Supplementary Material for "Partition Weighted Approach for Estimating the Marginal Posterior Density with Applications"

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Appendix A: Proof of Theorem 3.1

Using the Cauchy-Schwarz inequality, we have

$$1 = \left(\int_{\tilde{\Theta}_{\boldsymbol{\xi}}} \frac{w(\boldsymbol{\theta}|\boldsymbol{\xi})}{\sqrt{q(\boldsymbol{\theta}, \boldsymbol{\xi})}} \sqrt{q(\boldsymbol{\theta}, \boldsymbol{\xi})} d\boldsymbol{\theta} \right)^2 \leq \int_{\tilde{\Theta}_{\boldsymbol{\xi}}} \frac{w^2(\boldsymbol{\theta}|\boldsymbol{\xi})}{q(\boldsymbol{\theta}, \boldsymbol{\xi})} d\boldsymbol{\theta} \int_{\tilde{\Theta}_{\boldsymbol{\xi}}} q(\boldsymbol{\theta}, \boldsymbol{\xi}) d\boldsymbol{\theta}.$$

Subsequently, we obtain

$$\begin{aligned}
&\frac{1}{\int_{\tilde{\Theta}_{\boldsymbol{\xi}}} q(\boldsymbol{\theta}, \boldsymbol{\xi}) d\boldsymbol{\theta}} \leq \int_{\tilde{\Theta}_{\boldsymbol{\xi}}} \frac{w^2(\boldsymbol{\theta}|\boldsymbol{\xi})}{q(\boldsymbol{\theta}, \boldsymbol{\xi})} d\boldsymbol{\theta} \\
\Rightarrow &\int \left[q(\boldsymbol{\theta}_0, \boldsymbol{\xi}) \pi(\boldsymbol{\theta}_0, \boldsymbol{\xi}|D) \frac{1}{\int_{\tilde{\Theta}_{\boldsymbol{\xi}}} q(\boldsymbol{\theta}, \boldsymbol{\xi}) d\boldsymbol{\theta}} \right] d\boldsymbol{\xi} \leq \int \left[q(\boldsymbol{\theta}_0, \boldsymbol{\xi}) \pi(\boldsymbol{\theta}_0, \boldsymbol{\xi}|D) \int_{\tilde{\Theta}_{\boldsymbol{\xi}}} \frac{w^2(\boldsymbol{\theta}|\boldsymbol{\xi})}{q(\boldsymbol{\theta}, \boldsymbol{\xi})} d\boldsymbol{\theta} \right] d\boldsymbol{\xi} \\
\Rightarrow &\mathrm{Var}_{w_{\mathrm{opt}}}\{\hat{\pi}_t(\boldsymbol{\theta}_0|D)\} \leq \mathrm{Var}_w\{\hat{\pi}_t(\boldsymbol{\theta}_0|D)\},
\end{aligned}$$

which completes the proof.

Appendix B: An Additional Table

S1. Sensitivity analysis of the conditional working parameter space when setting $r = \sqrt{\chi^2_{3,0.5}}$.

	$\log \pi_{t_3}(0,0,0) = -3.911$			$\log \pi_{t_3}(1,0,0) = -4.326$		
PWMDE	Mean	MCSE	RMSE	Mean	MCSE	RMSE
K=5	-3.911	0.010	0.010	-4.325	0.010	0.010
K=10	-3.911	0.010	0.010	-4.325	0.009	0.009
K=15	-3.911	0.010	0.010	-4.325	0.009	0.009
K=20	-3.911	0.010	0.010	-4.325	0.009	0.009
	$\log \pi_{t_3}(0,1,0) = -4.421$			$\log \pi_{t_3}(0,0,1) = -4.326$		
PWMDE	Mean	MCSE	RMSE	Mean	MCSE	RMSE
K=5	-4.420	0.010	0.010	-4.325	0.011	0.011
K=10	-4.421	0.009	0.009	-4.325	0.011	0.011
K=15	-4.421	0.009	0.009	-4.325	0.011	0.011
K=20	-4.421	0.009	0.009	-4.325	0.010	0.010