First Online Supplement:

A Glimpse at The TMA(∞) Representation of The TARMA

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Define for convenience the vectors

$$\mathbf{vl}_p((l_1,\ldots,l_h)|(i_1,\ldots,i_h)) := (\mathbf{0}_{l_1-1}, i_1, \mathbf{0}_{l_2-l_1-1}, i_2 \ldots, i_h, \mathbf{0}_{p-l_h}),\ h = 1,\ldots,p:$$

$$f_x(X_t) = f_x(I_t) + \sum_{h_q^{(1)}=1}^{q} \sum_{\substack{n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)})=1 \\ n^{(1)}(1)<\ldots<n^{(1)}(h_q^{(1)})}}^{q} \sum_{j^{(1)}_{n^{(1)}(1)}(1),\ldots,j^{(1)}_{n^{(1)}(h_q^{(1)})}(h_q^{(1)})=v_1,\ldots,v_k}$$

$$d^{h_q^{(1)}} f_x(I_t^{(\mathbf{0}_p|\mathbf{vl}_q((n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)}))|(j^{(1)}_{n^{(1)}(1)}(1),\ldots,j^{(1)}_{n^{(1)}(h_q^{(1)})}(h_q^{(1)}))))}) \cdot \left(\prod_{r=1}^{h_q^{(1)}} f_{j^{(1)}_{n^{(1)}(r)}(r)}(I_{t-n^{(1)}(r)})\right) +$$

$$\sum_{h_p^{(1)}=1}^{p} \sum_{h_q^{(1)}=0}^{q} \sum_{\substack{l^{(1)}(1),\ldots,l^{(1)}(h_p^{(1)})=1 \\ l^{(1)}(1)<\ldots<l^{(1)}(h_p^{(1)})}}^{p} \sum_{\substack{n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)})=1 \\ n^{(1)}(1)<\ldots<n^{(1)}(h_q^{(1)})}}^{q}$$

$$\sum_{i^{(1)}_{l^{(1)}(1)}(1),\ldots,i^{(1)}_{l^{(1)}(h_p^{(1)})}(h_p^{(1)}),\ j^{(1)}_{n^{(1)}(1)}(1),\ldots,j^{(1)}_{n^{(1)}(h_q^{(1)})}(h_q^{(1)})=v_1,\ldots,v_k}$$

$$d^{h_p^{(1)}+h_q^{(1)}} f_x(I_t^{(\mathbf{vl}_p((l^{(1)}(1),\ldots,l^{(1)}(h_p^{(1)}))|(i^{(1)}_{l^{(1)}(1)}(1),\ldots,i^{(1)}_{l^{(1)}(h_p^{(1)})}(h_p^{(1)})))|\mathbf{vl}_q((n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)}))|(j^{(1)}_{n^{(1)}(1)}(1),\ldots)))})$$

$$\left(\prod_{r=1}^{h_p^{(1)}} f_{i^{(1)}_{l^{(1)}(r)}(r)}(I_{t-l^{(1)}(r)})\right) \left(\prod_{r=1}^{h_q^{(1)}} f_{j^{(1)}_{n^{(1)}(r)}(r)}(I_{t-n^{(1)}(r)})\right) +$$

$$\sum_{h_p^{(1)}=1}^{p} \sum_{h_q^{(1)}=0}^{q} \sum_{\substack{l^{(1)}(1),\ldots,l^{(1)}(h_p^{(1)})=1 \\ l^{(1)}(1)<\ldots<l^{(1)}(h_p^{(1)})}}^{p} \sum_{\substack{n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)})=1 \\ n^{(1)}(1)<\ldots<n^{(1)}(h_q^{(1)})}}^{q}$$

$$\sum_{w_q^{(2)}=1}^{h_p^{(1)}} \sum_{\substack{\tau^{(2)}(1),\ldots,\tau^{(2)}(w_q^{(2)})=1 \\ \tau^{(2)}(1)<\ldots<\tau^{(2)}(w_q^{(2)})}}^{h_p^{(1)}} \sum_{\substack{h_q^{(2)}\left(l^{(1)}(\tau^{(2)}(r))\right)=1 \\ r=1,\ldots,w_q^{(2)}}}^{q}$$

