Construction of Two-Level Nonregular Designs of Strength Three With Large Run Sizes

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This document includes the following sections:

A   Proofs

Proof of Lemma 1. The proof is straightforward and therefore omitted.

Proof of Theorem 1. Let $W_1, \ldots, W_q$ be the words of D_0, where $q = 2^p - 1$. Without loss of generality, assume that the first g words contain only generated factors and the other

$q-g$ words contain both basic and generated factors. For $i = g+1, \ldots, q$, let $W_i = \{H_i, G_i\}$ where $\{H_i\}$ and $\{G_i\}$ denote a set of basic and generated factors, respectively. Note that all sets $\{G_i\}$ are different. Otherwise, there are two distinct words $W_i = \{H_i, G\}$ and $W_j = \{H_j, G\}$ in the defining relation of D_0. This implies that the defining relation of D_0 has a word involving only basic factors, which is a contradiction.

For any $u \in Z_b$, let $H_i + u$ be the set formed by applying the permutation l_u to every element of H_i. The q words of D_u then are $W_1^u, \ldots, W_q^u$, where $W_i^u = W_i$ for $i = 1, \ldots, g$ and $W_i^u = \{H_i + u, G_i\}$ for $i = g+1, \ldots, q$. We will show that the words W_i^u are different for $u = 0, \ldots, b-1$, and $i = g+1, \ldots, q$. First, suppose that $W_i^u = W_j^v$ for some $0 \le u < v < b$ and $g + 1 \le i \le j \le q$. Since all sets $\{G_i\}$ are distinct, we must have $i = j$ and thus $W_i^u = W_i^v$. This implies that $H_i + u = H_i + v$ or that

$$H_i = H_i + w \pmod{b}, \tag{S.1}$$

where $w = v - u$. Let $r_i = |H_i|$, equation (S.1) implies that

$$H_i = \{x_0, x_0 + w, x_0 + 2w, \ldots, x_0 + (r_i - 1)w\}$$

for some x_0, and that

$$r_i w = 0 \pmod{b}. \tag{S.2}$$

Since Condition 1 implies that $1 \le r_i < b$, where b is a prime, equation (S.2) implies that $w = 0 \pmod{b}$. Therefore, since $0 \le w < b$, we have that $w = 0$ and so $u = v$, which is a contradiction.

Therefore, the complete words containing basic and generated factors in D_0, D_1, $\ldots$, D_{d-1} are different from each other. By Lemma 1, these complete words define $d(q - g)$ different partial words with an aliasing index of $1/d$ in the concatenated design. The g complete words containing only the generated factors in D_0 remain complete in the concatenated design because each such word remains the same after permutations on the set of basic factors. This completes the proof.

Proof of Theorem 2. We need to show that the $2^p - h - 1$ words containing the generated factors and the set of permuted basic factors in the defining relation of D_0 change after linear permutations on the set B. Since the number of permuted basic factors is a prime, the proof is similar to Theorem 1 and therefore omitted.

Proof of Lemma 2. Let $D_1, \ldots, D_d$ be d parent designs of strength t and with n runs. Let $T = (D_1^T, \ldots, D_d^T)^T$ and $\mathrm{b} = (11_n^T, \ldots, d1_n^T)^T$, where 1_n denotes a column of ones. Let $C_\mathrm{b} = (T, \mathrm{b})$ be the blocked concatenated design. Design C_b is a mixed-level orthogonal array of strength t since, for all sets of t columns, the level-combinations appear equally often. So, the $d-1$ block contrast vectors are orthogonal to the j-factor interaction contrast vectors for $j = 1, \ldots, t-1$. For any set of $t+1$ columns that include column b, all level-combinations in this set appear equally often in C_b. Thus, the block contrast vectors are also orthogonal to all t-factor interaction contrast vectors. As a result, the block effects are not confounded with the main effects and all the j-factor interactions for $j = 2, \ldots, t$.

B   Issues with permuting other sets of factors

Permuting sets of factors other than the basic factors in the regular design does not permit an easy characterization of the aliasing structure of the concatenated design. This is due to possibly repeat complete words involving basic and generated factors, in two or more isomorphic copies of the regular design. We illustrate this issue using a 64-run design constructed by concatenating isomorphic copies of a 2^{12-7} design involving linear permutations on the generated factors. To save space, we only consider words of length 4 in the regular and concatenated designs.

Example S.1. Let D_0 be the minimum aberration 2^{12-7} design with basic factors 1, 2, 3, 4 and 5, and generators $6 = 123$, $7 = 124$, $8 = 134$, $9 = 234$, $t_0 = 125$, $t_1 = 135$ and $t_2 = 145$. The partial defining relation of D_0 is

$$
\begin{aligned}
I &= 256t_1 = 257t_2 = 34t_1t_2 = 356t_0 = 358t_2 = 457t_0 = 458t_1 \\
&= 4789 = 49t_0t_1 = \cdots .
\end{aligned}
$$

Design D_0 has $GR(D_0) = 4$ and $B_4(D_0) = 38$. Let D_1 be an isomorphic copy of D_0 formed by applying the linear permutation l_1 to the set of generated factors; that is, $6 \to 7$, $7 \to 8$, $8 \to 9$, $9 \to t_0$, $t_0 \to t_1$, $t_1 \to t_2$ and $t_2 \to 6$. The partial defining relation of D_1 is

$$
\begin{aligned}
I &= 257t_2 = 2568 = 346t_2 = 357t_1 = 3569 = 458t_1 = 459t_2 \\
&= 489t_0 = 4t_0t_1t_2 = \cdots .
\end{aligned}
$$

The defining relations of D_0 and D_1 have two words in common, namely $257t_2$ and $458t_1$, which contain both basic and generated factors. This is because $257t_2$ is obtained from $256t_1$ by applying l_1 to the generated factors. Similarly, $458t_1$ is obtained from $457t_0$. Let C be the 64-run design constructed by concatenating D_0 and D_1. By Lemma 1, C has the factors sets $\{2, 5, 7, t_2\}$ and $\{4, 5, 8, t_1\}$ as complete words of length 4. It is easy to verify that the other 4-factor sets either form partial words with an aliasing index $\rho_4(S; C) = 1/2$ or do not form a word.

Example S.1 shows that complete words containing basic and generated factors may be repeated in the parent designs. A problem with these repeat words is that their presence depends on whether words in the defining relation of the regular design can be obtained from others via linear permutations, which hinders the characterization of the aliasing structure of the concatenated design. Permuting only the basic factors circumvents this issue because the words in the defining relation contain different sets of generated factors, which do not change; see Theorem 1 and its proof in Section A. Permuting other sets of factors such as all factors leads to a similar problem as permuting the generated factors.

C   Issues with 2^{m-p} designs when $m-p$ is not a prime

If the 2^{m-p} design has a number of basic factors $m-p$ that is not a prime, it is not possible to classify the words in the concatenated design into absent, complete, or partial with an aliasing index of $1/d$, where d is the number of concatenated copies. Depending on the properties of the 2^{m-p} design, other partial words with an aliasing index between $1/d$ and 1 may be present in the concatenated design. To illustrate this issue, we first introduce a general class of permutations called *cycles*, to which the linear permutations belong. Next, we use a simple but representative example where we concatenate three isomorphic copies of a regular design with 4 basic factors.

Let S_b denote the group of all permutations of the set $Z_b = \{0, \ldots, b-1\}$. A permutation $\sigma \in S_b$ is a *q-cycle* if there are q elements $a_1, \ldots, a_q \in Z_b$ such that

1. $\sigma(a_i) = a_{i+1}$ for $1 \le i < q - 1$;

2. $\sigma(a_q) = a_1$;

3. $\sigma(j) = j$ for all $j \notin \{a_1, a_2, \ldots, a_q\}$.

We use the notation $(a_1, \ldots, a_q)$ for a q-cycle σ. Two cycles $(a_1, \ldots, a_q)$ and $(b_1, \ldots, b_p)$ are said to be disjoint if the sets $\{a_1, \ldots, a_q\}$ and $\{b_1, \ldots, b_p\}$ do not have any element in common. Consider two permutations σ and τ, the composition of two permutations, denoted as $\sigma\tau$, is the operation such that $(\sigma\tau)(i) = \sigma(\tau(i))$. If b is a prime, the linear permutation $l_u \in S_b$ is a b-cycle over Z_b, also referred to as a *cyclic* permutation, for $u \neq 0 \pmod{b}$. If b is not a prime, l_u can be expressed as the composition of two or more disjoint cycles, except when u and b are relatively prime.

Example S.2. Suppose that we have a regular design with four basic factors, labeled as 0, 1, 2 and 3. Let $02G_1$ and $13G_2$ be two words in its defining relation, with $\{G_i\}$ a set of generated factors. Consider a design C constructed by concatenating three isomorphic copies of this regular design, each one formed by applying l_u, $u = 0, 1, 2$, to the set of basic factors. Note that $l_1 = (0, 1, 2, 3)$ whereas $l_2 = (0, 2)(1, 3)$, the composition of two disjoint 2-cycles. It is easy to see that the defining relations of the isomorphic copies formed from l_0 and l_2 share the words $02G_1$ and $13G_2$. On the contrary, the defining relation of the isomorphic copy formed from l_1 does not contain these words. By Lemma 1, the factor sets $\{0, 2, G_1\}$ and $\{1, 3, G_2\}$ form partial words with an aliasing index of $2/3$ in C.

Example S.2 shows that a concatenated design constructed from a 2^{m-p} design with $m - p$ not a prime may have partially aliased words with an aliasing index between $1/d$ and 1. A problem with these words is that their presence depends on whether the l_u used can be expressed as the composition of disjoint cycles, and on whether the 2^{m-p} design has complete words that include the cycles. For this reason, it is difficult to characterize the aliasing structure of the concatenated design constructed from a 2^{m-p} design with $m - p$ not a prime. Theorems 1 and 2 circumvent this issue by permuting a prime number of basic factors and restricting to regular designs that satisfy Condition 1; see Section A.

D   Regular designs used

Table S.1 shows the regular designs of strength 3 used to construct our concatenated designs. The table includes the labels, the run sizes and the partial wordlength patterns

of the designs. All the designs in the table have minimum aberration and were obtained from the FrF2 package (Grömping, 2014) in R. The package contains a collection of good regular designs from Chen et al. (1993), Block and Mee (2005), Xu (2009) and Ryan and Bulutoglu (2010). The designs in Table S.1 are even-odd, except for the 32-run designs with 11 factors or more and the 64-run designs with 21 factors or more, which are even designs.

E   Contributions of theorems and algorithm

In this section, we show the contributions of our theoretical results and the VNS algorithm for constructing nonregular strength-3 designs. To this end, we use the three design cases shown in Table S.2. The table shows the run sizes and numbers of factors of the concatenated designs, the regular designs used (see Section D) and the numbers of parent designs. The regular designs have a number of basic factors that is a prime. Case 1 requires the construction of a 96-run design from three isomorphic copies of the minimum aberration (MA) 2^{11-6} design with 5 basic factors. Case 2 requires the construction of a 384-run design from three isomorphic copies of the MA 2^{21-14} design with 7 basic factors. The last case in the table requires the construction of a 896-run design from seven isomorphic copies of the MA 2^{28-21} design with 7 basic factors. The construction settings for these designs are described in Section 4.1 in the main text.

