Familywise Error Rate Controlling Procedures for Discrete Data - Supplementary Materials

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August 3, 2019

S1 Results from Independence Simulation Settings

The simulation results under the independence setting for stepwise procedures comparisons are shown in this section. Tables S1, S2 and Tables S3, S4 respectively provide the results of numerical comparisons of single-step procedures using Fisher and Binomial Exact Tests (as plotted in Figures 1 and 2 in main paper).

Table S1: Simulated FWER comparisons for single-step procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.2 (Tarone), and the conventional Sidak (Sidak) and Bonferroni (Bonf) procedures.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$, $\pi_0 = 0.2$	MBonf	0.0025	0.0060	0.0035	0.0075	0.0075	0.0095
	Tarone	0.0015	0.0030	0.0015	0.0055	0.0045	0.0085
	Sidak	0.0010	0.0030	0.0015	0.0055	0.0045	0.0085
	Bonf	0.0010	0.0030	0.0015	0.0055	0.0045	0.0085
$m = 5$, $\pi_0 = 0.4$	MBonf	0.0100	0.0135	0.0145	0.0160	0.0160	0.0200
	Tarone	0.0080	0.0055	0.0105	0.0120	0.0125	0.0145
	Sidak	0.0030	0.0055	0.0105	0.0120	0.0135	0.0145
	Bonf	0.0030	0.0055	0.0105	0.0120	0.0125	0.0145
$m = 5$, $\pi_0 = 0.6$	MBonf	0.0110	0.0185	0.0185	0.0225	0.0245	0.0270
	Tarone	0.0060	0.0090	0.0095	0.0180	0.0150	0.0175
	Sidak	0.0035	0.0090	0.0095	0.0180	0.0160	0.0175
	Bonf	0.0035	0.0090	0.0095	0.0180	0.0150	0.0175
$m = 5$, $\pi_0 = 0.8$	MBonf	0.0190	0.0280	0.0265	0.0315	0.0370	0.0360
	Tarone	0.0125	0.0135	0.0170	0.0225	0.0250	0.0260
	Sidak	0.0030	0.0125	0.0170	0.0225	0.0260	0.0260
	Bonf	0.0030	0.0125	0.0170	0.0225	0.0250	0.0260
$m = 10$, $\pi_0 = 0.2$	MBonf	0.0025	0.0090	0.0075	0.0100	0.0075	0.0085
	Tarone	0.0010	0.0035	0.0045	0.0060	0.0055	0.0060
	Sidak	0.0005	0.0035	0.0045	0.0065	0.0060	0.0065
	Bonf	0.0005	0.0035	0.0045	0.0060	0.0055	0.0060
$m = 10$, $\pi_0 = 0.4$	MBonf	0.0060	0.0140	0.0170	0.0130	0.0170	0.0165
	Tarone	0.0030	0.0065	0.0105	0.0100	0.0075	0.0110
	Sidak	0.0020	0.0060	0.0105	0.0100	0.0095	0.0115
	Bonf	0.0020	0.0060	0.0105	0.0100	0.0075	0.0110
$m = 10$, $\pi_0 = 0.6$	MBonf	0.0130	0.0310	0.0245	0.0225	0.0230	0.0340
	Tarone	0.0045	0.0170	0.0175	0.0135	0.0140	0.0220
	Sidak	0.0025	0.0165	0.0175	0.0135	0.0150	0.0225
	Bonf	0.0025	0.0165	0.0175	0.0135	0.0140	0.0220
$m = 10$, $\pi_0 = 0.8$	MBonf	0.0200	0.0290	0.0320	0.0345	0.0365	0.0380
	Tarone	0.0110	0.0145	0.0170	0.0220	0.0225	0.0250
	Sidak	0.0065	0.0140	0.0170	0.0225	0.0245	0.0250
	Bonf	0.0065	0.0140	0.0170	0.0220	0.0225	0.0250
$m = 15$, $\pi_0 = 0.2$	MBonf	0.0025	0.0075	0.0050	0.0085	0.0095	0.0130
	Tarone	0.0010	0.0040	0.0010	0.0055	0.0065	0.0070
	Sidak	0.0005	0.0040	0.0010	0.0055	0.0065	0.0070
	Bonf	0.0005	0.0040	0.0010	0.0055	0.0065	0.0070
$m = 15$, $\pi_0 = 0.4$	MBonf	0.0075	0.0120	0.0150	0.0220	0.0185	0.0175
	Tarone	0.0035	0.0080	0.0050	0.0130	0.0120	0.0100
	Sidak	0.0015	0.0060	0.0050	0.0130	0.0120	0.0100
	Bonf	0.0015	0.0060	0.0050	0.0130	0.0120	0.0100
$m = 15$, $\pi_0 = 0.6$	MBonf	0.0105	0.0275	0.0255	0.0280	0.0285	0.0320
	Tarone	0.0050	0.0125	0.0075	0.0120	0.0170	0.0215
	Sidak	0.0015	0.0105	0.0075	0.0135	0.0170	0.0215
	Bonf	0.0015	0.0105	0.0075	0.0120	0.0170	0.0215
$m = 15$, $\pi_0 = 0.8$	MBonf	0.0240	0.0300	0.0260	0.0355	0.0355	0.0370
	Tarone	0.0080	0.0190	0.0120	0.0175	0.0170	0.0205
	Sidak	0.0025	0.0140	0.0120	0.0200	0.0170	0.0205
	Bonf	0.0025	0.0140	0.0120	0.0175	0.0170	0.0205

Table S2: Simulated minimal power comparisons for single-step procedures with independent p-values generated from Fisher's exact test statistics, including Procedure 3.1 (MBonf), Procedure 2.2 (Tarone), and the conventional Sidak (Sidak) and Bonferroni (Bonf) procedures.

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$, $\pi_0 = 0.2$	MBonf	0.2550	0.5060	0.6855	0.8195	0.9145	0.9505
	Tarone	0.1945	0.3900	0.5775	0.7680	0.8655	0.9275
	Sidak	0.1125	0.3825	0.5850	0.7680	0.8710	0.9340
	Bonf	0.1125	0.3825	0.5765	0.7680	0.8655	0.9275
$m = 5$, $\pi_0 = 0.4$	MBonf	0.2070	0.4295	0.5875	0.7310	0.8350	0.8975
	Tarone	0.1635	0.3180	0.4750	0.6440	0.7835	0.8600
	Sidak	0.0850	0.3065	0.4865	0.6440	0.7885	0.8675
	Bonf	0.0850	0.3065	0.4735	0.6440	0.7835	0.8600
$m = 5$, $\pi_0 = 0.6$	MBonf	0.1480	0.3120	0.4485	0.5685	0.7110	0.7765
	Tarone	0.1180	0.2400	0.3515	0.4870	0.6300	0.7250
	Sidak	0.0580	0.2275	0.3615	0.4870	0.6370	0.7350
	Bonf	0.0580	0.2275	0.3510	0.4870	0.6300	0.7250
$m = 5$, $\pi_0 = 0.8$	MBonf	0.0810	0.1665	0.2650	0.3575	0.4510	0.5240
	Tarone	0.0660	0.1235	0.2035	0.2965	0.3755	0.4665
	Sidak	0.0305	0.1155	0.2080	0.2965	0.3800	0.4750
	Bonf	0.0305	0.1155	0.2035	0.2965	0.3755	0.4665
$m = 10$, $\pi_0 = 0.2$	MBonf	0.3140	0.6070	0.8265	0.9260	0.9725	0.9950
	Tarone	0.1980	0.4625	0.7405	0.8615	0.9400	0.9810
	Sidak	0.1490	0.4605	0.7405	0.8665	0.9410	0.9830
	Bonf	0.1490	0.4605	0.7405	0.8615	0.9400	0.9810
$m = 10$, $\pi_0 = 0.4$	MBonf	0.2525	0.5180	0.7200	0.8510	0.9370	0.9595
	Tarone	0.1760	0.3785	0.6130	0.7685	0.8970	0.9355
	Sidak	0.1270	0.3700	0.6130	0.7760	0.8975	0.9390
	Bonf	0.1270	0.3700	0.6130	0.7685	0.8970	0.9355
$m = 10$, $\pi_0 = 0.6$	MBonf	0.1980	0.3825	0.5815	0.7060	0.8350	0.8990
	Tarone	0.1235	0.2460	0.4700	0.6030	0.7705	0.8485
	Sidak	0.0730	0.2390	0.4695	0.6110	0.7715	0.8585
	Bonf	0.0730	0.2390	0.4695	0.6030	0.7705	0.8485
$m = 10$, $\pi_0 = 0.8$	MBonf	0.1165	0.2180	0.3520	0.4790	0.5895	0.6850
	Tarone	0.0835	0.1410	0.2605	0.3780	0.4995	0.6105
	Sidak	0.0435	0.1325	0.2605	0.3865	0.5000	0.6225
	Bonf	0.0435	0.1325	0.2605	0.3780	0.4995	0.6105
$m = 15$, $\pi_0 = 0.2$	MBonf	0.3475	0.6695	0.8570	0.9630	0.9920	0.9975
	Tarone	0.2615	0.4980	0.7325	0.9025	0.9785	0.9925
	Sidak	0.1400	0.4790	0.7320	0.9055	0.9785	0.9925
	Bonf	0.1400	0.4790	0.7320	0.9020	0.9785	0.9925
$m = 15$, $\pi_0 = 0.4$	MBonf	0.2855	0.5660	0.7765	0.9020	0.9615	0.9890
	Tarone	0.2155	0.4175	0.6310	0.8090	0.9265	0.9730
	Sidak	0.0970	0.3865	0.6275	0.8150	0.9270	0.9730
	Bonf	0.0970	0.3865	0.6275	0.8090	0.9265	0.9730
$m = 15$, $\pi_0 = 0.6$	MBonf	0.2105	0.4400	0.6540	0.7990	0.9055	0.9525
	Tarone	0.1575	0.3125	0.4925	0.6845	0.8320	0.9160
	Sidak	0.0785	0.2845	0.4885	0.6915	0.8350	0.9160
	Bonf	0.0785	0.2845	0.4885	0.6845	0.8320	0.9160
$m = 15$, $\pi_0 = 0.8$	MBonf	0.1215	0.2375	0.4110	0.5495	0.6680	0.7945
	Tarone	0.0790	0.1555	0.2910	0.4265	0.5785	0.7180
	Sidak	0.0300	0.1300	0.2885	0.4315	0.5780	0.7180
	Bonf	0.0300	0.1300	0.2885	0.4260	0.5780	0.7180