$$\sum_{\substack{n^{(2)}(1),\ldots,n^{(2)}(h_q^{(2)}(l^{(1)}(\tau^{(2)}(r))))=1 \\ n^{(2)}(1)<\ldots<n^{(2)}(h_q^{(2)}(l^{(1)}(\tau^{(2)}(r)))), \\ r=1,\ldots,w_q^{(2)}}}^{q}$$

$$\sum_{\substack{i^{(1)}_{l^{(1)}(r)}(r)=v_1,\ldots,v_k \\ r=1,\ldots,h_p^{(1)}}} \sum_{\substack{j^{(1)}_{n^{(1)}(r)}(r)=v_1,\ldots,v_k \\ r=1,\ldots,h_q^{(1)}}} \sum_{\substack{j^{(2)}_{n^{(2)}(rr)}(rr)=v_1,\ldots,v_k \\ rr=1,\ldots,h_q^{(2)}(l^{(1)}(\tau^{(2)}(r))), \\ r=1,\ldots,w_q^{(2)}}}$$

$$\left\{ d^{h_p^{(1)}+h_q^{(1)}} f_x(I_t^{(\mathbf{vl}_p((l^{(1)}(1),\ldots,l^{(1)}(h_p^{(1)}))|(i^{(1)}_{l^{(1)}(1)}(1),\ldots,i^{(1)}_{l^{(1)}(h_p^{(1)})}(h_p^{(1)})))|\mathbf{vl}_q((n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)}))|(j^{(1)}_{n^{(1)}(1)}(1),\ldots)))}) \right\}$$

$$\left\{ \prod_{r=1}^{w_q^{(2)}} d^{h_q^{(2)}(l^{(1)}(\tau^{(2)}(r)))} f_{i^{(1)}_{l^{(1)}(\tau^{(2)}(r))}(\tau^{(2)}(r))} \right.$$

$$\left. (I_{t-l^{(1)}(\tau^{(2)}(r))}^{\left(\mathbf{0}_p|\mathbf{vl}_q((n^{(2)}(1),\ldots,n^{(2)}(h_q^{(2)}(l^{(1)}(\tau^{(2)}(r)))))|(j^{(2)}_{n^{(2)}(1)}(1),\ldots,j^{(2)}_{n^{(2)}\left(h_q^{(2)}(l^{(1)}(\tau^{(2)}(r)))\right)}(h_q^{(2)}(\ldots)))))\right)}) \right\}$$

$$\left(\prod_{r=1}^{h_q^{(1)}} f_{j^{(1)}_{n^{(1)}(r)}(r)}\left(I_{t-n^{(1)}(r)}\right)\right) \left(\prod_{r=1}^{w_q^{(2)}} \prod_{rr=1}^{h_q^{(2)}\left(l^{(1)}(\tau^{(2)}(r))\right)} f_{j^{(2)}_{n^{(2)}(rr)}(rr)} \left(I_{t-l^{(1)}(\tau^{(2)}(r))-n^{(2)}(rr)}\right)\right)$$

$$\prod_{\substack{r=1 \\ r\neq\tau^{(2)}(\rho), \\ \rho=1,\ldots,w_q^{(2)}}}^{h_p^{(1)}} f_{i^{(1)}_{l^{(1)}(r)}(r)}\left(I_{t-l^{(1)}(r)}\right) \;+$$

$$\sum_{h_p^{(1)}=1}^{p} \sum_{h_q^{(1)}=0}^{q} \sum_{\substack{l^{(1)}(1),\ldots,l^{(1)}(h_p^{(1)})=1 \\ l^{(1)}(1)<\ldots<l^{(1)}(h_p^{(1)})}}^{p} \sum_{\substack{n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)})=1 \\ n^{(1)}(1)<\ldots<n^{(1)}(h_q^{(1)})}}^{q}$$