Tables S.3, S.4 and S.5 show the construction of the concatenated designs from Cases 1, 2 and 3, respectively. The tables show the generalized resolution (GR), the F_4 vector and the B_4 value of the regular and concatenated designs. A dash as an element of the F_4 vector means that the corresponding J_4-characteristic does not exist.

Table S.3 shows the steps of our construction method to generate the 11-factor 96-run concatenated design from Case 1. The MA 2^{11-6} design has 25 complete words of length 4, three of which only include the generated factors. Theorem 1 generates a 9-factor 96-run concatenated design with 3 complete words of length 4 and 66 partial words of the same length with a J_4-characteristic of 32. This concatenated design has a B_4 value of 10.33. The improved concatenated design, resulting from our VNS algorithm, eliminates all the complete words of length 4. For this reason, it has a GR value of 4.66. The improved

Table S.1: Regular minimum aberration designs of strength 3.

Runs	Design	B_4	B_5	B_6	Runs	Design	B_4	B_5	B_6
32	2^{9-4}	6	8	0	128	2^{28-21}	210	840	2800
	2^{10-5}	10	16	0		2^{29-22}	266	945	3472
	2^{11-6}	25	0	27		2^{30-23}	335	972	4662
	2^{12-7}	38	0	52		2^{31-24}	391	1134	5826
	2^{13-8}	55	0	96		2^{32-25}	452	1322	7219
	2^{14-9}	77	0	168		2^{33-26}	518	1543	8863
	2^{15-10}	105	0	280		2^{34-27}	589	1800	10788
	2^{16-11}	140	0	448		2^{35-28}	665	2100	13020
64	2^{17-11}	59	108	150		2^{36-29}	756	2401	15736
	2^{18-12}	78	144	228		2^{37-30}	854	2744	18886
	2^{19-13}	100	192	336		2^{38-31}	959	3136	22512
	2^{20-14}	125	256	480		2^{39-32}	1071	3584	26656
	2^{21-15}	204	0	1680		2^{40-33}	1190	4096	31360
	2^{22-16}	250	0	2304	256	2^{24-16}	26	216	584
	2^{23-17}	304	0	3105		2^{25-17}	34	262	760
	2^{24-18}	365	0	4138		2^{26-18}	43	325	963
	2^{25-19}	435	0	5440		2^{27-19}	53	395	1224
	2^{26-20}	515	0	7062		2^{28-20}	64	476	1550
128	2^{20-13}	36	152	340		2^{29-21}	78	579	1908
	2^{21-14}	51	200	414		2^{30-22}	93	672	2400
	2^{22-15}	65	248	572		2^{31-23}	113	792	2928
	2^{23-16}	83	316	744		2^{32-24}	133	932	3576
	2^{24-17}	102	384	992		2^{33-25}	153	1095	4360
	2^{25-18}	124	482	1312		2^{34-26}	176	1280	5272
	2^{26-19}	152	568	1704		2^{35-27}	200	1488	6360
	2^{27-20}	180	690	2200		2^{36-28}	225	1728	7632

Table S.2: Design cases used to evaluate the components of our construction method.

Case	Factors	Runs	Regular design	Copies
1	11	96	2^{11-6}	3
2	21	384	2^{21-14}	3
3	28	896	2^{28-21}	7

concatenated design has 69 partial words of length 4 (three more than the concatenated design resulting from Theorem 1) with a J_4-characteristic of 32, and a B_4 value of 7.67. The improved concatenated design with 96 runs and 11 factors is listed in Table 1 in the main text.

Table S.3: Contributions of Theorem 1 and the VNS algorithm for constructing an 11-factor 96-run design.

Step	Runs	GR	$F_4(96)$	$F_4(32)$	B_4
Regular design	32	4	–	25	25
Theorem 1	96	4	3	66	10.33
VNS Algorithm		4.66	0	69	7.67

Table S.4 shows the steps of our construction method to generate the 21-factor 384-run concatenated design from Case 2. The MA 2^{21-14} design has 52 complete words of length 4, 12 of which include only the generated factors. Using Theorem 1, we generated a 21-factor 384-run concatenated design with a GR value of 4 and a B_4 value of 25. More specifically, this design has 12 complete words of length 4 and 117 partial words of length 4 with a J_4-characteristic of 128. The VNS algorithm resulted in an improved concatenated design that turned the 12 complete words into 12 partial words of length 4 with a J_4-characteristic of 128. The improved concatenated design has a GR value of 4.66, a B_4 value of 14.33 and it is listed in Table 3 in the main text.

Table S.5 shows the steps of our construction method to generate the concatenated design from Case 3. The MA 2^{28-21} design has a B_4 value of 210. Theorem 1 generated a 28-factor 896-run concatenated design with 65 complete words of length 4. The B_4 value of this design is 85.71. The VNS algorithm eliminated all the complete words and

Table S.4: Contributions of Theorem 1 and the VNS algorithm for constructing a 21-factor 384-run design.

Step	Runs	GR	$F_4(384)$	$F_4(128)$	B_4
Regular design	128	4	—	52	52
Theorem 1	384	4	12	117	25
VNS Algorithm		4.66	0	129	14.33

generated an improved concatenated design with a GR value of 4.857. The B_4 value of the improved concatenated design is almost four times smaller than the concatenated design from Theorem 1. The 896-run improved concatenated design is shown in Table 1 in the main text.

This small study showed that the theoretical results turn most of the complete words in the regular design into partial words in the concatenated design. The VNS algorithm further reduces the number of complete words and results in concatenated designs with a better F_4 vector, a better B_4 value and also a better GR than the starting designs.

Table S.5: Contributions of Theorem 1 and the VNS algorithm for constructing a 28-factor 896-run design.

Step	Runs	GR	$F_4(896)$	$F_4(128)$	B_4
Regular design	128	4	—	210	210
Theorem 1	896	4	65	1015	85.71
VNS Algorithm		4.857	0	1080	22.04

F   Tables of two-level strength-3 designs

We show the strength-3 nonregular designs obtained from 32-, 64-, 128- and 256-run regular designs in Tables S.6, S.7, S.8 and S.9, respectively. The tables report the regular design used and the number of parent designs (d) as well as the run size, the generalized resolution (GR), the F_4 vector, the B_4 value and the degrees of freedom (df) for estimating two-factor interactions (2FIs) of the concatenated designs. Tables S.7 and S.9 also include the best

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basic factor to keep fixed (f). In the tables, we use a boldface GR value and a boldface F_4 vector to indicate that these values are larger and sequentially smaller, respectively, than the corresponding values of all the benchmark designs available in the literature. The degrees of freedom for estimating 2FIs are calculated as the rank of the matrix consisting of the 2FI contrast vectors (Cheng et al., 2008). A referee pointed out that some concatenated designs in the tables have repeat runs. Repeat runs are attractive for physical experiments as they provide a pure error estimate of the error variance. For this reason, we also report the designs with repeat runs in Tables S.6-S.9.

Table S.6 shows the strength-3 designs constructed from 32-run minimum aberration (MA) designs. The 128-run designs in the table outperform the corresponding regular MA designs of Xu (2009) in terms of the G-aberration criterion, except for 9-11 factors. For these numbers of factors, 128-run regular designs with a strength strictly larger than 3 are available (Xu, 2009). For 15 and 16 factors, the 128-run designs in Table S.6 have a larger GR value than the 160-run designs. However, the 160-run designs provide a smaller B_4 value and also a larger number of degrees of freedom for estimating 2FIs than the smaller alternatives. The designs with 11 factors or more in Table S.6 are even because they are constructed from even parent designs; see Section D. This implies that they cannot estimate more than 15d 2FIs. For 9 and 10 factors, the designs in the table can estimate all the 2FIs. For 11 factors, the 128- and 160-run designs also permit the estimation of the 2FIs. For 12 factors, only the 160-run design shares this property. Note that, for 9-15 factors, strength-4 128-run designs are available in Hedayat et al. (1999) and Schoen et al. (2010), or from the quaternary linear codes (QLCs) of Xu and Wong (2007). Strength-4 designs with 9 factors and 160 runs are available in Bulutoglu and Ryan (2018). Our 16-factor 128-run design outperforms all the benchmark designs available in the literature in terms of the G-aberration criterion.

Table S.7 shows the strength-3 designs constructed from 64-run MA designs. For 18 factors or more, our 256-run designs have less G-aberration than the corresponding regular MA designs in Xu (2009). For 17-21 factors, our 256-run designs also have less G-aberration than the 256-run QLC designs of Xu and Wong (2007). For 20 and 21 factors, our 256-run designs have a larger GR value but a smaller B_4 value and also a smaller number of degrees

of freedom for estimating 2FIs, than the 320-run designs in Table S.7. The designs with 21 factors or more in the table are even because their parent designs are even too; see Section D. So, they cannot estimate more than $31d$ 2FIs. The 320-run designs with 17 and 19 factors in Table S.7 are the only ones that permit the estimation of all the 2FIs. For 17-19 factors, strength-4 256-run designs are available in Hedayat et al. (1999).

Table S.8 shows strength-3 designs constructed from 128-run MA designs. Our 512-run designs outperform the best 512-run regular designs of Xu (2009) in terms of the G-aberration criterion, except for 20-23 factors. For these numbers of factors, 512-run regular designs with a GR value of 5 exist (Xu, 2009). For 27-29 factors, the 640-run designs in Table S.8 have a smaller GR value than designs with 512 runs. Similarly, for 19-26 and 36 factors, the 768-run designs have smaller GR values than the designs with 640 or 512 runs. For 30 factors or more, the 896-run designs also provide smaller GR values than smaller alternatives with 768 runs, except for 36 and 37 factors. However, Table S.8 shows that, for each number of factors, the B_4 value decreases with the run size of the design. For 20-22 factors, the designs in the table permit the estimation of the 2FIs. For 23-25 factors, the designs with more than 384 runs share this property as well as the 26-factor 640-run design. For 26-29 factors, the 768- and 896-run designs in the table can also estimate all the 2FIs. All the designs in Table S.8 are even-odd designs.

Finally, Table S.9 shows the strength-3 nonregular designs constructed from 256-run MA designs. For 34, 35 and 36 factors, our 1024-run designs provide a larger GR value than the best 1024-run regular designs available in Xu (2009). All 1280-run designs in Table S.9 have a GR value of 4.8. For 24-29 factors, the designs in Table S.9 can estimate all the 2FIs. For 30 factors or more, only the designs with 1024 and 1280 runs share this property. Note that, for 24-33 factors, regular 1024-run designs of strength 4 are available in Xu (2009). For 24-36 factors, the 768-run designs in Table S.8 outperform the designs in Table S.9 in terms of the G-aberration criterion. However, the 768-run designs constructed from 256-run MA designs provide less G_2-aberration than the 768-run designs constructed from 128-run MA designs. All designs in Table S.9 are even-odd designs.