Table S3: Simulated FWER comparisons for single-step procedures with independent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.2 (Tarone), and the conventional Sidak (Sidak) and Bonferroni (Bonf) procedures.

		$\pi_0 = 0.2$	$\pi_0 = 0.4$	$\pi_0 = 0.6$	$\pi_0 = 0.8$
	MBonf	0.0020	0.0060	0.0075	0.0165
$m = 5$	Tarone	0.0010	0.0030	0.0055	0.0105
$\alpha = 0.05$	Sidak	0.0010	0.0020	0.0025	0.0030
	Bonf	0.0010	0.0020	0.0025	0.0030
	MBonf	0.0010	0.0045	0.0130	0.0160
$m = 10$	Tarone	0.0000	0.0010	0.0050	0.0115
$\alpha = 0.05$	Sidak	0.0000	0.0005	0.0025	0.0025
	Bonf	0.0000	0.0005	0.0025	0.0025
	MBonf	0.0010	0.0065	0.0045	0.0150
$m = 15$	Tarone	0.0000	0.0010	0.0020	0.0070
$\alpha = 0.05$	Sidak	0.0000	0.0005	0.0000	0.0000
	Bonf	0.0000	0.0005	0.0000	0.0000
	MBonf	0.0070	0.0125	0.0200	0.0365
$m = 5$	Tarone	0.0020	0.0065	0.0110	0.0285
$\alpha = 0.1$	Sidak	0.0020	0.0055	0.0065	0.0130
	Bonf	0.0020	0.0055	0.0065	0.0130
	MBonf	0.0040	0.0080	0.0275	0.0350
$m = 10$	Tarone	0.0000	0.0030	0.0165	0.0195
$\alpha = 0.1$	Sidak	0.0000	0.0015	0.0055	0.0060
	Bonf	0.0000	0.0015	0.0055	0.0060
	MBonf	0.0060	0.0155	0.0185	0.0315
$m = 15$	Tarone	0.0005	0.0060	0.0045	0.0200
$\alpha = 0.1$	Sidak	0.0000	0.0010	0.0020	0.0025
	Bonf	0.0000	0.0010	0.0020	0.0025

Table S4: Simulated minimal power comparisons for single-step procedures with independent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.2 (Tarone), and the conventional Sidak (Sidak) and Bonferroni (Bonf) procedures.

		$\pi_0 = 0.2$	$\pi_0 = 0.4$	$\pi_0 = 0.6$	$\pi_0 = 0.8$
	MBonf	0.9205	0.8805	0.7845	0.5565
$m = 5$	Tarone	0.8815	0.8240	0.7395	0.5235
$\alpha = 0.05$	Sidak	0.8735	0.8055	0.6610	0.4045
	Bonf	0.8735	0.8055	0.6610	0.4045
	MBonf	0.9850	0.9635	0.9035	0.7390
$m = 10$	Tarone	0.9470	0.9240	0.8630	0.6855
$\alpha = 0.05$	Sidak	0.9315	0.8635	0.7050	0.4775
	Bonf	0.9315	0.8635	0.7050	0.4775
	MBonf	0.9925	0.9810	0.9555	0.8210
$m = 15$	Tarone	0.9825	0.9500	0.9095	0.7845
$\alpha = 0.05$	Sidak	0.9820	0.9475	0.8560	0.6135
	Bonf	0.9820	0.9475	0.8560	0.6135
	MBonf	0.9680	0.9415	0.8615	0.6330
$m = 5$	Tarone	0.9410	0.9140	0.8240	0.5920
$\alpha = 0.1$	Sidak	0.9050	0.8375	0.7040	0.4520
	Bonf	0.9050	0.8375	0.7040	0.4520
	MBonf	0.9965	0.9875	0.9620	0.8315
$m = 10$	Tarone	0.9885	0.9660	0.9170	0.7835
$\alpha = 0.1$	Sidak	0.9870	0.9565	0.8690	0.6600
	Bonf	0.9870	0.9565	0.8690	0.6600
	MBonf	0.9995	0.9970	0.9830	0.9030
$m = 15$	Tarone	0.9960	0.9930	0.9605	0.8400
$\alpha = 0.1$	Sidak	0.9880	0.9615	0.8830	0.6515
	Bonf	0.9895	0.9635	0.8880	0.6590

Tables S5 and S6 provide numerical results of step-down procedures comparisons using Fisher Exact Test, which are also plotted as Figures S1 and S2. Tables S7 and S8 provide numerical results of step-up procedures comparisons using Fisher Exact Test, which are plotted as Figures S3 and S4.

Table S5: Simulated FWER comparisons for step-down procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$, $\pi_0 = 0.2$	MHolm	0.0030	0.0090	0.0065	0.0115	0.0150	0.0150
	TH	0.0015	0.0045	0.0030	0.0075	0.0090	0.0140
	Holm	0.0010	0.0045	0.0030	0.0075	0.0090	0.0140
$m = 5$, $\pi_0 = 0.4$	MHolm	0.0115	0.0150	0.0195	0.0215	0.0285	0.0300
	TH	0.0080	0.0070	0.0110	0.0145	0.0190	0.0235
	Holm	0.0035	0.0065	0.0110	0.0145	0.0190	0.0235
$m = 5$, $\pi_0 = 0.6$	MHolm	0.0110	0.0200	0.0220	0.0305	0.0315	0.0340
	TH	0.0060	0.0095	0.0110	0.0195	0.0195	0.0250
	Holm	0.0035	0.0090	0.0110	0.0195	0.0195	0.0250
$m = 5$, $\pi_0 = 0.8$	MHolm	0.0200	0.0300	0.0285	0.0335	0.0385	0.0405
	TH	0.0135	0.0135	0.0175	0.0250	0.0270	0.0295
	Holm	0.0030	0.0125	0.0175	0.0250	0.0270	0.0295
$m = 10$, $\pi_0 = 0.2$	MHolm	0.0025	0.0095	0.0080	0.0130	0.0130	0.0180
	TH	0.0010	0.0050	0.0045	0.0065	0.0075	0.0110
	Holm	0.0010	0.0045	0.0045	0.0065	0.0075	0.0110
$m = 10$, $\pi_0 = 0.4$	MHolm	0.0065	0.0160	0.0200	0.0185	0.0250	0.0220
	TH	0.0030	0.0065	0.0115	0.0100	0.0120	0.0150
	Holm	0.0025	0.0060	0.0115	0.0100	0.0120	0.0150
$m = 10$, $\pi_0 = 0.6$	MHolm	0.0135	0.0320	0.0275	0.0250	0.0285	0.0410
	TH	0.0045	0.0170	0.0175	0.0135	0.0185	0.0285
	Holm	0.0025	0.0165	0.0175	0.0135	0.0185	0.0285
$m = 10$, $\pi_0 = 0.8$	MHolm	0.0200	0.0290	0.0330	0.0385	0.0385	0.0410
	TH	0.0115	0.0145	0.0170	0.0225	0.0230	0.0280
	Holm	0.0065	0.0140	0.0170	0.0225	0.0230	0.0280
$m = 15$, $\pi_0 = 0.2$	MHolm	0.0025	0.0085	0.0060	0.0125	0.0160	0.0195
	TH	0.0010	0.0040	0.0020	0.0055	0.0090	0.0115
	Holm	0.0005	0.0040	0.0020	0.0055	0.0090	0.0115
$m = 15$, $\pi_0 = 0.4$	MHolm	0.0075	0.0135	0.0175	0.0265	0.0245	0.0230
	TH	0.0045	0.0080	0.0065	0.0145	0.0140	0.0145
	Holm	0.0015	0.0065	0.0065	0.0145	0.0140	0.0145
$m = 15$, $\pi_0 = 0.6$	MHolm	0.0105	0.0290	0.0280	0.0315	0.0355	0.0395
	TH	0.0050	0.0135	0.0080	0.0140	0.0180	0.0250
	Holm	0.0015	0.0105	0.0080	0.0140	0.0180	0.0250
$m = 15$, $\pi_0 = 0.8$	MHolm	0.0250	0.0310	0.0275	0.0385	0.0380	0.0400
	TH	0.0080	0.0190	0.0120	0.0185	0.0170	0.0230
	Holm	0.0025	0.0140	0.0120	0.0185	0.0170	0.0230