$$\sum_{w_p^{(2)}=1}^{h_p^{(1)}} \sum_{\substack{s^{(2)}(1),\ldots,s^{(2)}(w_p^{(2)})=1 \\ s^{(2)}(1)<\ldots<s^{(2)}(w_p^{(2)})}}^{h_p^{(1)}} \sum_{w_q^{(2)}=0}^{h_p^{(1)}-w_p^{(2)}} \sum_{\substack{\tau^{(2)}(1),\ldots,\tau^{(2)}(w_q^{(2)})=1 \\ \tau^{(2)}(1)<\ldots<\tau^{(2)}(w_q^{(2)}), \\ \tau^{(2)}(r_1)\neq s^{(2)}(r_2), \\ r_1=1,\ldots,w_q^{(2)}, \\ r_2=1,\ldots,w_p^{(2)}}}^{h_p^{(1)}}$$

$$\sum_{\substack{h_p^{(2)}\left(l^{(1)}(s^{(2)}(r))\right)=1 \\ r=1,\ldots,w_p^{(2)}}}^{p} \sum_{\substack{h_q^{(2)}\left(l^{(1)}(s^{(2)}(r))\right)=0 \\ r=1,\ldots,w_p^{(2)}}}^{q} \sum_{\substack{h_q^{(2),1}\left(l^{(1)}(\tau^{(2)}(r))\right)=1 \\ r=1,\ldots,w_q^{(2)}}}^{q}$$

$$\sum_{\substack{l^{(2)}(1),\ldots,l^{(2)}(h_p^{(2)}(l^{(1)}(s^{(2)}(r))))=1 \\ l^{(2)}(1)<\ldots<l^{(2)}(h_p^{(2)}(l^{(1)}(s^{(2)}(r)))), \\ r=1,\ldots,w_p^{(2)}}}^{p} \sum_{\substack{n^{(2)}(1),\ldots,n^{(2)}(h_q^{(2)}(l^{(1)}(s^{(2)}(r))))=1 \\ n^{(2)}(1)<\ldots<n^{(2)}(h_q^{(2)}(l^{(1)}(s^{(2)}(r)))), \\ r=1,\ldots,w_p^{(2)}}}^{q}$$

$$\sum_{\substack{n^{(2),1}(1),\ldots,n^{(2),1}(h_q^{(2),1}(l^{(1)}(\tau^{(2)}(r))))=1 \\ n^{(2),1}(1)<\ldots<n^{(2),1}(h_q^{(2),1}(l^{(1)}(\tau^{(2)}(r)))), \\ r=1,\ldots,w_q^{(2)}}}^{q}$$

$$\sum_{\substack{i^{(1)}_{l^{(1)}(r)}(r) = v_1, \ldots, v_k \\ r = 1, \ldots, h_p^{(1)}}} \quad \sum_{\substack{j^{(1)}_{n^{(1)}(r)}(r) = v_1, \ldots, v_k \\ r = 1, \ldots, h_q^{(1)}}}$$

$$\sum_{\substack{i^{(2)}_{l^{(2)}(rr)}(rr) = v_1, \ldots, v_k \\ rr = 1, \ldots, h_p^{(2)}(l^{(1)}(s^{(2)}(r))), \\ r = 1, \ldots, w_p^{(2)}}}$$

$$\sum_{\substack{j^{(2)}_{n^{(2)}(rr)}(rr) = v_1, \ldots, v_k \\ rr = 1, \ldots, h_q^{(2)}(l^{(1)}(s^{(2)}(r))), \\ r = 1, \ldots, w_p^{(2)}}} \quad \sum_{\substack{j^{(2),1}_{n^{(2),1}(rr)}(rr) = v_1, \ldots, v_k \\ rr = 1, \ldots, h_q^{(2),1}(l^{(1)}(\tau^{(2)}(r))), \\ r = 1, \ldots, w_q^{(2)}}}$$