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Table S.6: Strength-3 designs obtained from 32-run MA designs. A boldface F_4 vector indicates that it is sequentially smaller than the F_4 vectors of the benchmarks available in the literature. d: number of parent designs; df: degrees of freedom for estimating 2FIs; Runs$_{i,j,k}$: design has i duplicate, j triplicate and k quadruplicate runs.

Parent	Runs	GR	$F_4(96, 64, 32)$	B_4	df	d
2^{9-4}	$96_{4,0,0}$	4.66	$(0, 0, 18)$	2	36	3
	$128_{6,2,0}$	4.75	$(0, 0, 24)$	1.5	36	4
	$160_{12,0,2}$	4.8	$(0, 0, 30)$	1.2	36	5
2^{10-5}	$96_{2,0,0}$	4.66	$(0, 0, 30)$	3.33	45	3
	$128_{0,2,0}$	4.75	$(0, 0, 40)$	2.5	45	4
	$160_{0,0,2}$	4.8	$(0, 0, 50)$	2	45	5
2^{11-6}	96	4.66	$(0, 0, 69)$	7.67	45	3
	128	4.75	$(0, 0, 88)$	5.5	55	4
	$160_{4,0,0}$	4.8	$(0, 0, 113)$	4.52	55	5
2^{12-7}	96	4.66	$(0, 0, 108)$	12	45	3
	128	4.75	$(0, 0, 140)$	8.75	60	4
	$160_{2,0,0}$	4.8	$(0, 0, 178)$	7.12	66	5
2^{13-8}	96	4.66	$(0, 0, 155)$	17.22	45	3
	128	4.75	$(0, 0, 200)$	12.5	60	4
	$160_{2,0,0}$	4.8	$(0, 0, 255)$	10.2	74	5
2^{14-9}	96	4.66	$(0, 0, 213)$	23.67	45	3
	128	4.75	$(0, 0, 272)$	17	60	4
	$160_{2,0,0}$	4.8	$(0, 0, 349)$	13.96	74	5
2^{15-10}	96	4	$(1, 0, 284)$	32.56	45	3
	128	4.5	$(0, 6, 360)$	24	60	4
	160	4.4	$(1, 0, 464)$	18.92	75	5
2^{16-11}	96	4	$(3, 0, 367)$	43.78	45	3
	128	4.5	$\mathbf{(0, 13, 460)}$	32	60	4
	160	4.4	$(3, 0, 597)$	24.96	75	5

Table S.7: Strength-3 designs obtained from 64-run MA designs. A dash as an element of the F_4 vector means that the corresponding J_4-characteristic does not exist. A boldface F_4 vector indicates that it is sequentially smaller than the F_4 vectors of the benchmarks available in the literature. d: number of parent designs; f: best factor to fix; df: degrees of freedom for estimating 2FIs; Runs$_i$: design has i duplicate runs.

Parent	Runs	GR	$F_4(256, 192, 128, 64)$	B_4	df	d	f
2^{17-11}	192	4.66	$(-, 0, 0, 153)$	17	119	3	6
	256	4.75	$(0, 0, 0, 188)$	11.75	135	4	6
	320_4	4.8	$(0, 0, 0, 247)$	9.88	136	5	6
2^{18-12}	192	4.66	$(-, 0, 0, 198)$	22	126	3	6
	256	4.75	$(0, 0, 0, 240)$	15	141	4	6
	320_4	4.8	$(0, 0, 0, 318)$	12.72	149	5	6
2^{19-13}	192	4	$(-, 1, 0, 243)$	28	129	3	6
	256	4.5	$(0, 0, 4, 312)$	20.50	156	4	3
	320	4.8	$(0, 0, 0, 412)$	16.48	171	5	3
2^{20-14}	192	4	$(-, 2, 0, 291)$	34.33	129	3	6
	256	4.5	$\mathbf{(0, 0, 14, 368)}$	26.50	170	4	1
	320	4.4	$(0, 2, 0, 459)$	19.08	187	5	6
2^{21-15}	192	4	$(-, 7, 0, 483)$	60.67	93	3	1
	256	4.5	$\mathbf{(0, 0, 31, 572)}$	43.50	124	4	1
	320	4.4	$(0, 7, 0, 769)$	33.28	155	5	1
2^{22-16}	192	4	$(-, 11, 0, 579)$	75.33	93	3	6
	256	4	$(1, 0, 42, 680)$	54.00	124	4	6
	320	4.4	$(0, 13, 0, 917)$	41.36	155	5	1
2^{23-17}	192	4	$(-, 15, 0, 685)$	91.11	93	3	1
	256	4	$(2, 0, 55, 792)$	65.25	124	4	1
	320	4.4	$(0, 19, 0, 1077)$	49.92	155	5	1
2^{24-18}	192	4	$(-, 21, 0, 806)$	110.56	93	3	1
	256	4	$(4, 0, 71, 924)$	79.50	124	4	1
	320	4.4	$(0, 30, 0, 1267)$	61.48	155	5	6
2^{25-19}	192	4	$(-, 26, 0, 949)$	131.44	93	3	6
	256	4	$(6, 0, 88, 1080)$	95.50	124	4	6
	320	4.4	$(0, 40, 0, 1475)$	73.4	155	5	6
2^{26-20}	192	4	$(-, 38, 0, 1097)$	159.89	93	3	6
	256	4	$(11, 0, 108, 1240)$	115.50	124	4	6
	320	4.4	$(0, 63, 0, 1692)$	90.36	155	5	6

Table S.8: Strength-3 designs obtained from 128-run MA designs. A dash as an element of the F_4 vector means that the corresponding J_4-characteristic does not exist. A boldface GR value and F_4 vector indicate that these values are larger and sequentially smaller, respectively, than the corresponding values of the benchmarks available in the literature. d: number of parent designs; df: degrees of freedom for estimating 2FIs; Runs$_i$: design has i duplicate runs.

Parent	Runs	GR	$F_4(640, 512, 384, 256, 128)$	B_4	df	d
2^{20-13}	384	4.66	$(-, -, 0, 0, 92)$	10.22	190	3
	512	4.75	$(-, 0, 0, 0, 112)$	7	190	4
	640_2	4.8	$(0, 0, 0, 0, 148)$	5.92	190	5
	768_2	4.66	$(0, 0, 0, 2, 168)$	4.89	190	6
	896_2	4.857	$(0, 0, 0, 0, 204)$	4.16	190	7
2^{21-14}	384	4.66	$(-, -, 0, 0, 129)$	14.33	210	3
	512	4.75	$(-, 0, 0, 0, 156)$	9.75	210	4
	640_2	4.8	$(0, 0, 0, 0, 207)$	8.28	210	5
	768_4	4.66	$(0, 0, 0, 3, 234)$	6.83	210	6
	896_2	4.857	$(0, 0, 0, 0, 285)$	5.82	210	7
2^{22-15}	384	4.66	$(-, -, 0, 0, 173)$	19.22	231	3
	512	4.75	$(-, 0, 0, 0, 216)$	13.5	231	4
	640_2	4.8	$(0, 0, 0, 0, 281)$	11.24	231	5
	768_2	4.66	$(0, 0, 0, 2, 324)$	9.22	231	6
	896_2	4.857	$(0, 0, 0, 0, 389)$	7.94	231	7
2^{23-16}	384	4.66	$(-, -, 0, 0, 211)$	23.44	251	3
	512	4.75	$(-, 0, 0, 0, 256)$	16	253	4
	640_2	4.8	$(0, 0, 0, 0, 339)$	13.56	253	5
	768_2	4.66	$(0, 0, 0, 5, 384)$	11.22	253	6
	896_4	4.857	$(0, 0, 0, 0, 467)$	9.53	253	7
2^{24-17}	384	4.66	$(-, -, 0, 0, 258)$	28.67	269	3
	512	**4.75**	$\mathbf{(-, 0, 0, 0, 312)}$	19.5	276	4
	640_2	4.8	$(0, 0, 0, 0, 414)$	16.56	276	5
	768_2	4.66	$(0, 0, 0, 7, 468)$	13.78	276	6
	896_2	4.857	$(0, 0, 0, 0, 570)$	11.63	276	7
2^{25-18}	384	4.66	$(-, -, 0, 0, 316)$	35.11	269	3
	512	**4.75**	$\mathbf{(-, 0, 0, 0, 384)}$	24	300	4

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Table S.8 (continued)

Parent	Runs	GR	$F_4(640, 512, 384, 256, 128)$	B_4	df	d
	640	4.8	$(0, 0, 0, 0, 508)$	20.32	300	5
	768_2	4.66	$(0, 0, 0, 9, 576)$	17	300	6
	896_2	4.857	$(0, 0, 0, 0, 700)$	14.29	300	7
2^{26-19}	384	4	$(-, -, 1, 0, 373)$	42.44	276	3
	512	**4.5**	$(-, 0, 0, 8, 444)$	29.75	324	4
	640	4.8	$(0, 0, 0, 0, 596)$	23.84	325	5
	768	4.66	$(0, 0, 0, 14, 666)$	20.06	325	6
	896	4.857	$(0, 0, 0, 0, 818)$	16.69	325	7
2^{27-20}	384	4	$(-, -, 3, 0, 431)$	50.89	282	3
	512	**4.5**	$(-, 0, 0, 18, 508)$	36.25	347	4
	640	4.4	$(0, 0, 2, 0, 686)$	28.16	350	5
	768	4.66	$(0, 0, 0, 19, 762)$	23.28	351	6
	896	4.857	$(0, 0, 0, 0, 942)$	19.22	351	7
2^{28-21}	384	4	$(-, -, 5, 0, 495)$	60	289	3
	512	**4.5**	$(-, 0, 0, 27, 580)$	43	365	4
	640	4.4	$(0, 0, 3, 0, 787)$	32.56	377	5
	768	4.66	$(0, 0, 0, 25, 870)$	26.94	378	6
	896	4.857	$(0, 0, 0, 0, 1080)$	22.04	378	7
2^{29-22}	384	4	$(-, -, 9, 0, 611)$	76.89	291	3
	512	**4.5**	$(-, 0, 0, 44, 708)$	55.25	367	4
	640	4.4	$(0, 0, 9, 0, 965)$	41.84	401	5
	768	4.66	$(0, 0, 0, 41, 1062)$	34.06	406	6
	896	4.857	$(0, 0, 0, 0, 1328)$	27.10	406	7
2^{30-23}	384	4	$(-, -, 12, 0, 777)$	98.33	261	3
	512	4	$(-, 1, 0, 56, 908)$	71.75	348	4
	640	4.4	$(0, 0, 16, 0, 1227)$	54.84	425	5
	768	4.66	$(0, 0, 0, 90, 1362)$	47.83	426	6
	896	4.571	$(0, 0, 15, 0, 1682)$	37.08	433	7
2^{31-24}	384	4	$(-, -, 17, 0, 890)$	115.89	261	3
	512	4	$(-, 2, 0, 70, 1032)$	84	348	4
	640	4.4	$(0, 0, 26, 0, 1397)$	65.2	431	5
	768	4.66	$(0, 0, 0, 119, 1548)$	56.22	457	6
	896	4.571	$(0, 0, 30, 0, 1909)$	44.47	462	7
2^{32-25}	384	4	$(-, -, 22, 0, 1004)$	133.56	261	3
	512	4	$(-, 5, 0, 84, 1148)$	97.75	348	4