Table S6: Simulated minimal power comparisons for step-down procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$, $\pi_0 = 0.2$	MHolm	0.2555	0.5070	0.6855	0.8200	0.9145	0.9505
	TH	0.1945	0.3905	0.5780	0.7680	0.8660	0.9280
	Holm	0.1130	0.3830	0.5770	0.7680	0.8660	0.9280
$m = 5$, $\pi_0 = 0.4$	MHolm	0.2070	0.4300	0.5875	0.7310	0.8350	0.8975
	TH	0.1635	0.3180	0.4760	0.6440	0.7835	0.8605
	Holm	0.0850	0.3065	0.4745	0.6440	0.7835	0.8605
$m = 5$, $\pi_0 = 0.6$	MHolm	0.1480	0.3125	0.4490	0.5685	0.7110	0.7780
	TH	0.1180	0.2410	0.3530	0.4880	0.6325	0.7255
	Holm	0.0580	0.2285	0.3525	0.4880	0.6325	0.7255
$m = 5$, $\pi_0 = 0.8$	MHolm	0.0815	0.1680	0.2665	0.3585	0.4510	0.5255
	TH	0.0660	0.1240	0.2035	0.2965	0.3785	0.4685
	Holm	0.0305	0.1155	0.2035	0.2965	0.3785	0.4685
$m = 10$, $\pi_0 = 0.2$	MHolm	0.3140	0.6070	0.8265	0.9260	0.9725	0.9950
	TH	0.1980	0.4625	0.7405	0.8615	0.9400	0.9815
	Holm	0.1490	0.4605	0.7405	0.8615	0.9400	0.9815
$m = 10$, $\pi_0 = 0.4$	MHolm	0.2540	0.5190	0.7205	0.8510	0.9370	0.9595
	TH	0.1760	0.3785	0.6130	0.7685	0.8970	0.9355
	Holm	0.1275	0.3700	0.6130	0.7685	0.8970	0.9355
$m = 10$, $\pi_0 = 0.6$	MHolm	0.1985	0.3835	0.5815	0.7065	0.8350	0.8990
	TH	0.1235	0.2460	0.4700	0.6030	0.7710	0.8495
	Holm	0.0730	0.2390	0.4695	0.6030	0.7710	0.8495
$m = 10$, $\pi_0 = 0.8$	MHolm	0.1165	0.2185	0.3530	0.4795	0.5910	0.6850
	TH	0.0835	0.1410	0.2605	0.3780	0.4995	0.6105
	Holm	0.0435	0.1325	0.2605	0.3780	0.4995	0.6105
$m = 15$, $\pi_0 = 0.2$	MHolm	0.3475	0.6695	0.8570	0.9630	0.9920	0.9975
	TH	0.2615	0.4980	0.7325	0.9025	0.9785	0.9925
	Holm	0.1400	0.4790	0.7320	0.9020	0.9785	0.9925
$m = 15$, $\pi_0 = 0.4$	MHolm	0.2855	0.5660	0.7765	0.9020	0.9615	0.9890
	TH	0.2155	0.4175	0.6310	0.8090	0.9265	0.9735
	Holm	0.0970	0.3865	0.6275	0.8090	0.9265	0.9735
$m = 15$, $\pi_0 = 0.6$	MHolm	0.2105	0.4400	0.6540	0.7990	0.9060	0.9540
	TH	0.1575	0.3130	0.4925	0.6845	0.8325	0.9160
	Holm	0.0785	0.2845	0.4885	0.6845	0.8325	0.9160
$m = 15$, $\pi_0 = 0.8$	MHolm	0.1220	0.2380	0.4110	0.5500	0.6690	0.7950
	TH	0.0790	0.1555	0.2910	0.4275	0.5785	0.7185
	Holm	0.0300	0.1300	0.2885	0.4270	0.5780	0.7185

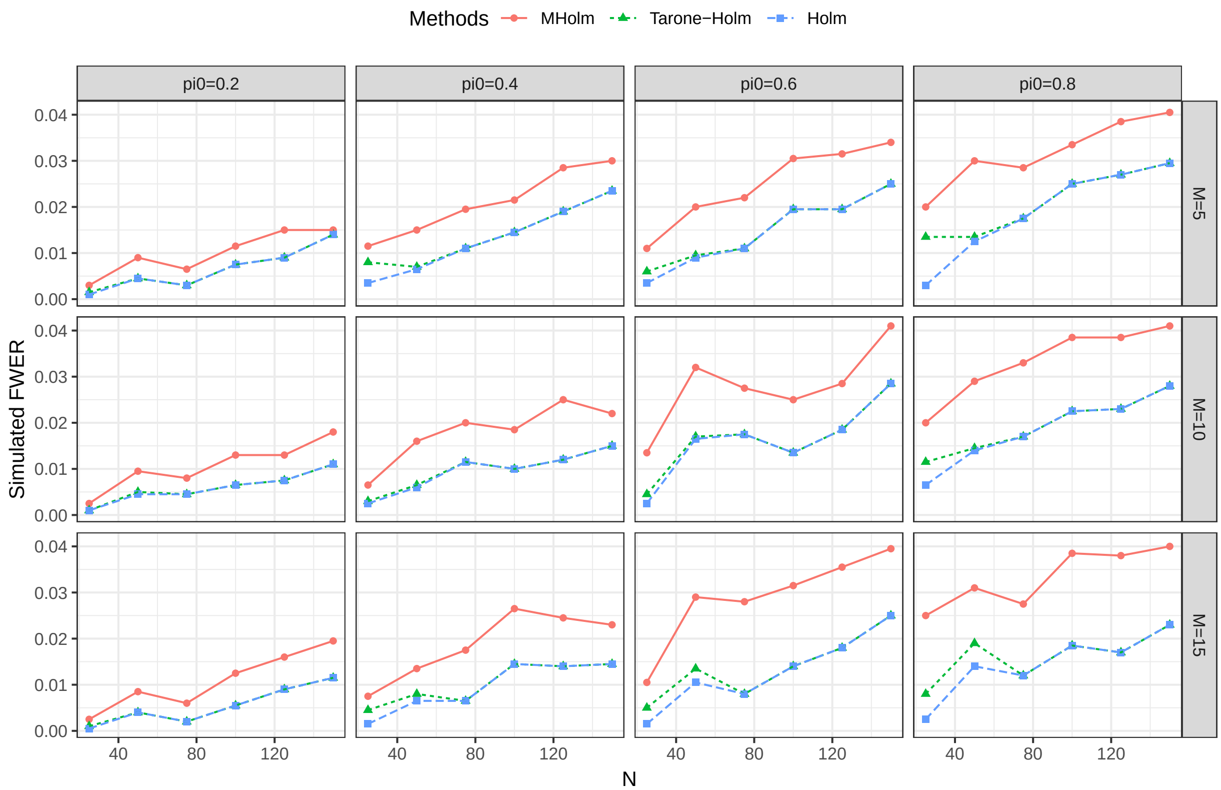

Figure S1: Simulated FWER comparisons for different step-down procedures based on FET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

