Appendix A: Model Development

The following sections detail the development of the model equations presented in the article, and provides a dimensional analysis. Source code can be found at: https://github.com/margaretgarcia/PerCapWaterUse. For an interactive web version of this model see: https://mgarcia.shinyapps.io/demandmodel/.

Price

Per Griffin (2006, p31) the price elasticity of demand, ε, is given by the following equation, where D is the per capita water use and R is the unit price:

$$\varepsilon = \frac{\Delta D / D}{\Delta R / R} \tag{9}$$

Rearranging this equation, the change in demand is given by the following equation:

$$\Delta D = \varepsilon D \frac{\Delta R}{R} \tag{10}$$

Adding subscript t to denote time and subscript a to denote the average per capita water use (i.e. no seasonal variation), the equation becomes:

$$D_{a,t+1} - D_{a,t} = \varepsilon D_{a,t} \frac{(R_t - R_{t-1})}{R_t} \tag{11}$$

Rearranging this equation, the per capita water use at time t+1 is given by the following equation:

$$D_{a,t+1} = \varepsilon D_{a,t} \frac{(R_t - R_{t-1})}{R_t} + D_{a,t} \tag{12}$$

In other words:

$$per\ cap.use\ at\ t+1 = (price\ elasticity) * (per\ cap.use\ at\ t)\frac{(Unit\ Price\ at\ t - Unit\ Price\ at\ t-1)}{Unit\ Price\ at\ t}$$

(13)

To confirm the consistency of the equation the following dimensional analysis is presented:

$$[lpcpd] = [-][lpcpd]\frac{[\$-\$]}{[\$]} \qquad (14)$$

Code Change and Population Growth

The mass balance equation for a conserved solute in a liquid is used as a starting point where C_t is concentration, V_t is volume and C_{in} is inflow concentration, all at time t:

$$C_{t+1}V_{t+1} = C_tV_t + C_{in}(V_{t+1} - V_t) \qquad (15)$$

Changes in per capita water demand can be computed analogously. Population (P_t) is analogous to volume, per capita water use (D_t) to concentration and average water use for code compliance (D_{code}) to inflow concentration.

$$D_{t+1}P_{t+1} = D_tP_t + D_{code}(P_{t+1} - P_t) \qquad (16)$$

Rearranging the equation becomes:

$$D_{t+1} = \frac{D_tP_t + D_{code}(P_{t+1} - P_t)}{P_{t+1}} \qquad (17)$$

To make the equation consistent with the form of the price equation, the equation is rearranged to add the per capita water use at time t term, and subscribe a is added, yielding the final form of the equation:

$$D_{a,t+1} = D_{a,t}\left[\frac{D_{a,t}P_t + D_{code}(P_{t+1} - P_t)}{D_{a,t}P_{t+1}} - 1\right] + D_{a,t} \qquad (18)$$

In other words:

$$per\ cap.use\ at\ t+1 = per\ cap.use\ at\ t$$

$$\left\{\frac{[per\ cap.use\ at\ t*pop.at\ t - code\ per\ cap.use*(pop.at\ t+1 - pop.at\ t]}{per\ cap.use\ at\ t*pop.at\ t+1} - 1\right\} + \ (per\ cap.use\ at\ t)$$

(19)

To confirm the consistency of the equation the following dimensional analysis is presented:

$$[lpcpd] = [lpcpd]\left\{\frac{[lpcpd*people - lpcpd*people]}{[lpcpd*people]} - [-]\right\} + \ lpcpd \qquad (20)$$

Water Stress Response

As detailed in the text, the water stress response equation builds upon the theoretical model introduced by Garcia et al. (2016) and recently applied to model San Francisco Bay area water use by Gonzales and Ajami (2017):

$$\frac{dD}{dt} = -D_t M_t \alpha \left(1 - \frac{D_{min}}{D_t}\right) \qquad (21)$$

Rearranging this equation to be consistent with the forms of the price and code change equations, and adding subscript a, yields:

$$D_{a,t+1} = D_{a,t} M_t \alpha \left(\frac{D_{min}}{D_{a,t}} - 1\right) + D_{a,t} \qquad (22)$$

In other words:

$$per\ cap.use\ at\ t+1 = (per\ cap.use\ at\ t) * (salience) * (response\ intensity)$$

$$\left[\frac{minimum\ per\ cap.use}{per\ cap.use\ at\ t} - 1\right] + per\ cap.use\ at\ t \qquad (23)$$

To confirm the consistency of the equation the following dimensional analysis is presented:

$$[lpcpd] = [lpcpd][-][-]\left\{\frac{[lpcpd]}{[lpcpd]} - [-]\right\} + \ lpcpd \qquad (24)$$

Seasonal Water Use

The equations presented above model average annual per capita water use change. To utilize these equations at a monthly or smaller time-step the seasonal pattern of water use must be either removed from the data or added to the model. Here we choose to model the seasonal patterns so that the impact of temperature trends could be explored in the future.

Following Nash and Barsi (1982) analogously, we relate deviations in daily per capita water use to monthly variations in temperature. First, we compute monthly deviations in water use, relative to the annual average:

$$MD_t = Dobs_t - \frac{1}{12}\sum_{i=0}^{11} Dobs_{t-i} \qquad (26)$$

We then perform a linear regression with the monthly deviations, MD, as the dependent variable and the product of the average annual per capita use and observed monthly average temperature as the independent variable. The product of average annual per capita use and observed monthly average temperature is used because the amplitude of the seasonal cycle has decreased as consumption has declined due to the reduction in outdoor water usage. (In other words, summer water use has decline more than winter water use and this alters the strength of the seasonal signal.) The regression results in the following equation where T is observed monthly average temperature, β1 is the regression coefficient and β0 is the regression intercept:

$$MD_t = \beta_1 T_t * \frac{1}{12}\sum_{i=0}^{11} D_{a,t-i} \ + \beta_0 \qquad (27)$$

Replacing MD with Ds,t – Da,t and rearranging the equation

$$D_{s,t} = D_{a,t} + \beta_1 T_t \frac{1}{12} \sum_{t-11}^{t} D_{a,t} + \beta_0 \qquad (28)$$