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Parent	Runs	GR	$F_4(640, 512, 384, 256, 128)$	B_4	df	d
	640	4.4	$(0, 0, 38, 0, 1562)$	76.16	432	5
	768	4.66	$(0, 0, 0, 165, 1722)$	66.17	482	6
	896	4.571	$(0, 0, 51, 0, 2123)$	52.69	489	7
2^{33-26}	384	4	$(-, -, 29, 0, 1125)$	154	261	3
	512	4	$\mathbf{(-, 7, 0, 104, 1272)}$	112.5	348	4
	640	4.4	$(0, 0, 51, 0, 1739)$	87.92	430	5
	768	4.66	$(0, 0, 0, 200, 1908)$	75.22	507	6
	896	4.571	$(0, 0, 67, 0, 2359)$	60.45	518	7
2^{34-27}	384	4	$(-, -, 35, 0, 1264)$	175.44	261	3
	512	4	$\mathbf{(-, 9, 0, 124, 1420)}$	128.75	348	4
	640	4.4	$(0, 0, 65, 0, 1944)$	101.16	435	5
	768	4.66	$(0, 0, 0, 234, 2130)$	85.17	519	6
	896	4.571	$(0, 0, 86, 0, 2633)$	69.53	546	7
2^{35-28}	384	4	$(-, -, 40, 0, 1439)$	199.89	261	3
	512	4	$\mathbf{(-, 9, 0, 142, 1628)}$	146.25	348	4
	640	4.4	$(0, 0, 73, 0, 2220)$	115.08	435	5
	768	4.66	$(0, 0, 0, 258, 2442)$	96.5	521	6
	896	4.571	$(0, 0, 97, 0, 3010)$	79.245	576	7
2^{36-29}	384	4	$(-, -, 50, 0, 1604)$	228.22	261	3
	512	4	$\mathbf{(-, 13, 0, 170, 1796)}$	167.75	348	4
	640	4.4	$(0, 0, 98, 0, 2454)$	133.44	435	5
	768	4.33	$(0, 13, 0, 264, 2694)$	109.94	521	6
	896	4.571	$(0, 0, 136, 0, 3314)$	92.61	604	7
2^{37-30}	384	4	$(-, -, 56, 0, 1818)$	258	261	3
	512	4	$\mathbf{(-, 15, 0, 186, 2040)}$	189	348	4
	640	4	$(1, 0, 103, 0, 2790)$	149.68	435	5
	768	4.33	$(0, 15, 0, 298, 3060)$	124.78	520	6
	896	4.571	$(0, 0, 181, 0, 3733)$	109.43	606	7
2^{38-31}	384	4	$(-, -, 68, 0, 2011)$	291.44	261	3
	512	4	$\mathbf{(-, 20, 0, 206, 2240)}$	211.5	348	4
	640	4	$(2, 0, 122, 0, 3075)$	168.92	435	5
	768	4.33	$(0, 32, 0, 301, 3360)$	141	521	6
	896	4.286	$(2, 0, 175, 0, 4142)$	117.69	608	7
2^{39-32}	384	4	$(-, -, 80, 0, 2213)$	325.89	261	3
	512	4	$\mathbf{(-, 24, 0, 249, 2444)}$	239	348	4

Table S.8 (continued)

Parent	Runs	GR	$F_4(640, 512, 384, 256, 128)$	B_4	df	d
	640	4	$(4, 0, 139, 0, 3372)$	188.92	435	5
	768	4.33	$(0, 44, 0, 309, 3666)$	155.72	522	6
	896	4.286	$(5, 0, 197, 0, 4535)$	131.29	609	7
2^{40-33}	384	4	$(-, -, 93, 0, 2423)$	362.22	261	3
	512	4	$\mathbf{(-, 30, 0, 282, 2652)}$	266.25	348	4
	640	4	$(5, 0, 167, 0, 3670)$	211.92	435	5
	768	4.33	$(0, 64, 0, 342, 3978)$	176.94	522	6
	896	4.286	$(6, 0, 242, 0, 4920)$	147.92	609	7

Table S.9: Strength-3 designs obtained from 256-run MA designs. A boldface GR value and F_4 vector indicate that these values are larger and sequentially smaller, respectively, than the corresponding values of the benchmarks available in the literature. d: number of parent designs; f: best factor to fix; df: degrees of freedom for estimating 2FIs; $\text{Runs}_{i,j}$: design has i duplicate and j triplicate runs.

Parent	Runs	GR	$F_4(768, 512, 256)$	B_4	df	d	f
2^{24-16}	$768_{4,0}$	4.66	$(0, 0, 66)$	7.33	276	3	6
	$1024_{4,0}$	4.75	$(0, 0, 80)$	5	276	4	6
	$1280_{4,4}$	4.8	$(0, 0, 106)$	4.24	276	5	6
2^{25-17}	768	4.66	$(0, 0, 84)$	9.33	300	3	6
	1024	4.75	$(0, 0, 100)$	6.25	300	4	6
	$1280_{4,0}$	4.8	$(0, 0, 134)$	5.36	300	5	6
2^{26-18}	768	4.66	$(0, 0, 105)$	11.67	325	3	8
	1024	4.75	$(0, 0, 124)$	7.75	325	4	8
	$1280_{4,0}$	4.8	$(0, 0, 167)$	6.68	325	5	8
2^{27-19}	768	4.66	$(0, 0, 127)$	14.11	351	3	7
	1024	4.75	$(0, 0, 148)$	9.25	351	4	7
	$1280_{4,0}$	4.8	$(0, 0, 201)$	8.04	351	5	7
2^{28-20}	768	4.66	$(0, 0, 152)$	16.89	378	3	8
	1024	4.75	$(0, 0, 172)$	10.75	378	4	7
	$1280_{4,0}$	4.8	$(0, 0, 236)$	9.44	378	5	7
2^{29-21}	768	4.66	$(0, 0, 184)$	20.44	406	3	8
	1024	4.75	$(0, 0, 212)$	13.25	406	4	8

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Table S.9 (continued)

Parent	Runs	GR	$F_4(768, 512, 256)$	B_4	df	d	f
	1280	4.8	$(0, 0, 290)$	11.6	406	5	8
2^{30-22}	768	4.66	$(0, 0, 215)$	23.89	433	3	5
	1024	4.75	$(0, 0, 244)$	15.25	435	4	5
	1280	4.8	$(0, 0, 333)$	13.32	435	5	4
2^{31-23}	768	4.66	$(0, 0, 255)$	28.33	461	3	5
	1024	4.75	$(0, 0, 284)$	17.75	465	4	5
	1280	4.8	$(0, 0, 393)$	15.72	465	5	4
2^{32-24}	768	4.66	$(0, 0, 297)$	33	491	3	5
	1024	4.75	$(0, 0, 328)$	20.5	496	4	5
	1280	4.8	$(0, 0, 461)$	18.44	496	5	5
2^{33-25}	768	4.66	$(0, 0, 347)$	38.56	522	3	2
	1024	4.75	$(0, 0, 384)$	24	528	4	7
	1280	4.8	$(0, 0, 525)$	21	528	5	5
2^{34-26}	768	4.66	$(0, 0, 404)$	44.89	553	3	6
	1024	**4.75**	**$(0, 0, 456)$**	28.5	561	4	6
	1280	4.8	$(0, 0, 608)$	24.32	561	5	2
2^{35-27}	768	4	$(1, 0, 455)$	51.56	576	3	6
	1024	**4.5**	**$(0, 10, 496)$**	33.5	595	4	1
	1280	4.8	$(0, 0, 692)$	27.68	595	5	3
2^{36-28}	768	4	$(3, 0, 492)$	57.67	601	3	3
	1024	**4.5**	**$(0, 19, 560)$**	39.75	630	4	6
	1280	4.8	$(0, 0, 773)$	30.92	630	5	7

G Sets of factors for sign switching

Tables S.10, S.11 and S.17 include the best sets of factors in which to switch the signs in the parent designs with 32, 64 and 256 runs, respectively. Due to the length of the results, we show the best sets of factors in which to switch the signs in the 128-run parent designs in Tables S.12-S.16. More specifically, Tables S.12, S.13, S.14, S.15 and S.16 show the sets of factors for the 128-run parents used to construct 384-, 512-, 640-, 768- and 896-run designs, respectively.

The steps required to construct the strength-3 designs from Tables S.6-S.9 in Section F are:

1. Obtain the 2^{m-p} design with $n = 2^{m-p}$ runs and m factors in Table S.1 in Section D using the following command of the FrF2 package in R:

   `FrF2(nruns = n, nfactors = m, randomize = FALSE)`

   The output is a minimum aberration (MA) design where its first $b = m - p$ columns correspond to the basic factors and the rest to the generated factors.

2. Generate d isomorphic copies, $D_0, \ldots, D_{d-1}$, of the 2^{m-p} design as in Theorem 1 if b is a prime. Otherwise, generate the copies as in Theorem 2 with B the subset of basic factors that does not include factor f. Tables S.7 and S.9 in Section F show f for the 64- and 256-run regular designs, respectively.

3. Switch the signs in the factor columns of $D_0, \ldots, D_{d-1}$ according to Tables S.10-S.17.

4. Concatenate $D_0, \ldots, D_{d-1}$ to create the final design with $N = nd$ runs and m factors.

Example S.3. To generate the 18-factor 256-run design in Table S.7 in Section F, we start from the MA 2^{18-12} design with 6 basic factors in Section D. We generate the MA design using the command `FrF2(nruns = 64, nfactors = 18, randomize = FALSE)` of the FrF2 package in R. The first 6 columns of the resulting design correspond to the basic factors while the other columns to the generated factors. Since the number of basic factors of the regular design is not a prime, we use Theorem 2 to generate the four isomorphic copies, D_0, D_1, D_2 and D_3, of the 2^{18-12} design. To this end, Table S.7 shows that $f = 6$ and so the subset of basic factors to permute is $B = \{1, 2, 3, 4, 5\}$. Design D_0 is the regular

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design without modification. Design D_1 is formed by shifting the columns $1, \ldots, 5$ one position to the right and placing column 5 in the first position. Designs D_2 and D_3 are formed in a similar fashion by shifting the factor columns in B two and three positions to the right, respectively. According to Table S.11, we switch the signs of the columns in the sets $\{11, 13, 15, 16\}$, $\{13, 14, 18\}$ and $\{8, 14, 16\}$ in D_1, D_2 and D_3, respectively. Finally, we concatenate D_0, D_1, D_2 and D_3, to generate a 256-run design for 18 factors with a generalized resolution of 4.75, an $F_4(256, 128, 64) = (0, 0, 240)$ and a B_4 value of 15.