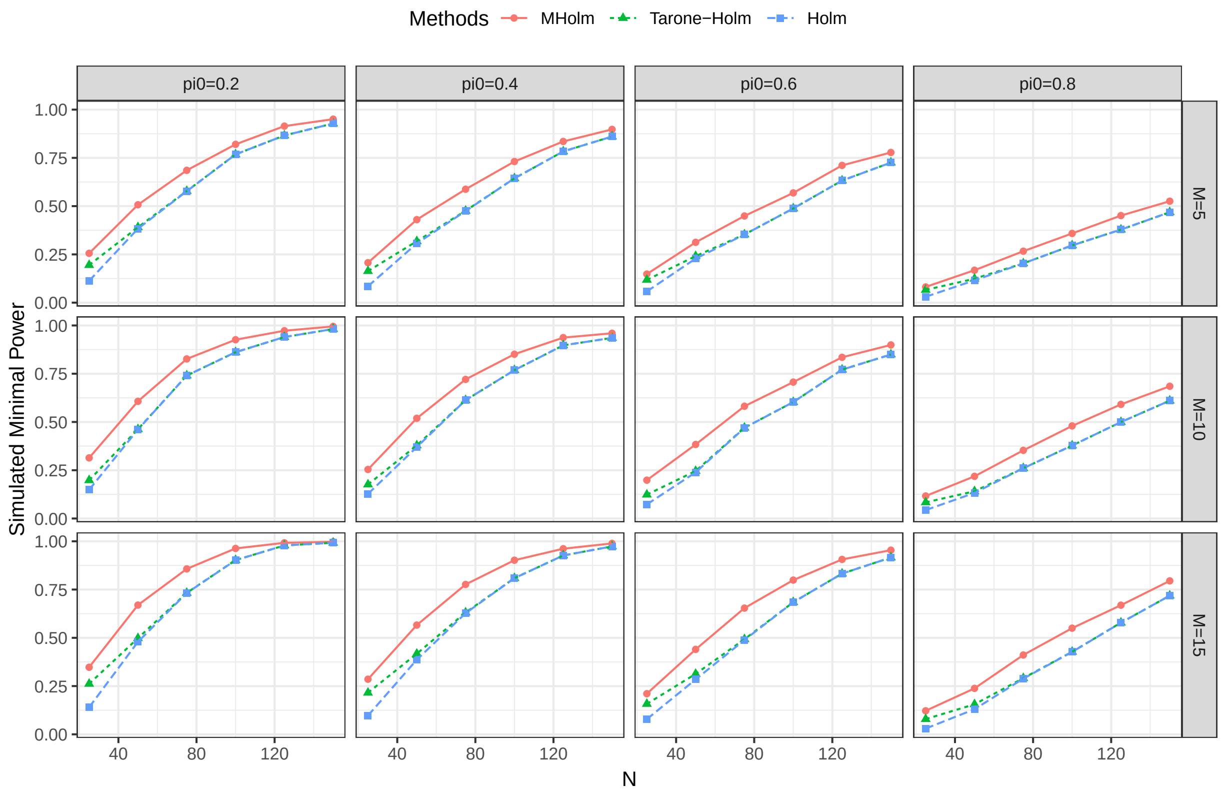

Figure S2: Simulated minimal power comparisons for different step-down procedures based on FET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

Table S7: Simulated FWER comparisons for step-up procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$, $\pi_0 = 0.2$	MHoch	0.0030	0.0090	0.0070	0.0115	0.0150	0.0155
	Roth	0.0020	0.0045	0.0040	0.0085	0.0115	0.0155
	Hoch	0.0015	0.0045	0.0040	0.0085	0.0115	0.0155
$m = 5$, $\pi_0 = 0.4$	MHoch	0.0115	0.0150	0.0200	0.0215	0.0290	0.0325
	Roth	0.0080	0.0070	0.0120	0.0145	0.0200	0.0245
	Hoch	0.0040	0.0065	0.0120	0.0145	0.0205	0.0245
$m = 5$, $\pi_0 = 0.6$	MHoch	0.0120	0.0210	0.0225	0.0310	0.0325	0.0340
	Roth	0.0055	0.0095	0.0110	0.0200	0.0190	0.0260
	Hoch	0.0040	0.0090	0.0110	0.0200	0.0200	0.0260
$m = 5$, $\pi_0 = 0.8$	MHoch	0.0210	0.0300	0.0290	0.0340	0.0385	0.0405
	Roth	0.0120	0.0135	0.0175	0.0250	0.0270	0.0295
	Hoch	0.0030	0.0125	0.0175	0.0250	0.0270	0.0295
$m = 10$, $\pi_0 = 0.2$	MHoch	0.0035	0.0100	0.0085	0.0130	0.0135	0.0205
	Roth	0.0010	0.0050	0.0045	0.0070	0.0070	0.0120
	Hoch	0.0010	0.0045	0.0045	0.0070	0.0080	0.0120
$m = 10$, $\pi_0 = 0.4$	MHoch	0.0070	0.0175	0.0210	0.0195	0.0250	0.0220
	Roth	0.0030	0.0065	0.0115	0.0100	0.0115	0.0155
	Hoch	0.0025	0.0060	0.0115	0.0100	0.0120	0.0155
$m = 10$, $\pi_0 = 0.6$	MHoch	0.0135	0.0320	0.0275	0.0250	0.0285	0.0410
	Roth	0.0045	0.0165	0.0175	0.0140	0.0185	0.0285
	Hoch	0.0025	0.0165	0.0175	0.0140	0.0185	0.0285
$m = 10$, $\pi_0 = 0.8$	MHoch	0.0205	0.0290	0.0330	0.0390	0.0390	0.0415
	Roth	0.0110	0.0145	0.0170	0.0225	0.0230	0.0280
	Hoch	0.0065	0.0140	0.0170	0.0225	0.0230	0.0280
$m = 15$, $\pi_0 = 0.2$	MHoch	0.0025	0.0085	0.0065	0.0125	0.0160	0.0205
	Roth	0.0010	0.0040	0.0020	0.0055	0.0090	0.0130
	Hoch	0.0005	0.0040	0.0020	0.0055	0.0090	0.0130
$m = 15$, $\pi_0 = 0.4$	MHoch	0.0075	0.0135	0.0175	0.0270	0.0245	0.0240
	Roth	0.0040	0.0080	0.0065	0.0145	0.0140	0.0145
	Hoch	0.0015	0.0070	0.0065	0.0145	0.0140	0.0145
$m = 15$, $\pi_0 = 0.6$	MHoch	0.0120	0.0290	0.0280	0.0320	0.0355	0.0395
	Roth	0.0050	0.0135	0.0080	0.0140	0.0180	0.0250
	Hoch	0.0015	0.0105	0.0080	0.0140	0.0180	0.0250
$m = 15$, $\pi_0 = 0.8$	MHoch	0.0255	0.0310	0.0280	0.0385	0.0385	0.0400
	Roth	0.0080	0.0190	0.0120	0.0185	0.0175	0.0230
	Hoch	0.0025	0.0140	0.0120	0.0185	0.0175	0.0230

Table S8: Simulated minimal power comparisons for step-up procedures with independent p-values generated from Fisher's Exact Test statistics, including Procedure 3.1 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

		$N = 25$	$N = 50$	$N = 75$	$N = 100$	$N = 125$	$N = 150$
$m = 5$, $\pi_0 = 0.2$	MHoch	0.2600	0.5075	0.6885	0.8240	0.9170	0.9525
	Roth	0.1900	0.3915	0.5820	0.7685	0.8695	0.9300
	Hoch	0.1170	0.3845	0.5810	0.7685	0.8695	0.9300
$m = 5$, $\pi_0 = 0.4$	MHoch	0.2070	0.4310	0.5905	0.7335	0.8370	0.9015
	Roth	0.1570	0.3195	0.4785	0.6475	0.7860	0.8625
	Hoch	0.0870	0.3085	0.4770	0.6475	0.7860	0.8625
$m = 5$, $\pi_0 = 0.6$	MHoch	0.1495	0.3140	0.4495	0.5705	0.7125	0.7785
	Roth	0.1100	0.2420	0.3530	0.4895	0.6350	0.7280
	Hoch	0.0585	0.2295	0.3525	0.4895	0.6350	0.7280
$m = 5$, $\pi_0 = 0.8$	MHoch	0.0825	0.1680	0.2670	0.3590	0.4510	0.5255
	Roth	0.0580	0.1240	0.2035	0.2965	0.3785	0.4685
	Hoch	0.0305	0.1155	0.2035	0.2965	0.3785	0.4685
$m = 10$, $\pi_0 = 0.2$	MHoch	0.3195	0.6115	0.8280	0.9270	0.9730	0.9955
	Roth	0.1995	0.4620	0.7405	0.8615	0.9410	0.9820
	Hoch	0.1495	0.4605	0.7405	0.8615	0.9410	0.9820
$m = 10$, $\pi_0 = 0.4$	MHoch	0.2540	0.5210	0.7225	0.8520	0.9375	0.9600
	Roth	0.1765	0.3750	0.6135	0.7690	0.8975	0.9365
	Hoch	0.1275	0.3700	0.6130	0.7690	0.8975	0.9365
$m = 10$, $\pi_0 = 0.6$	MHoch	0.1990	0.3845	0.5830	0.7070	0.8370	0.8995
	Roth	0.1230	0.2425	0.4700	0.6030	0.7715	0.8510
	Hoch	0.0730	0.2395	0.4695	0.6030	0.7715	0.8510
$m = 10$, $\pi_0 = 0.8$	MHoch	0.1170	0.2185	0.3535	0.4800	0.5925	0.6865
	Roth	0.0825	0.1370	0.2605	0.3780	0.4995	0.6105
	Hoch	0.0435	0.1325	0.2605	0.3780	0.4995	0.6105
$m = 15$, $\pi_0 = 0.2$	MHoch	0.3505	0.6700	0.8575	0.9640	0.9920	0.9980
	Roth	0.2615	0.5000	0.7330	0.9030	0.9785	0.9930
	Hoch	0.1400	0.4795	0.7325	0.9025	0.9785	0.9930
$m = 15$, $\pi_0 = 0.4$	MHoch	0.2875	0.5675	0.7785	0.9030	0.9615	0.9890
	Roth	0.2160	0.4195	0.6320	0.8095	0.9265	0.9740
	Hoch	0.0970	0.3875	0.6285	0.8095	0.9265	0.9740
$m = 15$, $\pi_0 = 0.6$	MHoch	0.2135	0.4400	0.6555	0.8005	0.9065	0.9545
	Roth	0.1580	0.3130	0.4925	0.6850	0.8325	0.9165
	Hoch	0.0785	0.2845	0.4890	0.6850	0.8325	0.9165
$m = 15$, $\pi_0 = 0.8$	MHoch	0.1225	0.2380	0.4110	0.5520	0.6690	0.7950
	Roth	0.0790	0.1550	0.2910	0.4270	0.5780	0.7195
	Hoch	0.0300	0.1300	0.2885	0.4270	0.5780	0.7195

