)^2\right) \quad 12$$

Where W_{Reg} is the weight of the regularization term, N_{st} is the number of states, and N_{con} is the number of controls. Note that an extra node $(N+1)$ was added for the periodicity constraint.

Predictive Gait Simulations

Figure 1 shows the five results with the highest objective that were found when minimizing metabolic rate. Figure 2 shows the five results with the highest objective that were found when minimizing effort.

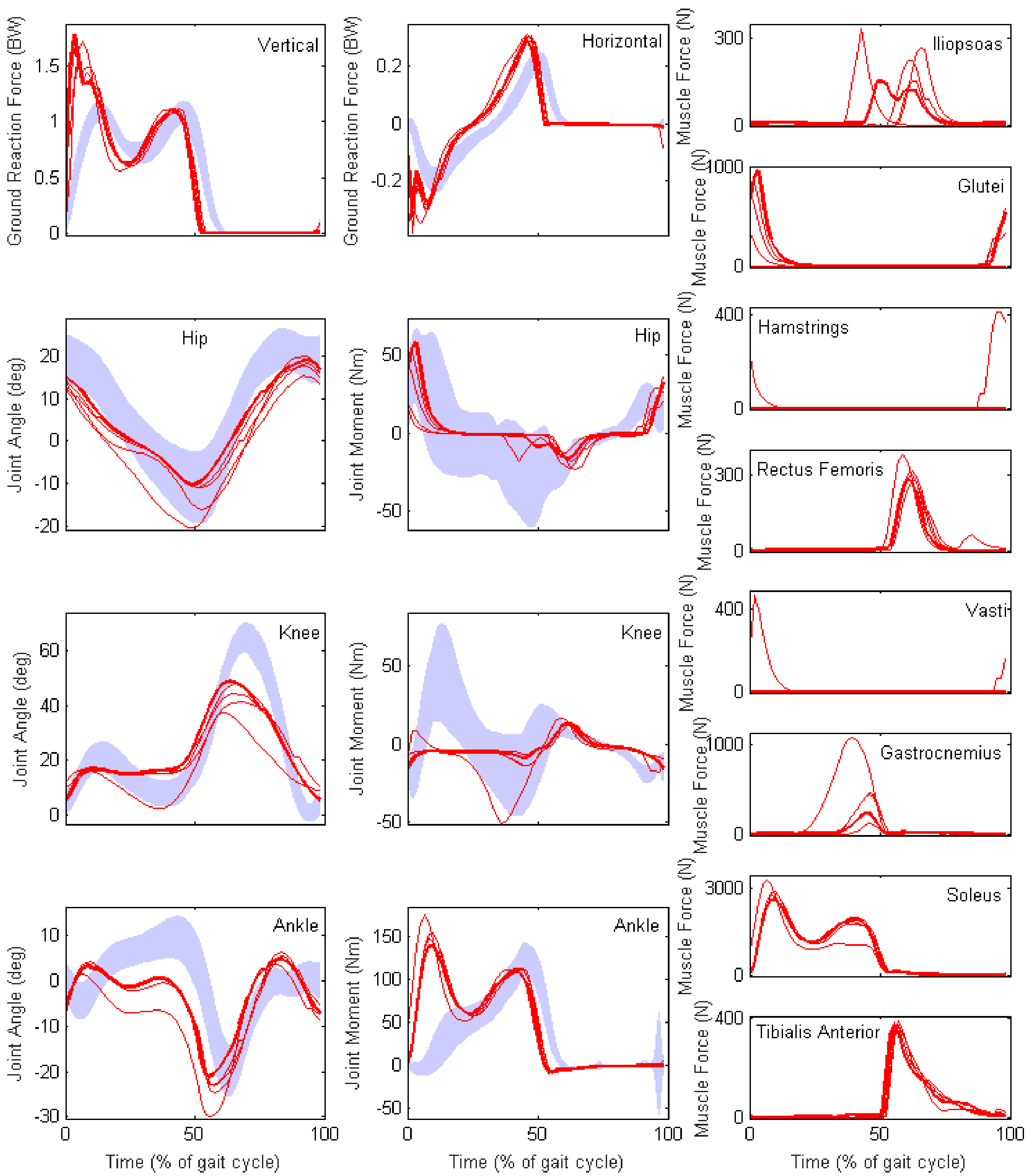

Figure 1 - Ground reaction force, joint angles, moments, and muscle forces of the five solutions with the lowest metabolic rate. The fill shows normal data from Winter [5]