Table S.10: Factors in which to switch the signs in 32-run parent designs.

Parent	N	D_1	D_2	D_3	D_4
2^{9-4}	96	8			
	128	8			
	160	8			
2^{10-5}	96	6			
	128	8			
	160	6			
2^{11-6}	96	8	7		
	128	9	10	11	
	160	6	9, 11	9	
2^{12-7}	96	12	11		
	128	6	8	7	
	160	12	7	10	
2^{13-8}	96	7, 8	8		
	128	7, 8	7, 13	11	
	160	6, 9	11, 13	10, 13	
2^{14-9}	96	9, 13	8, 12		
	128	7, 8, 14	7, 13	6, 8	
	160	8, 12	7, 9	12, 13, 14	
2^{15-10}	96	10, 11, 14	8, 13		
	128	8, 11, 13, 14	9, 13, 14	8, 9	
	160	7, 9, 11, 13	6, 10, 13	6, 8, 12	11
2^{16-11}	96	8, 10, 11, 13, 14, 16	6, 15		
	128	8, 10, 14, 15	6, 8, 15, 16	10, 11, 16	
	160	8, 10, 11, 13, 14	6, 11, 13	7, 11, 13, 14, 16	6, 13, 14

Table S.11: Factors in which to switch the signs in 64-run parent designs.

Parent	N	D_1	D_2	D_3	D_4
2^{17-11}	192	8, 11, 13, 14	8, 16	9, 11, 15	
	256	9, 14	9, 13, 14, 17	9, 14, 15	
	320	12, 13, 15	8, 11, 15		
2^{18-12}	192	10, 11, 14	10, 12, 18	8, 14, 16	
	256	11, 13, 15, 16	13, 14, 18	13, 15, 16, 18	
	320	9, 15, 16	9, 13, 18		
2^{19-13}	192	9, 10, 11, 13, 14, 15	8, 11, 12, 18		
	256	11, 14, 16, 17	9, 14, 17	7, 10, 15, 17, 18	
	320	7, 9, 10, 11, 17	7, 9, 14, 15, 17, 18	11, 16, 18	7, 8, 12, 16
2^{20-14}	192	8, 12, 18, 19	8, 9, 11, 17, 19		
	256	8, 11, 12, 14, 15, 16, 18, 20	7, 9, 11, 17, 20	10, 12, 17	
	320	10, 15, 16, 18, 20	8, 10, 14, 15, 19, 20	12, 13, 14, 15, 17, 19	16, 20
2^{21-15}	128	7, 9, 14, 15, 17, 18			
	192	10, 12, 18, 20, 21	7, 11, 12, 16, 18		
	256	7, 8, 9, 11, 14, 15, 21	7, 8, 9, 15, 16, 18, 20	7, 9, 10, 12, 14, 15	
	320	7, 8, 9, 12, 13, 17, 18, 19, 20, 21	7, 10, 13, 15, 16, 19, 20	7, 10, 11, 12, 13, 18, 20	11, 13, 15, 19, 21
2^{22-16}	192	10, 11, 17, 18, 19, 21, 22	10, 12, 14, 16, 18, 21		
	256	13, 14, 15, 17, 18, 20, 21, 22	7, 10, 14, 18, 22	12, 13, 14, 17, 22	
	320	8, 10, 14, 15, 18, 20, 22	8, 9, 10, 13, 19, 22	9, 10, 13, 14, 18	7, 8, 9, 10, 13, 21
2^{23-17}	192	7, 9, 13, 19, 20, 21, 22	8, 13, 15, 17, 23		
	256	7, 8, 9, 14, 15, 17, 23	9, 10, 13, 14, 20, 22	7, 8, 9, 10, 11	
	320	8, 10, 12, 13, 20, 23	11, 13, 14, 17, 19, 21, 22	8, 12, 14, 16, 18, 19, 20	9, 14, 20, 21, 22

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Table S.11 (continued)

Parent	N	D_1	D_2	D_3	D_4
2^{24-18}	192	9, 10, 11, 12, 16, 18, 21, 22, 23	7, 10, 14, 17, 19, 20, 21, 23	9, 13, 14, 15, 17, 22, 23	
	256	7, 10, 12, 17, 18, 19, 21, 23	7, 8, 13, 16, 18, 19, 20, 21, 22	7, 8, 10, 12, 20	7, 11, 14, 16, 21, 24
	320	10, 13, 16, 17, 18, 19, 20, 24	9, 11, 12, 13, 15, 16, 17, 19, 23		
2^{25-19}	192	8, 12, 13, 17, 19, 23, 24, 25	9, 11, 14, 15, 19, 20, 21		
	256	7, 8, 11, 13, 16, 17, 18, 19, 21, 23, 24, 25	8, 9, 10, 12, 14, 18, 19	8, 11, 15, 16, 17, 19, 22	
	320	11, 12, 16, 17, 20, 21, 22, 24	9, 10, 11, 12, 15, 23, 25	13, 16, 18, 20, 24, 25	7, 9, 10, 11, 17, 21, 24
2^{26-20}	192	14, 15, 16, 18, 22, 24, 26	10, 13, 15, 19, 22, 23, 24, 25		
	256	7, 10, 11, 19, 21, 22, 25	8, 11, 14, 17, 19, 20, 21, 26	7, 8, 11, 15, 20, 22, 23	
	320	8, 10, 14, 15, 18, 20, 22, 23, 24	10, 12, 15, 16, 19, 22, 23, 24	8, 9, 10, 11, 15, 17, 20, 22, 25	7, 8, 9, 11, 13, 17, 25

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Table S.12: Factors in which to switch the signs in 128-run parent designs to construct 384-run designs.

Parent	D_1	D_2
2^{20-13}	11, 17, 19, 20	15
2^{21-14}	9, 13, 15, 17	9, 18
2^{22-15}	10, 11, 12, 15	11, 16
2^{23-16}	8, 11, 14, 15, 17, 23	8, 9, 10, 11
2^{24-17}	12, 19, 20, 21, 22, 23	9, 11, 16, 20
2^{25-18}	8, 9, 17, 21, 25	9, 11, 14, 20, 23, 24
2^{26-19}	10, 12, 13, 15, 20, 22, 26	8, 10, 14, 17, 22
2^{27-20}	9, 10, 11, 17, 20, 22, 23, 27	9, 11, 13, 16, 19, 21, 27
2^{28-21}	8, 9, 11, 12, 13, 16, 18, 19, 22, 23, 28	12, 14, 15, 21, 22, 28
2^{29-22}	8, 13, 16, 17, 24, 27, 28, 29	21, 23, 25, 26, 28, 29
2^{30-23}	10, 12, 17, 22, 25, 27, 30	9, 13, 14, 15, 16, 18, 21, 24
2^{31-24}	8, 11, 15, 16, 25, 26, 27, 28, 30	10, 11, 13, 14, 15, 16, 22, 24, 28, 30
2^{32-25}	9, 11, 12, 15, 18, 28, 32	8, 11, 12, 14, 18, 19, 22, 28, 29, 31
2^{33-26}	10, 12, 14, 16, 20, 23, 24, 31, 33	9, 10, 11, 12, 13, 15, 20, 24, 25, 30
2^{34-27}	13, 14, 15, 18, 19, 21, 22, 24, 25, 26, 28, 30, 32, 34	8, 15, 17, 18, 19, 20, 23, 28, 32
2^{35-28}	8, 10, 11, 12, 13, 17, 19, 20, 21, 24, 26, 27, 33	8, 14, 20, 21, 24, 26, 27, 28, 30, 35
2^{36-29}	12, 18, 19, 24, 26, 28, 30, 33, 35, 36	9, 11, 19, 20, 25, 26, 29, 32, 33
2^{37-30}	10, 11, 12, 14, 19, 20, 21, 22, 24, 26, 27, 28, 30, 31, 35	8, 11, 14, 16, 20, 23, 28, 29, 30, 32, 33, 34
2^{38-31}	15, 17, 19, 20, 23, 27, 28, 29, 34, 35, 36, 37	10, 13, 15, 20, 22, 23, 25, 26, 27, 30, 35
2^{39-32}	13, 14, 17, 19, 20, 22, 23, 27, 30, 31, 38, 39	12, 17, 18, 19, 20, 23, 27, 32, 34, 35, 36
2^{40-33}	8, 10, 12, 13, 18, 19, 21, 25, 26, 28, 29, 30, 36, 40	15, 16, 21, 22, 24, 27, 30, 33, 34, 36, 37, 40

Table S.13: Factors in which to switch the signs in 128-run parent designs to construct 512-run designs.

Parent	Copy	Factors
2^{20-13}	D_1	8, 13, 16, 18, 19, 20
	D_2	9, 10, 18
	D_3	12, 16
2^{21-14}	D_1	8, 9, 12, 19, 20
	D_2	10, 14, 21
	D_3	17, 21
2^{22-15}	D_1	8, 9, 10, 14, 22
	D_2	10, 15, 21
	D_3	11, 17
2^{23-16}	D_1	10, 12, 14, 18, 20, 22
	D_2	11, 14, 15, 17, 22
	D_3	12, 14, 19
2^{24-17}	D_1	8, 9, 10, 12, 14, 15, 22, 23, 24
	D_2	9, 13, 14, 15, 22
	D_3	10, 11, 17, 18
2^{25-18}	D_1	13, 16, 17, 20, 23, 24
	D_2	8, 9, 14, 17, 20, 21, 22
	D_3	9, 11, 15, 18, 20, 25
2^{26-19}	D_1	8, 13, 14, 16, 20, 22, 23, 25, 26
	D_2	14, 17, 19, 20, 21, 25, 26
	D_3	12, 13, 14, 19, 20
2^{27-20}	D_1	8, 16, 19, 23, 25, 27
	D_2	8, 13, 14, 17, 20, 25
	D_3	14, 18, 20, 21, 25, 26