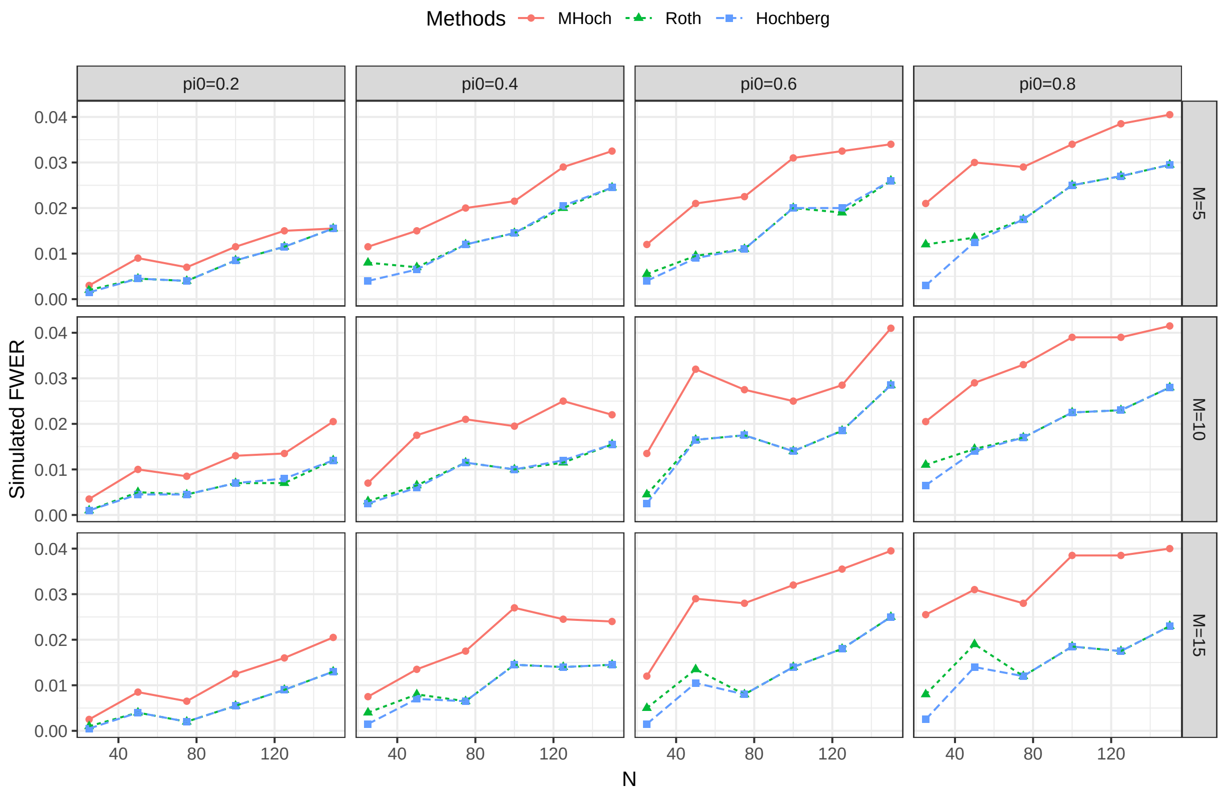

Figure S3: Simulated FWER comparisons for different step-up procedures based on FET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).

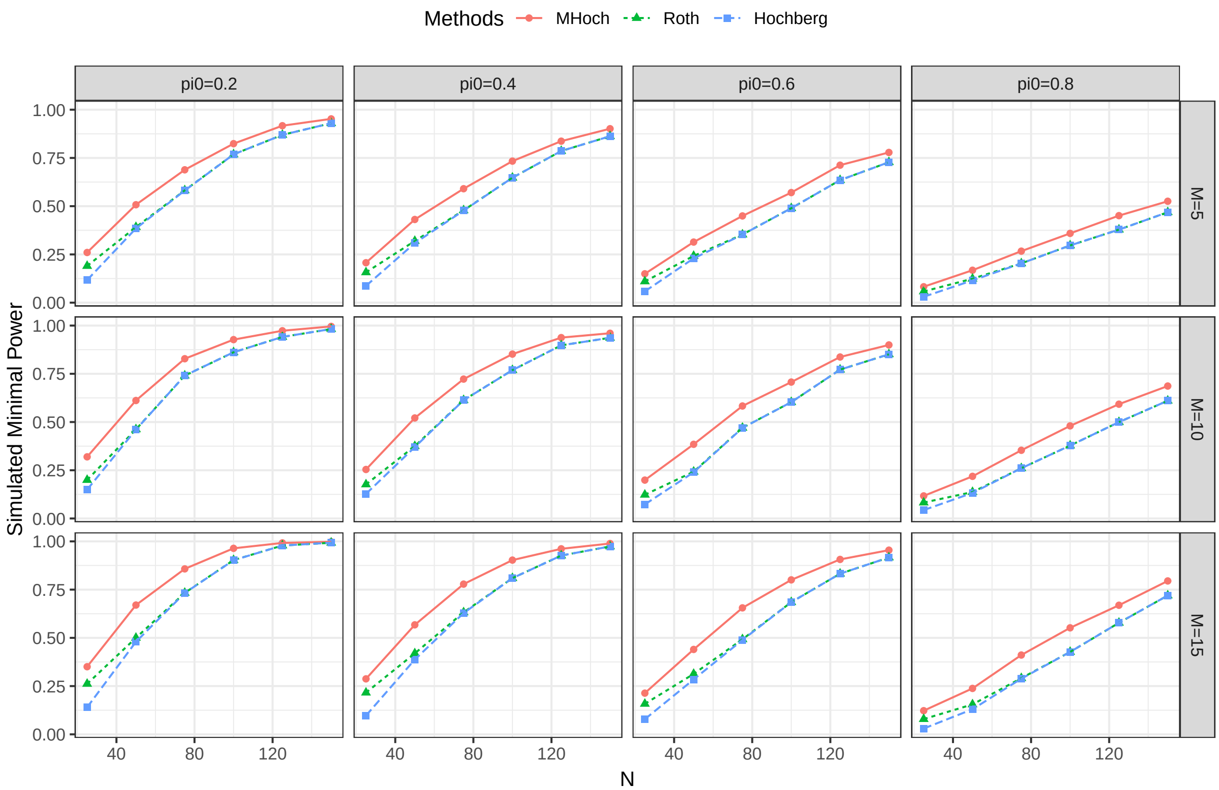

Figure S4: Simulated minimal power comparisons for different step-up procedures based on FET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).

S2 Results from Dependence Simulation Settings

In this section, we provide the details for simulating the block dependent binomial exact test (BET) statistics and the simulation results for the stepwise procedures comparisons. The following steps illustrate how to generate the dependent BET statistics and corresponding p-values.

Step 1. Generate dependent Poisson observed counts for each group

In order to generate m dependent BET statistics T_i, we use the following algorithm to generate m dependent Poisson random variables within each group, noting that the Poisson random variables between two groups are independent.

1. Let $\lambda_{i1} = 2$ for $i = 1, \ldots, m$, generate m independent Poisson random variable $Y_{i1} \sim Poi((1-\rho)\lambda_{i1})$ and one $Y_{01} \sim Poi(2\rho)$.

2. Let $X_{i1} = Y_{i1} + Y_{01}$ for $i = 1, \ldots, m$, then $X_{i1} \sim Poi(2)$ and the correlation between X_{i1} and X_{j1} is $\dfrac{Cov(X_{i1}, X_{j1})}{\sqrt{Var(X_{i1})}\sqrt{Var(X_{j1})}} = \dfrac{Var(Y_{01})}{\sqrt{2}\sqrt{2}} = \dfrac{2\rho}{2} = \rho$ for $i, j = 1, \ldots, m$ and $i \neq j$.