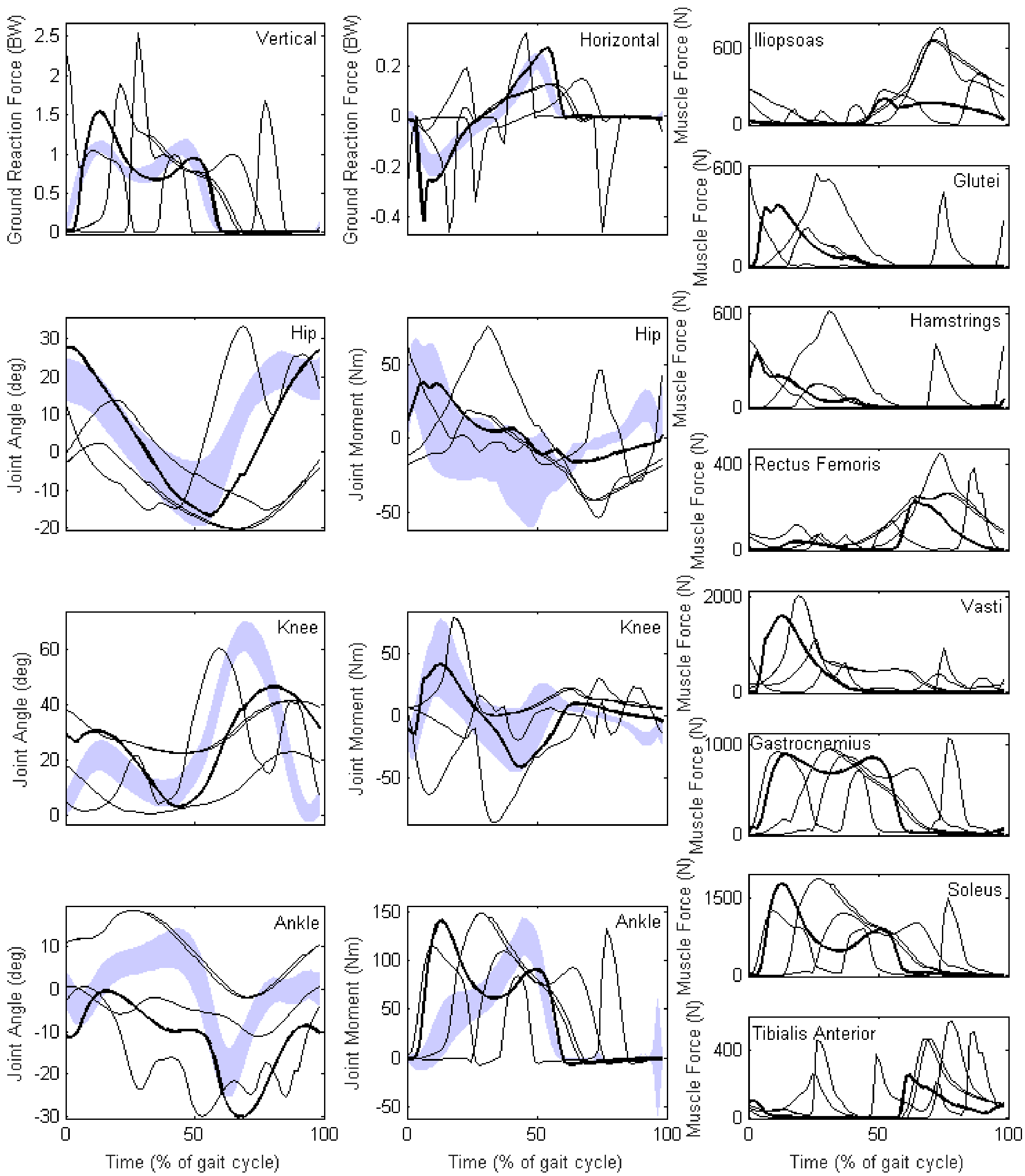

Figure 2 - Ground reaction force, joint angles, moments, and muscle forces of the five solutions with the lowest effort. The fill shows normal data from Winter [5]

References

[1] Brian R Umberger, Karin GM Gerritsen, and Philip E Martin. A model of human muscle energy expenditure. *Computer methods in biomechanics and biomedical engineering*, 6(2): 99-111, 2003.

[2] Brian R Umberger. Stance and swing phase costs in human walking. *Journal of the Royal Society Interface*, 7(5): 1329-1340, 2010.

[3] Jose Gonzalez-Alonso, Bjørn Quistorff, Peter Krustrup, Jens Bangsbo, and Bengt Saltin. Heat production in human skeletal muscle at the onset of intense dynamic exercise. *The Journal of physiology*, 524(2): 603-615, 2000.

[4] Earl Homsher and Charles J Kean. Skeletal muscle energetics and metabolism. *Annual Review of Physiology*, 40(1): 93-131, 1978.

[5] David A Winter. *Biomechanics and motor control of human gait: normal, elderly and pathological*. University of Waterloo Press, 1991.