Continued on next page

Parent	Copy	Factors
2^{28-21}	D_1	9, 12, 13, 14, 15, 16, 19, 20, 27, 28
	D_2	11, 15, 21, 26, 28
	D_3	8, 14, 21, 23, 28
2^{29-22}	D_1	9, 13, 15, 16, 19, 21, 23, 24, 25, 26
	D_2	10, 13, 14, 21, 25, 26, 28, 29
	D_3	10, 12, 14, 19, 24, 25, 29
2^{30-23}	D_1	8, 9, 10, 16, 17, 21, 23, 24, 26, 27, 29
	D_2	9, 10, 12, 14, 16, 17, 23, 28, 29, 30
	D_3	14, 15, 16, 17, 29
2^{31-24}	D_1	8, 12, 13, 14, 18, 20, 21, 23, 25, 26, 28, 29
	D_2	9, 10, 12, 14, 15, 16, 17, 21, 27
	D_3	8, 10, 12, 16, 18, 23, 24, 27
2^{32-25}	D_1	9, 11, 12, 13, 15, 18, 28, 31, 32
	D_2	14, 17, 18, 19, 20, 21, 26, 27, 29, 30
	D_3	8, 10, 11, 17, 20, 21, 27, 29
2^{33-26}	D_1	9, 11, 12, 21, 22, 24, 25, 29, 32, 33
	D_2	9, 11, 12, 14, 15, 19, 24, 25, 26, 27, 31
	D_3	15, 16, 19, 23, 25, 26, 27, 30, 31, 32, 33
2^{34-27}	D_1	12, 13, 16, 19, 23, 24, 25, 26, 27, 28, 29, 31, 33
	D_2	8, 9, 10, 12, 13, 14, 15, 18, 23, 24, 25, 28, 30, 32, 33, 34
	D_3	8, 9, 10, 14, 18, 20, 23, 29
2^{35-28}	D_1	8, 12, 15, 17, 18, 19, 26, 28, 32
	D_2	13, 19, 23, 24, 31, 33, 34, 35
	D_3	8, 9, 16, 19, 20, 26, 29, 30, 32, 35
2^{36-29}	D_1	8, 11, 12, 14, 15, 17, 18, 21, 23, 24, 27, 29, 30, 36
	D_2	12, 14, 15, 17, 19, 20, 26, 27, 34, 36
	D_3	12, 14, 16, 20, 21, 24, 25, 26, 29, 34, 36

Parent	Copy	Factors
2^{37-30}	D_1	8, 10, 15, 20, 21, 22, 23, 26, 27, 29, 31, 35
	D_2	8, 9, 10, 13, 14, 15, 16, 18, 20, 23, 24, 27, 29, 35, 36
	D_3	10, 13, 15, 17, 21, 27, 33, 34
2^{38-31}	D_1	11, 16, 17, 18, 19, 22, 24, 25, 28, 32, 33, 35, 37, 38
	D_2	9, 10, 14, 15, 18, 20, 22, 23, 24, 25, 26, 27, 28, 31, 36, 38
	D_3	13, 16, 19, 20, 24, 27, 29, 30, 31, 32, 33, 35
2^{39-32}	D_1	10, 12, 17, 20, 21, 22, 23, 26, 28, 29, 30, 31, 34, 37
	D_2	9, 11, 12, 14, 15, 20, 22, 23, 24, 26, 28, 30, 31, 33, 39
	D_3	10, 11, 14, 16, 17, 18, 21, 22, 23, 27, 30, 35, 39
2^{40-33}	D_1	9, 10, 11, 12, 15, 16, 19, 20, 22, 24, 25, 26, 28, 32, 34
	D_2	8, 9, 13, 14, 16, 17, 24, 29, 31, 34, 36, 37, 38, 39
	D_3	8, 11, 13, 16, 17, 21, 27, 28, 30, 31, 33, 36

Table S.14: Factors in which to switch the signs in 128-run parent designs to construct 640-run designs.

Parent	Copy	Factors
2^{20-13}	D_1	9, 14, 15, 17
	D_2	11, 13, 15, 20
	D_3	12, 18, 20
2^{21-14}	D_1	8, 12, 15, 16, 19
	D_2	9, 11, 15, 21
	D_3	14, 17, 21
2^{22-15}	D_1	16, 19, 21, 22
	D_2	11, 15, 16, 18
	D_3	9, 13, 21
2^{23-16}	D_1	10, 11, 12, 14, 15, 20, 22

Continued on next page

Parent	Copy	Factors
	D_2	12, 15, 16, 17, 21, 23
	D_3	14, 17, 21, 22
2^{24-17}	D_1	10, 11, 12, 14, 16, 20, 22
	D_2	8, 11, 12, 16, 24
	D_3	17, 18, 20, 22, 23
2^{25-18}	D_1	13, 16, 17, 20, 23, 24
	D_2	10, 14, 16, 18, 19, 21, 25
	D_3	8, 16, 25
	D_4	8, 15, 21, 25
2^{26-19}	D_1	8, 9, 10, 11, 12, 17, 19, 20, 24, 26
	D_2	9, 13, 20, 23, 24, 25
	D_3	8, 10, 16, 21, 22, 23
	D_4	9, 11, 14, 16, 24
2^{27-20}	D_1	10, 12, 13, 14, 18, 24, 26
	D_2	8, 12, 13, 18, 22, 26
	D_3	9, 11, 15, 17, 20, 25
	D_4	9, 11, 13, 24
2^{28-21}	D_1	9, 13, 15, 16, 18, 20, 22, 26
	D_2	9, 10, 11, 15, 20, 22, 25
	D_3	9, 10, 12, 16, 19, 22, 23, 24, 25
	D_4	8, 12, 19, 20, 22
2^{29-22}	D_1	9, 10, 11, 12, 13, 14, 16, 17, 18, 21, 22, 23, 25
	D_2	10, 12, 13, 15, 18, 20, 23, 28, 29
	D_3	9, 14, 15, 16, 20, 21, 22, 28
	D_4	8, 10, 17, 19, 20, 22, 26
2^{30-23}	D_1	8, 9, 13, 15, 16, 17, 22, 23, 26, 27, 28
	D_2	8, 12, 15, 18, 21, 22, 23, 25, 26, 27, 28, 29, 30
	D_3	11, 12, 14, 21, 23, 26, 29

Parent	Copy	Factors
	D_4	11, 14, 15, 19, 22, 28, 29, 30
2^{31-24}	D_1	8, 10, 11, 12, 13, 14, 15, 17, 18, 20, 22, 26, 27, 30
	D_2	10, 13, 14, 15, 16, 17, 19, 21, 22, 25, 31
	D_3	14, 17, 19, 21, 22, 23, 24, 26, 31
	D_4	8, 9, 13, 16, 19, 22, 24, 31
2^{32-25}	D_1	8, 11, 12, 15, 16, 21, 23, 24, 25, 26, 29
	D_2	8, 10, 11, 12, 13, 15, 17, 27, 30, 32
	D_3	9, 14, 17, 18, 19, 22, 26, 31
	D_4	12, 13, 14, 17, 18, 19, 25, 28
2^{33-26}	D_1	8, 10, 14, 15, 16, 19, 20, 29, 30
	D_2	9, 13, 15, 16, 18, 19, 20, 21, 29
	D_3	8, 11, 12, 14, 15, 18, 19, 23, 32
	D_4	14, 16, 17, 24, 29, 31, 32
2^{34-27}	D_1	10, 14, 15, 16, 17, 19, 20, 25, 27, 28, 33, 34
	D_2	8, 10, 11, 13, 21, 23, 25, 26, 27, 32, 33
	D_3	8, 10, 13, 14, 17, 20, 23, 24, 30, 33
	D_4	12, 13, 19, 21, 23, 26, 30, 32
2^{35-28}	D_1	10, 14, 15, 16, 17, 18, 21, 23, 31, 32, 33, 34, 35
	D_2	8, 11, 15, 16, 18, 20, 21, 22, 25, 26, 27
	D_3	8, 10, 12, 14, 24, 26, 27, 32, 34, 35
	D_4	8, 13, 17, 18, 26, 27, 28
2^{36-29}	D_1	8, 9, 10, 11, 13, 18, 19, 21, 24, 26, 28, 29, 31, 32, 34, 35
	D_2	9, 11, 12, 15, 16, 18, 19, 21, 25, 29, 31, 32, 33, 34, 35
	D_3	9, 10, 12, 15, 17, 18, 19, 23, 24, 27, 28, 29
	D_4	9, 16, 19, 20, 21, 23, 24, 25, 28, 33, 35
2^{37-30}	D_1	10, 11, 14, 16, 20, 24, 28, 29, 31, 32, 34, 37
	D_2	12, 13, 15, 19, 21, 23, 24, 25, 27, 28, 36
	D_3	9, 11, 17, 19, 20, 23, 28, 29

Continued on next page

Parent	Copy	Factors
	D_4	18, 20, 23, 24, 25, 26, 30, 31, 32
2^{38-31}	D_1	8, 14, 15, 16, 19, 20, 23, 24, 26, 29, 35, 36
	D_2	8, 10, 15, 17, 18, 25, 27, 29, 30, 32, 35, 37, 38
	D_3	9, 10, 12, 15, 16, 20, 22, 29, 31, 33, 36, 38
	D_4	8, 9, 23, 27, 28, 29, 31, 33, 36
2^{39-32}	D_1	8, 9, 11, 14, 17, 18, 20, 23, 25, 27, 28, 29, 32, 34, 38, 39
	D_2	11, 15, 18, 19, 20, 21, 22, 26, 33, 34, 37
	D_3	9, 10, 15, 17, 18, 19, 22, 24, 26, 27, 29, 34
	D_4	8, 10, 16, 18, 19, 24, 28, 29, 33, 34, 35
2^{40-33}	D_1	13, 15, 16, 17, 22, 27, 28, 29, 30, 31, 34, 35, 39
	D_2	9, 12, 15, 19, 20, 26, 27, 28, 30, 31, 33, 38, 39
	D_3	8, 10, 11, 12, 15, 17, 20, 24, 25, 28, 32, 36, 40
	D_4	11, 14, 15, 16, 21, 29, 30, 32, 40

Table S.15: Factors in which to switch the signs in 128-run parent designs to construct 768-run designs.