3. Let $\lambda_{i2} = 2$ for $i = 1, \ldots, m_0$ and $\lambda_{i2} = 10$ for $i = m_0 + 1, \ldots, m$, generate m independent Poisson random variable $Y_{i2} \sim Poi((1-\rho)\lambda_{i2})$ for $i = 1, \ldots, m$, one $Y_{02} \sim Poi(2\rho)$, and one $Y'_{02} \sim Poi(10\rho)$.

4. Let $X_{i2} = Y_{i2} + Y_{02}$ for $i = 1, \ldots, m_0$ and $X_{i2} = Y_{i2} + Y'_{02}$ for $i = m_0 + 1, \ldots, m$, then $X_{i2} \sim Poi(2)$ for $i = 1, \ldots, m_0$ and $X_{i2} \sim Poi(10)$ for $i = m_0 + 1, \ldots, m$. For $i, j = 1, \ldots, m_0$ and $i \neq j$, the correlation between X_{i2} and X_{j2} is $\dfrac{Cov(X_{i2}, X_{j2})}{\sqrt{Var(X_{i2})}\sqrt{Var(X_{j2})}} = \dfrac{Var(Y_{02})}{\sqrt{2}\sqrt{2}} = \dfrac{2\rho}{2} = \rho$. Similarly, for $i, j = m_0 + 1, \ldots, m$ and $i \neq j$, the correlation between X_{i2} and X_{j2} is also equal to ρ; for $i = 1, \ldots, m_0$ and $j = m_0 + 1, \ldots, m$, the correlation between X_{i2} and X_{j2} is equal to zero.

Step 2. Obtain the conditional test statistics

Since the generated Poisson random variables between two groups are independent, we can directly conduct BET for each hypothesis. After generating Poisson observed counts x_{i1} and x_{i2}, let $c_i = x_{i1} + x_{i2}$ be the total observed count for two groups. Then the test statistics T_i is conditional test statistics X_{i1} given $X_{i1} + X_{i2} = c_i$ and the critical value is the observed count x_{i1} for Group 1.

Step 3. Conditional distribution of the test statistics

Based on the conditional inference in Lehmann and Romano [1], which is the BET in our paper, the conditional distribution of X_{i1} given $X_{i1} + X_{i2} = c_i$ is Binomial, $Bin(c_i, p_i)$, where $p_i = \dfrac{\lambda_{i1}}{\lambda_{i1} + \lambda_{i2}}$.

Step 4. Calculate available p-value P_i and attainable p-values

When H_i is true, i.e., $\lambda_{i1} = \lambda_{i2}$, $p_i = 0.5$. Thus, $X_{i1} | X_{i1} + X_{i2} = c_i \sim Bin(c_i, 0.5)$ under H_i. Therefore, the available conditional p-value for H_i can be calculated by

$$\begin{aligned} P_i &= \Pr_{H_i}\{X_{i1} \geq x_{i1} | X_{i1} + X_{i2} = c_i\} \\ &= \sum_{j=x_{i1}}^{c_i} \binom{c_i}{j} 0.5^j (1-0.5)^{c_i - j} \\ &= \sum_{j=x_{i1}}^{c_i} \binom{c_i}{j} 0.5^{c_i}. \end{aligned} \tag{1}$$

The corresponding attainable p-values can be calculated by

$$\Pr_{H_i}\{X_{i1} \geq x | X_{i1} + X_{i2} = c_i\} = \sum_{j=x}^{c_i} \binom{c_i}{j} 0.5^{c_i} \quad \text{for } x = 0, 1, \ldots, c_i. \tag{2}$$

The simulation results under the above simulation setting for stepwise procedures comparisons are shown in Tables S9 - S14 and Figures S5 - S8. It is easy to see that in such block dependence simulation setting, the p-values calculated based on the Poisson outcomes satisfies the PRDS Assumption 2.2, since $\rho \geq 0$ and the tests are one-sided.

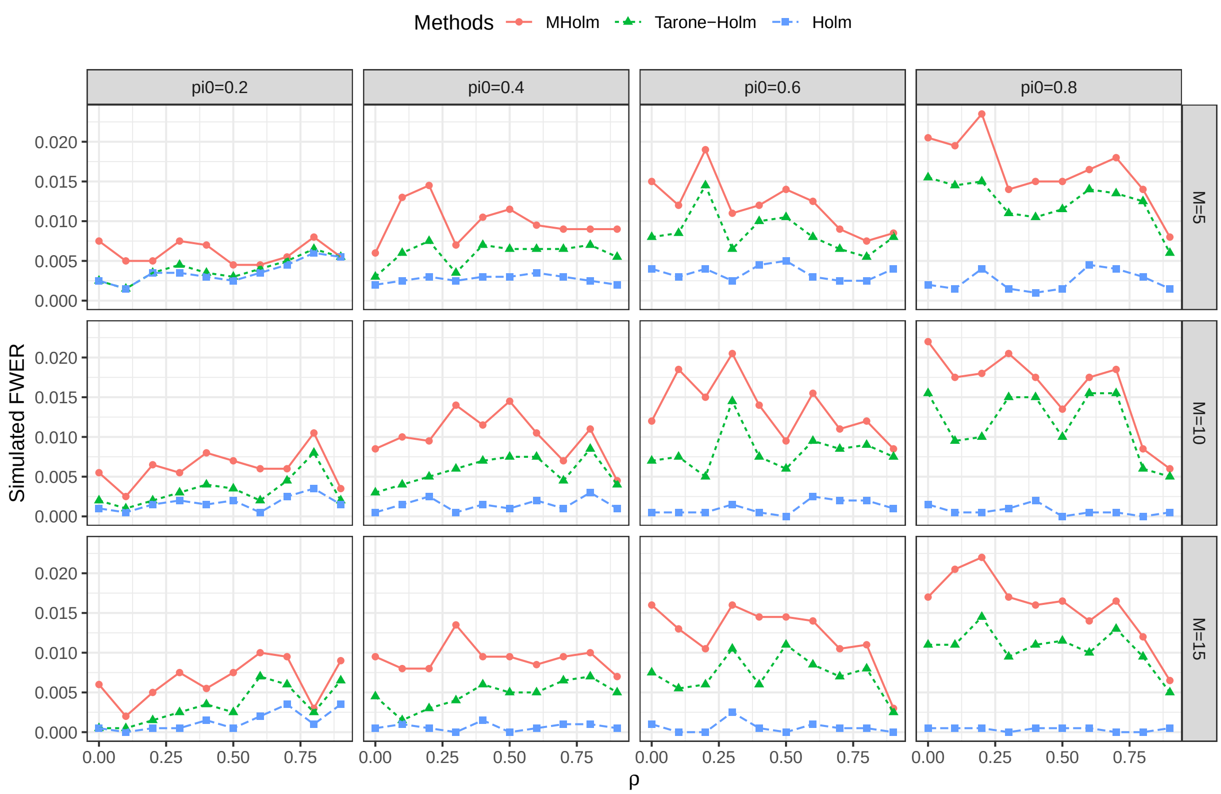

Figure S5: Simulated FWER comparisons for different step-down procedures based on the blocking dependent BET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

R-package for MHTdiscrete: R-package MHTdiscrete [3] contains R code to implement our proposed methods and several existing FWER controlling procedures for discrete data, which are described in this paper. The package can be downloaded from `https://cran.r-project.org/web/packages/MHTdiscrete`.

Web Application for MHTdiscrete: A web application containing the proposed procedures and several comparable procedures can be accessed at `https://allen.shinyapps.io/MTPs`.