$$\left\{ d^{h_p^{(1)}+h_q^{(1)}} f_x(I_t^{(\mathbf{vl}_p((l^{(1)}(1),\ldots,l^{(1)}(h_p^{(1)}))|(i^{(1)}_{l^{(1)}(1)}(1),\ldots,i^{(1)}_{l^{(1)}(h_p^{(1)})}(h_p^{(1)})))|\mathbf{vl}_q((n^{(1)}(1),\ldots,n^{(1)}(h_q^{(1)}))|(j^{(1)}_{n^{(1)}(1)}(1),\ldots)))}) \right\}$$

$$\left\{ \prod_{r=1}^{w_p^{(2)}} d^{h_p^{(2)}(l^{(1)}(s^{(2)}(r)))+h_q^{(2)}\left(l^{(1)}(s^{(2)}(r))\right)} f_{i^{(1)}_{l^{(1)}(s^{(2)}(r))}(s^{(2)}(r))} \right.$$
$$\left. (I_{t-l^{(1)}(s^{(2)}(r))}^{(\mathbf{vl}_p((l^{(2)}(1),\ldots,l^{(2)}(h_p^{(2)}(l^{(1)}(s^{(2)}(r)))))|(i^{(2)}_{l^{(2)}(1)}(1),\ldots))|\mathbf{vl}_q((n^{(2)}(1),\ldots,n^{(2)}(h_q^{(2)}(l^{(1)}(s^{(2)}(r)))))|))}) \right\}$$

$$\left\{ \prod_{r=1}^{w_q^{(2)}} d^{h_q^{(2),1}(l^{(1)}(\tau^{(2)}(r)))} \; f_{i^{(1)}_{l^{(1)}(\tau^{(2)}(r))}(\tau^{(2)}(r))} \right.$$
$$\left. (I_{t-l^{(1)}(\tau^{(2)}(r))}^{(\mathbf{0}_p|\mathbf{vl}_q((n^{(2),1}(1),\ldots,n^{(2),1}(h_q^{(2),1}(l^{(1)}(\tau^{(2)}(r)))))|(j^{(2),1}_{n^{(2),1}(1)}(1),\ldots,j^{(2),1}_{n^{(2),1}\left(h_q^{(2),1}(l^{(1)}(\tau^{(2)}(r)))\right)}\left(h_q^{(2),1}(\ldots)\right))))}) \right\}$$

$$\left(\prod_{r=1}^{h_q^{(1)}} f_{j^{(1)}_{n^{(1)}(r)}(r)}\left(I_{t-n^{(1)}(r)}\right) \right) \quad \prod_{\substack{r = 1 \\ r \neq s^{(2)}(\rho_1), \tau^{(2)}(\rho_2), \\ \rho_1 = 1, \ldots, w_p^{(2)}, \\ \rho_2 = 1, \ldots, w_q^{(2)}}}^{h_p^{(1)}} f_{i^{(1)}_{l^{(1)}(r)}(r)}\left(I_{t-l^{(1)}(r)}\right)$$

$$\left(\prod_{r=1}^{w_p^{(2)}} \prod_{rr=1}^{h_q^{(2)}\left(l^{(1)}(s^{(2)}(r))\right)} f_{j^{(2)}_{n^{(2)}(rr)}(rr)} \left(I_{t-l^{(1)}(s^{(2)}(r))-n^{(2)}(rr)}\right) \right)$$

$$\left(\prod_{r=1}^{w_q^{(2)}} \prod_{rr=1}^{h_q^{(2),1}\left(l^{(1)}(\tau^{(2)}(r))\right)} f_{j^{(2),1}_{n^{(2),1}(rr)}(rr)} \left(I_{t-l^{(1)}(\tau^{(2)}(r))-n^{(2),1}(rr)}\right) \right)$$

$$\left(\prod_{r=1}^{w_p^{(2)}} \prod_{rr=1}^{h_p^{(2)}\left(l^{(1)}(s^{(2)}(r))\right)} f_{i^{(2)}_{l^{(2)}(rr)}(rr)}\left(I_{t-l^{(1)}(s^{(2)}(r))-l^{(2)}(rr)}\right)\right) + \dots$$

and it keeps going in a way that should be obvious by now.