Parent	Copy	Factors
2^{20-13}	D_1	8, 10, 13, 16, 19
	D_2	9, 11, 16, 18
	D_3	9, 15
	D_4	15, 20
2^{21-14}	D_1	8, 9, 10, 17, 19, 20
	D_2	8, 9, 20
	D_3	11, 14, 18
	D_4	11, 14, 18
2^{22-15}	D_1	11, 21, 22
	D_2	9, 11, 15, 16, 19, 21
	D_3	10, 11, 16, 21, 22

Continued on next page

Parent	Copy	Factors
	D_4	9, 17, 19
2^{23-16}	D_1	8, 13, 14, 16, 20, 21
	D_2	8, 9, 14, 15, 16
	D_3	13, 15, 17, 19, 22
	D_4	8, 11, 13, 23
2^{24-17}	D_1	9, 12, 17, 21, 23
	D_2	13, 14, 15, 21
	D_3	9, 11, 12, 15, 19, 21
	D_4	8, 10, 14, 15, 20
2^{25-18}	D_1	10, 11, 12, 14, 15, 16, 22
	D_2	11, 12, 13, 17, 19, 23, 24
	D_3	10, 11, 13, 15, 18, 23
	D_4	8, 10, 12, 21, 23, 25
2^{26-19}	D_1	9, 10, 11, 12, 15, 18, 22, 24, 26
	D_2	8, 16, 17, 18, 21, 22
	D_3	8, 14, 19, 21, 22, 23, 26
	D_4	9, 10, 12, 14, 15, 20
	D_5	10, 22, 23
2^{27-20}	D_1	8, 10, 11, 15, 16, 23, 25
	D_2	8, 10, 11, 16, 19, 25, 26, 27
	D_3	8, 13, 23, 24, 26
	D_4	8, 9, 11, 18, 24
	D_5	12, 18, 21
2^{28-21}	D_1	8, 9, 16, 17, 18, 22, 23, 27, 28
	D_2	8, 9, 13, 14, 16, 19, 20
	D_3	12, 15, 16, 17, 21, 23, 27, 28
	D_4	10, 15, 16, 18, 24, 25, 27
	D_5	8, 12, 14, 18

Table S.15 (continued)

Parent	Copy	Factors
2^{29-22}	D_1	9, 10, 11, 12, 13, 18, 19, 22, 26, 28, 29
	D_2	8, 13, 14, 19, 20, 25
	D_3	11, 14, 18, 19, 21, 22, 25
	D_4	9, 12, 17, 19, 20, 21, 25, 27
	D_5	12, 16, 19, 21, 26
2^{30-23}	D_1	17, 18, 19, 21, 24, 25, 27, 28, 30
	D_2	8, 9, 13, 15, 24, 25, 26, 29
	D_3	8, 10, 11, 15, 20, 22, 23, 26, 28
	D_4	9, 13, 14, 19, 24, 27
	D_5	9, 14, 30
2^{31-24}	D_1	12, 14, 16, 17, 19, 20, 21, 22, 23, 26, 27, 29
	D_2	9, 10, 14, 15, 17, 18, 21, 22, 26, 28, 30, 31
	D_3	13, 14, 19, 20, 22, 24, 26, 28
	D_4	9, 10, 13, 18, 19, 21, 26
	D_5	8, 9, 10, 21, 29
2^{32-25}	D_1	8, 10, 12, 13, 14, 15, 16, 17, 21, 22, 24, 26, 28, 29, 30
	D_2	12, 14, 15, 17, 18, 19, 22, 24, 26, 29
	D_3	8, 15, 17, 19, 22, 23, 25, 29, 30, 31, 32
	D_4	10, 11, 17, 20, 23, 27, 32
	D_5	9, 18, 21, 28, 32
2^{33-26}	D_1	11, 13, 16, 17, 19, 20, 21, 26, 27, 30
	D_2	8, 9, 13, 14, 16, 20, 26, 28, 29, 32
	D_3	8, 10, 15, 18, 20, 21, 24, 25, 27, 28, 32
	D_4	9, 12, 13, 20, 21, 23, 27, 32
	D_5	9, 10, 11, 15, 17, 19, 27
2^{34-27}	D_1	11, 14, 16, 19, 22, 26, 27, 30, 31, 32, 33, 34
	D_2	16, 19, 20, 21, 22, 25, 26, 29, 32
	D_3	13, 16, 18, 20, 23, 24, 25, 31, 32, 34
	D_4	9, 11, 13, 20, 21, 25, 26, 34
	D_5	13, 16, 22, 23, 24, 25, 30, 33

Parent	Copy	Factors
2^{35-28}	D_1	8, 14, 21, 22, 25, 26, 32, 34, 35
	D_2	9, 16, 17, 21, 22, 23, 24, 27, 29, 30, 31
	D_3	8, 9, 11, 12, 14, 19, 24, 26, 27, 29, 34
	D_4	8, 9, 22, 24, 26, 30, 34
	D_5	9, 13, 17, 20, 24, 34, 35
2^{36-29}	D_1	8, 14, 15, 16, 18, 20, 24, 25, 26, 27, 29, 34, 35, 36
	D_2	9, 10, 11, 12, 13, 14, 16, 19, 20, 23, 25, 26, 27, 28, 29, 30, 35
	D_3	9, 13, 14, 15, 17, 22, 24, 25, 29, 30, 32, 34, 36
	D_4	10, 13, 20, 21, 24, 26, 28, 31, 32, 33, 34
	D_5	10, 11, 12, 13, 17, 20, 23, 24, 27, 33
2^{37-30}	D_1	10, 11, 13, 15, 16, 18, 20, 22, 23, 29, 33, 34, 35
	D_2	10, 11, 13, 17, 18, 22, 23, 24, 25, 27, 33, 34, 36, 37
	D_3	8, 9, 12, 13, 15, 17, 18, 22, 23, 24, 26, 27, 30, 31, 33, 36
	D_4	8, 12, 13, 16, 17, 18, 19, 32, 35, 36
	D_5	13, 18, 20, 25, 27, 28, 29, 30
2^{38-31}	D_1	8, 9, 11, 12, 15, 16, 17, 18, 20, 22, 26, 27, 29, 30, 32, 35, 38
	D_2	8, 11, 13, 14, 21, 23, 24, 25, 27, 30, 36
	D_3	9, 10, 11, 13, 14, 16, 30, 34, 37
	D_4	10, 11, 12, 17, 21, 22, 23, 32, 35, 38
	D_5	10, 12, 23, 24, 28, 30, 36, 37
2^{39-32}	D_1	9, 14, 18, 20, 22, 23, 24, 27, 29, 30, 33, 35, 36, 38
	D_2	8, 10, 11, 13, 14, 15, 18, 21, 22, 25, 27, 29, 30, 36, 38, 39
	D_3	12, 13, 14, 15, 16, 20, 21, 25, 28, 30, 31, 37, 39
	D_4	9, 10, 15, 19, 20, 21, 23, 24, 26, 27, 29, 35, 36, 37, 38, 39
	D_5	9, 19, 22, 24, 29, 30, 32, 37, 38, 39
2^{40-33}	D_1	11, 12, 15, 18, 22, 24, 25, 26, 27, 33, 34, 37, 39, 40
	D_2	8, 14, 17, 19, 20, 23, 25, 26, 29, 30, 31, 34, 35, 37, 40
	D_3	11, 13, 15, 17, 18, 20, 25, 26, 30, 38, 39, 40
	D_4	11, 14, 15, 16, 17, 21, 23, 24, 27, 30, 34, 35, 38

Continued on next page

Parent	Copy	Factors
	D_5	11, 15, 18, 19, 21, 22, 24, 31, 32, 33, 34, 35, 36, 37, 38

Table S.16: Factors in which to switch the signs in 128-run parent designs to construct 896-run designs.

Parent	Copy	Factors
2^{20-13}	D_1	11, 12, 14, 18
	D_2	11, 19
	D_3	12, 13, 16
	D_4	14, 18
	D_5	15
2^{21-14}	D_1	11, 14, 15, 21
	D_2	12, 18, 21
	D_3	8, 11, 13, 20
	D_4	8, 13, 18
	D_5	8, 9
2^{22-15}	D_1	11, 14, 15, 18, 19, 22
	D_2	8, 15, 17, 18, 21, 22
	D_3	10, 14, 21
	D_4	9, 15, 20
	D_5	16, 19
2^{23-16}	D_1	8, 11, 12, 14, 16, 18, 19, 22, 23
	D_2	11, 14, 19, 20, 21
	D_3	10, 17, 19, 23
	D_4	13, 17, 22, 23
	D_5	10, 17, 19, 23
2^{24-17}	D_1	15, 17, 18, 20, 23, 24
	D_2	8, 9, 10, 12, 21
	D_3	8, 13, 14, 16, 21
	D_4	8, 11, 12, 20, 23

Continued on next page

Parent	Copy	Factors
	D_5	9, 11, 16, 20
2^{25-18}	D_1	8, 9, 13, 15, 16, 20, 24
	D_2	8, 9, 12, 13, 15, 17, 19, 23
	D_3	10, 11, 12, 15, 19
	D_4	12, 15, 17, 18, 21, 23, 24
	D_5	9, 13, 15, 18, 20
2^{26-19}	D_1	8, 10, 12, 13, 14, 15, 19, 23
	D_2	8, 14, 17, 18, 20, 22, 23, 24
	D_3	11, 12, 14, 21, 23, 25
	D_4	11, 13, 14, 19, 20, 22
	D_5	9, 16, 19, 23
	D_6	11, 16
2^{27-20}	D_1	8, 9, 10, 12, 18, 19, 20, 24
	D_2	8, 16, 18, 20, 21, 22, 24, 26
	D_3	9, 11, 14, 16, 21, 22, 25, 26, 27
	D_4	12, 13, 17, 18, 20, 21, 24, 25, 26
	D_5	13, 14, 16, 17, 18, 19, 26
	D_6	14, 16, 22
2^{28-21}	D_1	8, 10, 11, 12, 13, 14, 19, 20, 28
	D_2	9, 10, 12, 16, 18, 23, 24, 26
	D_3	8, 12, 13, 16, 18, 20, 27
	D_4	10, 11, 14, 18, 21, 24, 27, 28
	D_5	9, 13, 15, 17, 18, 24, 26
	D_6	18, 22, 27, 28
2^{29-22}	D_1	8, 9, 12, 14, 16, 17, 19, 21, 22, 28, 29
	D_2	9, 11, 16, 18, 23, 27, 28, 29
	D_3	12, 14, 15, 18, 20, 22, 23, 24, 25, 28
	D_4	12, 13, 19, 25, 28, 29
	D_5	8, 9, 10, 12, 15, 16, 19, 22, 23, 28
	D_6	10, 12, 19, 23, 26, 27

Table S.16 (continued)

Parent	Copy	Factors
2^{30-23}	D_1	13, 16, 17, 18, 21, 22, 23, 24, 26, 29, 30
	D_2	15, 17, 20, 21, 22, 26, 27, 29, 30
	D_3	8, 9, 13, 14, 20, 26, 27
	D_4	9, 11, 14, 18, 19, 27, 28, 30
	D_5	10, 13, 14, 20, 21, 24, 26, 30
	D_6	9, 12, 16, 17, 20, 23
2^{31-24}	D_1	8, 10, 12, 14, 15, 16, 17, 18, 19, 23, 25, 27, 28, 29, 31
	D_2	8, 10, 12, 13, 16, 17, 18, 28, 29, 30
	D_3	9, 10, 11, 19, 20, 21, 22, 23, 24, 27, 29
	D_4	14, 19, 21, 23, 24, 28, 30, 31
	D_5	8, 9, 16, 20, 27, 30
	D_6	13, 17, 19, 24, 26, 29
2^{32-25}	D_1	8, 10, 11, 17, 18, 19, 24, 30, 31
	D_2	9, 10, 12, 14, 16, 19, 20, 21, 24, 27, 30, 32
	D_3	12, 16, 20, 24, 28, 31, 32
	D_4	8, 9, 11, 12, 15, 26, 27, 32
	D_5	10, 12, 15, 19, 23, 30, 31, 32
	D_6	13, 15, 23, 24, 25, 28, 29, 31
2^{33-26}	D_1	9, 13, 15, 16, 22, 23, 25, 27, 29, 30, 31, 32, 33
	D_2	10, 12, 13, 14, 17, 19, 22, 23, 27, 32, 33
	D_3	9, 11, 12, 14, 15, 16, 17, 22, 23, 25, 26, 33
	D_4	8, 9, 16, 17, 20, 21, 27, 28, 31
	D_5	18, 20, 24, 26, 27, 31, 32
	D_6	14, 16, 17, 21, 24, 29, 31, 32
2^{34-27}	D_1	8, 10, 12, 13, 15, 16, 17, 18, 20, 25, 26, 27, 28, 30, 31, 33
	D_2	15, 17, 19, 23, 24, 27, 29
	D_3	10, 13, 14, 16, 17, 20, 21, 24, 25, 27, 28, 31, 33, 34
	D_4	10, 12, 16, 17, 18, 20, 24, 25, 29