Table S9: Simulated FWER comparisons for single-step procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Sidak (Sidak) and Bonferroni (Bonf) procedures.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$	MBonf	0.0025	0.0040	0.0055	0.0035	0.0055	0.0030	0.0035	0.0030	0.0035	0.0020
	Tarone	0.0015	0.0010	0.0030	0.0010	0.0005	0.0005	0.0015	0.0025	0.0020	0.0015
$\pi_0 = 0.4$	Sidak	0.0015	0.0005	0.0025	0.0010	0.0005	0.0005	0.0015	0.0020	0.0015	0.0010
	Bonf	0.0015	0.0005	0.0025	0.0010	0.0005	0.0005	0.0015	0.0020	0.0015	0.0010
$m = 5$	MBonf	0.0090	0.0070	0.0115	0.0055	0.0085	0.0090	0.0060	0.0050	0.0060	0.0045
	Tarone	0.0055	0.0055	0.0075	0.0020	0.0045	0.0050	0.0020	0.0025	0.0040	0.0035
$\pi_0 = 0.6$	Sidak	0.0030	0.0025	0.0030	0.0015	0.0025	0.0015	0.0010	0.0010	0.0005	0.0015
	Bonf	0.0030	0.0025	0.0030	0.0015	0.0025	0.0015	0.0010	0.0010	0.0005	0.0015
$m = 5$	MBonf	0.0160	0.0155	0.0185	0.0115	0.0105	0.0120	0.0155	0.0140	0.0115	0.0055
	Tarone	0.0105	0.0075	0.0105	0.0065	0.0065	0.0075	0.0095	0.0070	0.0070	0.0040
$\pi_0 = 0.8$	Sidak	0.0020	0.0015	0.0040	0.0010	0.0010	0.0010	0.0045	0.0035	0.0030	0.0015
	Bonf	0.0020	0.0015	0.0040	0.0010	0.0010	0.0010	0.0045	0.0035	0.0030	0.0015
$m = 10$	MBonf	0.0010	0.0030	0.0040	0.0035	0.0040	0.0030	0.0050	0.0020	0.0050	0.0010
	Tarone	0.0000	0.0010	0.0015	0.0000	0.0010	0.0020	0.0020	0.0005	0.0005	0.0000
$\pi_0 = 0.4$	Sidak	0.0000	0.0010	0.0015	0.0000	0.0005	0.0010	0.0015	0.0005	0.0000	0.0000
	Bonf	0.0000	0.0010	0.0015	0.0000	0.0005	0.0010	0.0015	0.0005	0.0000	0.0000
$m = 10$	MBonf	0.0065	0.0085	0.0065	0.0095	0.0045	0.0040	0.0085	0.0045	0.0060	0.0035
	Tarone	0.0040	0.0035	0.0015	0.0055	0.0025	0.0015	0.0045	0.0030	0.0020	0.0020
$\pi_0 = 0.6$	Sidak	0.0005	0.0005	0.0000	0.0010	0.0005	0.0000	0.0005	0.0010	0.0005	0.0005
	Bonf	0.0005	0.0005	0.0000	0.0010	0.0005	0.0000	0.0005	0.0010	0.0005	0.0005
$m = 10$	MBonf	0.0165	0.0090	0.0120	0.0140	0.0130	0.0085	0.0120	0.0125	0.0055	0.0025
	Tarone	0.0100	0.0055	0.0060	0.0085	0.0100	0.0035	0.0090	0.0085	0.0030	0.0020
$\pi_0 = 0.8$	Sidak	0.0015	0.0005	0.0005	0.0010	0.0020	0.0000	0.0005	0.0005	0.0000	0.0005
	Bonf	0.0015	0.0005	0.0005	0.0010	0.0020	0.0000	0.0005	0.0005	0.0000	0.0005

Table S10: Simulated minimal power comparisons for single-step procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.1 (MBonf), Procedure 2.1 (Tarone), and the conventional Sidak (Sidak) and Bonferroni (Bonf) procedures.

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$m = 5$	MBonf	0.9200	0.8710	0.8575	0.8445	0.8275	0.7690	0.7395	0.7375	0.6935	0.6425
	Tarone	0.8755	0.8150	0.8000	0.7840	0.7610	0.7040	0.6755	0.6745	0.6235	0.5680
$\pi_0 = 0.4$	Sidak	0.7880	0.7250	0.7060	0.6915	0.6680	0.6130	0.5835	0.5730	0.5320	0.4760
	Bonf	0.7880	0.7250	0.7060	0.6915	0.6680	0.6130	0.5835	0.5730	0.5320	0.4760
$m = 5$	MBonf	0.8085	0.8065	0.7870	0.7650	0.7445	0.7255	0.7205	0.6965	0.6610	0.6195
	Tarone	0.7610	0.7545	0.7355	0.7175	0.7030	0.6790	0.6675	0.6515	0.6185	0.5790
$\pi_0 = 0.6$	Sidak	0.5985	0.6060	0.5875	0.5710	0.5475	0.5395	0.5070	0.5095	0.4680	0.4200
	Bonf	0.5985	0.6060	0.5875	0.5710	0.5475	0.5395	0.5070	0.5095	0.4680	0.4200
$m = 5$	MBonf	0.6030	0.6085	0.5920	0.6175	0.5875	0.5865	0.6330	0.6295	0.6110	0.6110
	Tarone	0.5635	0.5730	0.5615	0.5875	0.5610	0.5680	0.6105	0.6115	0.5905	0.6000
$\pi_0 = 0.8$	Sidak	0.3750	0.3910	0.3595	0.3865	0.3680	0.3420	0.3890	0.3860	0.3940	0.3750
	Bonf	0.3750	0.3910	0.3595	0.3865	0.3680	0.3420	0.3890	0.3860	0.3940	0.3750
$m = 10$	MBonf	0.9755	0.9465	0.9205	0.8900	0.8580	0.8195	0.7765	0.7395	0.7025	0.6255
	Tarone	0.9435	0.8970	0.8695	0.8285	0.7905	0.7565	0.7000	0.6560	0.6200	0.5425
$\pi_0 = 0.4$	Sidak	0.8805	0.8225	0.7915	0.7425	0.6965	0.6565	0.6115	0.5550	0.5230	0.4475
	Bonf	0.8805	0.8225	0.7915	0.7425	0.6965	0.6565	0.6115	0.5550	0.5230	0.4475
$m = 10$	MBonf	0.9210	0.8970	0.8760	0.8575	0.8175	0.7980	0.7535	0.7160	0.6720	0.6200
	Tarone	0.8670	0.8465	0.8240	0.8015	0.7545	0.7370	0.7025	0.6535	0.6205	0.5750
$\pi_0 = 0.6$	Sidak	0.7265	0.7165	0.6815	0.6600	0.6045	0.5775	0.5315	0.4940	0.4415	0.4025
	Bonf	0.7265	0.7165	0.6815	0.6600	0.6045	0.5775	0.5315	0.4940	0.4415	0.4025
$m = 10$	MBonf	0.7720	0.7710	0.7250	0.7240	0.7085	0.6790	0.6750	0.6685	0.6265	0.6125
	Tarone	0.7175	0.7230	0.6870	0.6710	0.6705	0.6270	0.6310	0.6295	0.5895	0.5765
$\pi_0 = 0.8$	Sidak	0.4950	0.4915	0.4735	0.4580	0.4650	0.4105	0.4215	0.4140	0.3710	0.3420
	Bonf	0.4950	0.4915	0.4735	0.4580	0.4650	0.4105	0.4215	0.4140	0.3710	0.3420

Table S11: Simulated FWER comparisons for step-down procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHolm	0.0060	0.0130	0.0145	0.0070	0.0105	0.0115	0.0095	0.0090	0.0090	0.0090
$m = 5$	TH	0.0030	0.0060	0.0075	0.0035	0.0070	0.0065	0.0065	0.0065	0.0070	0.0055
$\pi_0 = 0.4$	Holm	0.0020	0.0025	0.0030	0.0025	0.0030	0.0030	0.0035	0.0030	0.0025	0.0020
	MHolm	0.0150	0.0120	0.0190	0.0110	0.0120	0.0140	0.0125	0.0090	0.0075	0.0085
$m = 5$	TH	0.0080	0.0085	0.0145	0.0065	0.0100	0.0105	0.0080	0.0065	0.0055	0.0080
$\pi_0 = 0.6$	Holm	0.0040	0.0030	0.0040	0.0025	0.0045	0.0050	0.0030	0.0025	0.0025	0.0040
	MHolm	0.0205	0.0195	0.0235	0.0140	0.0150	0.0150	0.0165	0.0180	0.0140	0.0080
$m = 5$	TH	0.0155	0.0145	0.0150	0.0110	0.0105	0.0115	0.0140	0.0135	0.0125	0.0060
$\pi_0 = 0.8$	Holm	0.0020	0.0015	0.0040	0.0015	0.0010	0.0015	0.0045	0.0040	0.0030	0.0015
	MHolm	0.0085	0.0100	0.0095	0.0140	0.0115	0.0145	0.0105	0.0070	0.0110	0.0045
$m = 10$	TH	0.0030	0.0040	0.0050	0.0060	0.0070	0.0075	0.0075	0.0045	0.0085	0.0040
$\pi_0 = 0.4$	Holm	0.0005	0.0015	0.0025	0.0005	0.0015	0.0010	0.0020	0.0010	0.0030	0.0010
	MHolm	0.0120	0.0185	0.0150	0.0205	0.0140	0.0095	0.0155	0.0110	0.0120	0.0085
$m = 10$	TH	0.0070	0.0075	0.0050	0.0145	0.0075	0.0060	0.0095	0.0085	0.0090	0.0075
$\pi_0 = 0.6$	Holm	0.0005	0.0005	0.0005	0.0015	0.0005	0.0000	0.0025	0.0020	0.0020	0.0010
	MHolm	0.0220	0.0175	0.0180	0.0205	0.0175	0.0135	0.0175	0.0185	0.0085	0.0060
$m = 10$	TH	0.0155	0.0095	0.0100	0.0150	0.0150	0.0100	0.0155	0.0155	0.0060	0.0050
$\pi_0 = 0.8$	Holm	0.0015	0.0005	0.0005	0.0010	0.0020	0.0000	0.0005	0.0005	0.0000	0.0005