Continued on next page

Table S.16 (continued)

Parent	Copy	Factors
	D_5	11, 13, 17, 25, 29, 31, 33
	D_6	15, 17, 18, 20, 21, 22, 24, 25, 31, 33, 34
2^{35-28}	D_1	9, 10, 12, 17, 20, 24, 27, 28, 29, 32, 34, 35
	D_2	8, 9, 14, 15, 16, 17, 19, 21, 24, 28, 31, 33, 34, 35
	D_3	9, 12, 13, 16, 19, 23, 27, 33
	D_4	10, 11, 15, 17, 19, 20, 28, 34, 35
	D_5	8, 13, 26, 28, 30, 31, 35
	D_6	9, 10, 13, 14, 15, 18, 23, 25, 26, 27, 30, 33
2^{36-29}	D_1	8, 9, 10, 13, 14, 16, 17, 18, 23, 24, 26,
		27, 29, 31, 32, 33, 34, 35, 36
	D_2	10, 12, 16, 20, 24, 26, 28, 32, 35
	D_3	8, 10, 12, 19, 20, 24, 26, 29, 31, 32, 33, 36
	D_4	8, 11, 13, 15, 16, 17, 21, 25, 26, 28, 33, 35
	D_5	10, 11, 17, 18, 21, 26, 28, 32, 33, 36
	D_6	8, 9, 12, 14, 16, 21, 22, 25, 28, 32, 33
2^{37-30}	D_1	8, 11, 12, 13, 14, 16, 17, 21, 24,
		26, 32, 33, 35, 37
	D_2	8, 10, 12, 14, 18, 19, 20, 24, 26, 29, 30
	D_3	9, 10, 14, 15, 20, 24, 28, 29, 30, 33, 34
	D_4	10, 12, 14, 15, 17, 22, 26, 27, 30, 31, 34, 37
	D_5	12, 15, 18, 19, 21, 22, 24, 29, 33, 34
	D_6	12, 14, 27, 35, 37
2^{38-31}	D_1	8, 9, 11, 14, 19, 21, 27, 29, 35, 36
	D_2	9, 12, 15, 17, 23, 24, 25, 27, 35, 36
	D_3	8, 10, 11, 13, 16, 17, 20, 25, 31, 33, 34, 36
	D_4	9, 17, 18, 21, 22, 32, 33, 34, 35, 37
	D_5	11, 14, 15, 16, 25, 34, 35, 36
	D_6	8, 12, 17, 18, 24, 25, 28, 29, 30, 31, 33, 34, 35
2^{39-32}	D_1	11, 13, 14, 17, 18, 20, 22, 23, 24, 32, 35
	D_2	13, 14, 27, 30, 31, 32, 33, 34, 35, 37, 38, 39

Continued on next page

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Parent	Copy	Factors
	D_3	11, 20, 24, 26, 27, 28, 30, 33, 37, 38
	D_4	10, 13, 16, 20, 25, 26, 28, 29, 30, 31, 36, 39
	D_5	10, 14, 15, 19, 21, 24, 26, 27, 28, 34, 36
	D_6	8, 13, 14, 16, 19, 20, 21, 23, 32, 34, 35, 39
2^{40-33}	D_1	13, 18, 19, 20, 21, 23, 24, 29, 30, 31,
		32, 33, 34, 36, 39
	D_2	9, 10, 13, 14, 17, 18, 20, 21, 22, 23, 25, 26, 27, 29,
		30, 31, 32, 33, 34, 36
	D_3	9, 10, 11, 12, 18, 21, 23, 26, 29, 30, 31, 35
	D_4	9, 10, 13, 17, 19, 24, 25, 27, 30, 31, 34, 35, 37
	D_5	8, 9, 10, 12, 13, 16, 25, 27, 31, 33
	D_6	8, 9, 10, 12, 13, 20, 21, 23, 26, 33, 34, 35, 38

Table S.17: Factors in which to switch the signs in 256-run parent designs.

Parent	N	D_1	D_2	D_3	D_4
2^{24-16}	768	10, 13, 19, 20, 24	9, 10, 12, 13, 21		
	1024	11, 12, 13, 19, 20, 23	9, 10, 12, 13, 21		
	1280	9, 12, 14, 22, 24	9, 10, 12, 13, 21		
2^{25-17}	768	14, 20, 21, 25	19, 24		
	1024	9, 11, 13, 14, 20	11, 14, 19, 24, 25	15, 19	
	1280	9, 12, 13, 16, 17, 21, 24	13, 19, 20, 23	9, 15	
2^{26-18}	768	11, 15, 16, 18, 20, 24	9, 13		
	1024	9, 10, 11, 13, 14, 17, 20	13, 16, 22, 24, 25	17	
	1280	10, 19, 20, 22, 23, 25	12, 15, 16, 20, 24	12, 16	
2^{27-19}	768	9, 13, 17, 18, 23, 26	12, 19, 23		
	1024	9, 13, 14, 18, 22, 24, 25	9, 17, 18, 19, 22, 26	15, 17	
	1280	9, 10, 11, 15, 16, 20, 22, 24	11, 13, 19, 20, 24	16, 22, 24, 25	
2^{28-20}	768	9, 13, 14, 19, 23, 25, 26	12, 14, 22		
	1024	10, 13, 17, 19, 22, 25, 27, 28	9, 15, 18, 19, 27	16, 19, 22, 27, 28	
	1280	10, 12, 14, 17, 21, 22, 23, 24, 25, 28	9, 11, 12, 18, 21, 22, 23, 24, 26	16, 17, 22, 25	
2^{29-21}	768	9, 10, 13, 16, 17, 18, 20, 22, 23, 24	9, 12, 14, 20		
	1024	9, 11, 16, 22, 26, 28	10, 12, 18, 23, 24, 28	10, 13, 20, 29	
	1280	10, 16, 20, 21, 22, 26, 27, 29	10, 12, 20, 21, 26, 28	11, 13, 15, 16, 17, 20	15
2^{30-22}	768	11, 15, 18, 26, 29, 30	9, 11, 12, 13, 19, 25, 30		
	1024	9, 10, 14, 15, 19, 21, 22, 23, 24, 30	10, 15, 20, 22, 24, 30	11, 12, 15, 16, 25, 30	
	1280	10, 13, 18, 19, 21, 24, 27, 30	12, 14, 19, 26, 27, 28, 29	9, 16, 22, 26, 29, 30	9, 10

Table S.17 (continued)

Parent	N	D_1	D_2	D_3	D_4
2^{31-23}	768	9, 10, 12, 17, 18, 19, 23, 27	10, 11, 14, 16, 23, 27, 30, 31		
	1024	12, 13, 18, 20, 21, 22, 24, 25, 29, 30, 31	10, 14, 16, 18, 19, 23, 24, 25, 26, 27, 28, 31	11, 13, 16, 18, 20, 27, 31	
	1280	9, 10, 11, 16, 17, 18, 20, 26	12, 13, 16, 19, 20, 21, 23, 25, 31	13, 16, 21, 22, 23, 24, 26	10, 21, 28, 29
2^{32-24}	768	11, 13, 17, 18, 20, 24, 27, 29, 32	9, 11, 16, 25, 26, 28, 29, 32		
	1024	10, 14, 15, 17, 20, 21, 22, 26, 28, 32	9, 11, 12, 13, 17, 18, 24, 25, 29	12, 15, 17, 18, 19, 22, 24	
	1280	10, 12, 14, 22, 27, 28, 30, 31	9, 15, 19, 22, 23, 30, 31, 32	9, 13, 17, 21, 29, 31, 32	9, 13, 17, 21, 29, 31, 32
2^{33-25}	768	19, 20, 26, 27, 29, 30, 33	10, 16, 20, 23, 28, 29, 31		
	1024	10, 11, 12, 14, 18, 21, 29, 30, 33	10, 16, 19, 23, 24, 25, 28, 30, 31	15, 19, 21, 24, 25, 26, 30, 31, 33	
	1280	9, 14, 16, 17, 24, 28, 29, 30, 33	13, 16, 18, 20, 23, 26, 28, 29, 31	9, 10, 12, 13, 14, 15, 18, 21, 23, 24, 29	14, 19, 20, 25, 28, 30
2^{34-26}	768	13, 14, 16, 18, 26, 27, 28, 29, 30, 32, 33	12, 13, 16, 18, 25, 27, 30, 31, 32		
	1024	12, 17, 20, 21, 22, 23, 26, 28, 32	17, 18, 19, 23, 25, 26, 31, 32	12, 18, 19, 20, 21, 22, 25, 28, 31	
	1280	10, 11, 14, 15, 16, 19, 26, 28, 32, 33	9, 10, 15, 24, 28, 31, 33	18, 20, 25, 26, 27, 28, 33, 34	9, 11, 14, 25, 27, 28, 29
2^{35-27}	768	10, 19, 21, 22, 23, 25, 26, 29, 32, 35	13, 17, 26, 28, 30, 31, 32, 34		

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Table S.17 (continued)

Parent	N	D_1	D_2	D_3	D_4
	1024	12, 17, 18, 19, 24, 26, 28, 30, 31, 33	11, 16, 21, 22, 24, 25, 26, 33	10, 11, 17, 20, 26, 28, 31, 32	
	1280	9, 10, 17, 18, 20, 22, 26, 28, 30, 32, 34	11, 12, 16, 20, 21, 22, 24, 26, 28	10, 11, 13, 17, 20, 23, 26, 27, 33	13, 19, 20, 22, 23, 30, 32
2^{36-28}	768	11, 13, 17, 24, 25, 27, 28, 33, 34	10, 16, 18, 20, 24, 28, 33, 34, 36		
	1024	11, 12, 15, 16, 17, 19, 20, 22, 23, 29, 30, 33	13, 14, 15, 16, 24, 29, 30, 32, 33, 36	14, 16, 17, 19, 21, 25, 27, 31, 35, 36	
	1280	9, 12, 13, 18, 20, 27, 30, 31, 35, 36	10, 12, 14, 16, 17, 20, 23, 30, 31, 34, 36	9, 11, 14, 15, 17, 18, 20, 21, 23, 24, 28, 32, 33	9, 15, 17, 20, 21, 25, 26

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