Table S12: Simulated minimal power comparisons for step-down procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.2 (MHolm), Procedure 2.3 (TH), and the conventional Holm procedure (Holm).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHolm	0.9200	0.8715	0.8575	0.8445	0.8275	0.7690	0.7395	0.7375	0.6935	0.6425
$m = 5$	TH	0.8755	0.8155	0.8000	0.7840	0.7610	0.7040	0.6755	0.6745	0.6235	0.5680
$\pi_0 = 0.4$	Holm	0.7880	0.7250	0.7065	0.6915	0.6680	0.6130	0.5835	0.5730	0.5320	0.4760
	MHolm	0.8100	0.8065	0.7870	0.7650	0.7450	0.7260	0.7205	0.6965	0.6615	0.6195
$m = 5$	TH	0.7610	0.7545	0.7355	0.7175	0.7035	0.6795	0.6680	0.6520	0.6185	0.5790
$\pi_0 = 0.6$	Holm	0.5990	0.6060	0.5875	0.5710	0.5480	0.5395	0.5075	0.5095	0.4680	0.4200
	MHolm	0.6070	0.6105	0.5940	0.6185	0.5885	0.5870	0.6335	0.6300	0.6120	0.6110
$m = 5$	TH	0.5650	0.5730	0.5630	0.5880	0.5615	0.5685	0.6105	0.6115	0.5915	0.6000
$\pi_0 = 0.8$	Holm	0.3755	0.3910	0.3595	0.3865	0.3680	0.3420	0.3890	0.3860	0.3940	0.3750
	MHolm	0.9755	0.9465	0.9205	0.8900	0.8580	0.8195	0.7765	0.7395	0.7025	0.6255
$m = 10$	TH	0.9435	0.8970	0.8695	0.8285	0.7905	0.7565	0.7000	0.6560	0.6200	0.5425
$\pi_0 = 0.4$	Holm	0.8805	0.8225	0.7915	0.7425	0.6965	0.6565	0.6115	0.5550	0.5230	0.4475
	MHolm	0.9210	0.8970	0.8760	0.8575	0.8175	0.7980	0.7535	0.7170	0.6720	0.6205
$m = 10$	TH	0.8680	0.8465	0.8245	0.8020	0.7545	0.7370	0.7025	0.6535	0.6205	0.5750
$\pi_0 = 0.6$	Holm	0.7265	0.7165	0.6815	0.6600	0.6045	0.5775	0.5315	0.4940	0.4415	0.4025
	MHolm	0.7735	0.7715	0.7255	0.7240	0.7085	0.6790	0.6750	0.6685	0.6265	0.6130
$m = 10$	TH	0.7180	0.7230	0.6870	0.6710	0.6710	0.6275	0.6310	0.6295	0.5895	0.5770
$\pi_0 = 0.8$	Holm	0.4950	0.4915	0.4735	0.4580	0.4650	0.4105	0.4215	0.4140	0.3710	0.3425

Table S13: Simulated FWER comparisons for step-up procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHoch	0.0060	0.0135	0.0150	0.0070	0.0110	0.0120	0.0105	0.0090	0.0115	0.0115
$m = 5$	Roth	0.0035	0.0080	0.0075	0.0035	0.0085	0.0085	0.0080	0.0070	0.0095	0.0080
$\pi_0 = 0.4$	Hoch	0.0020	0.0030	0.0035	0.0025	0.0040	0.0040	0.0050	0.0030	0.0045	0.0040
	MHoch	0.0155	0.0120	0.0195	0.0110	0.0120	0.0150	0.0130	0.0130	0.0105	0.0135
$m = 5$	Roth	0.0090	0.0100	0.0155	0.0055	0.0105	0.0095	0.0090	0.0075	0.0090	0.0115
$\pi_0 = 0.6$	Hoch	0.0050	0.0030	0.0040	0.0025	0.0050	0.0050	0.0045	0.0035	0.0050	0.0065
	MHoch	0.0205	0.0195	0.0235	0.0150	0.0155	0.0150	0.0185	0.0195	0.0160	0.0100
$m = 5$	Roth	0.0165	0.0160	0.0155	0.0110	0.0115	0.0110	0.0150	0.0145	0.0135	0.0070
$\pi_0 = 0.8$	Hoch	0.0020	0.0015	0.0040	0.0015	0.0010	0.0025	0.0045	0.0045	0.0045	0.0040
	MHoch	0.0095	0.0110	0.0100	0.0160	0.0115	0.0170	0.0130	0.0080	0.0130	0.0095
$m = 10$	Roth	0.0020	0.0045	0.0055	0.0080	0.0060	0.0080	0.0095	0.0050	0.0100	0.0060
$\pi_0 = 0.4$	Hoch	0.0005	0.0015	0.0025	0.0005	0.0015	0.0010	0.0025	0.0015	0.0040	0.0030
	MHoch	0.0140	0.0195	0.0160	0.0210	0.0165	0.0115	0.0165	0.0140	0.0145	0.0130
$m = 10$	Roth	0.0070	0.0075	0.0055	0.0135	0.0080	0.0050	0.0105	0.0085	0.0100	0.0090
$\pi_0 = 0.6$	Hoch	0.0005	0.0005	0.0005	0.0015	0.0005	0.0000	0.0025	0.0020	0.0025	0.0040
	MHoch	0.0220	0.0180	0.0195	0.0215	0.0190	0.0160	0.0190	0.0195	0.0115	0.0090
$m = 10$	Roth	0.0135	0.0085	0.0100	0.0130	0.0140	0.0095	0.0150	0.0140	0.0075	0.0050
$\pi_0 = 0.8$	Hoch	0.0015	0.0005	0.0005	0.0010	0.0020	0.0000	0.0005	0.0010	0.0020	0.0020

Table S14: Simulated minimal power comparisons for step-up procedures with dependent p-values generated from Binomial Exact Test statistics, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hoch).

	ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	MHoch	0.9235	0.8745	0.8620	0.8475	0.8315	0.7740	0.7455	0.7475	0.7040	0.6775
$m = 5$	Roth	0.8815	0.8240	0.8090	0.7955	0.7705	0.7135	0.6885	0.6930	0.6470	0.6045
$\pi_0 = 0.4$	Hoch	0.7975	0.7330	0.7175	0.7010	0.6785	0.6215	0.5970	0.5860	0.5475	0.5005
	MHoch	0.8140	0.8095	0.7915	0.7695	0.7505	0.7285	0.7260	0.7055	0.6690	0.6410
$m = 5$	Roth	0.7665	0.7555	0.7375	0.7245	0.7125	0.6835	0.6740	0.6595	0.6220	0.5955
$\pi_0 = 0.6$	Hoch	0.6040	0.6160	0.5910	0.5775	0.5545	0.5490	0.5145	0.5190	0.4810	0.4390
	MHoch	0.6070	0.6105	0.5940	0.6190	0.5885	0.5870	0.6340	0.6310	0.6120	0.6115
$m = 5$	Roth	0.5545	0.5625	0.5455	0.5735	0.5470	0.5490	0.5925	0.5930	0.5745	0.5760
$\pi_0 = 0.8$	Hoch	0.3755	0.3910	0.3595	0.3865	0.3680	0.3420	0.3890	0.3860	0.3950	0.3760
	MHoch	0.9765	0.9475	0.9220	0.8940	0.8620	0.8220	0.7830	0.7470	0.7215	0.6495
$m = 10$	Roth	0.9460	0.9015	0.8730	0.8305	0.7940	0.7580	0.7070	0.6645	0.6365	0.5775
$\pi_0 = 0.4$	Hoch	0.8805	0.8225	0.7915	0.7425	0.6970	0.6570	0.6120	0.5585	0.5270	0.4560
	MHoch	0.9245	0.9010	0.8790	0.8600	0.8230	0.8045	0.7595	0.7260	0.6865	0.6425
$m = 10$	Roth	0.8745	0.8510	0.8275	0.8075	0.7570	0.7425	0.7085	0.6615	0.6275	0.5930
$\pi_0 = 0.6$	Hoch	0.7265	0.7170	0.6820	0.6600	0.6055	0.5775	0.5320	0.4940	0.4430	0.4100
	MHoch	0.7760	0.7730	0.7275	0.7255	0.7095	0.6815	0.6775	0.6720	0.6305	0.6280
$m = 10$	Roth	0.7160	0.7215	0.6880	0.6725	0.6730	0.6305	0.6350	0.6345	0.5950	0.5885
$\pi_0 = 0.8$	Hoch	0.4950	0.4915	0.4735	0.4580	0.4650	0.4105	0.4215	0.4140	0.3710	0.3425

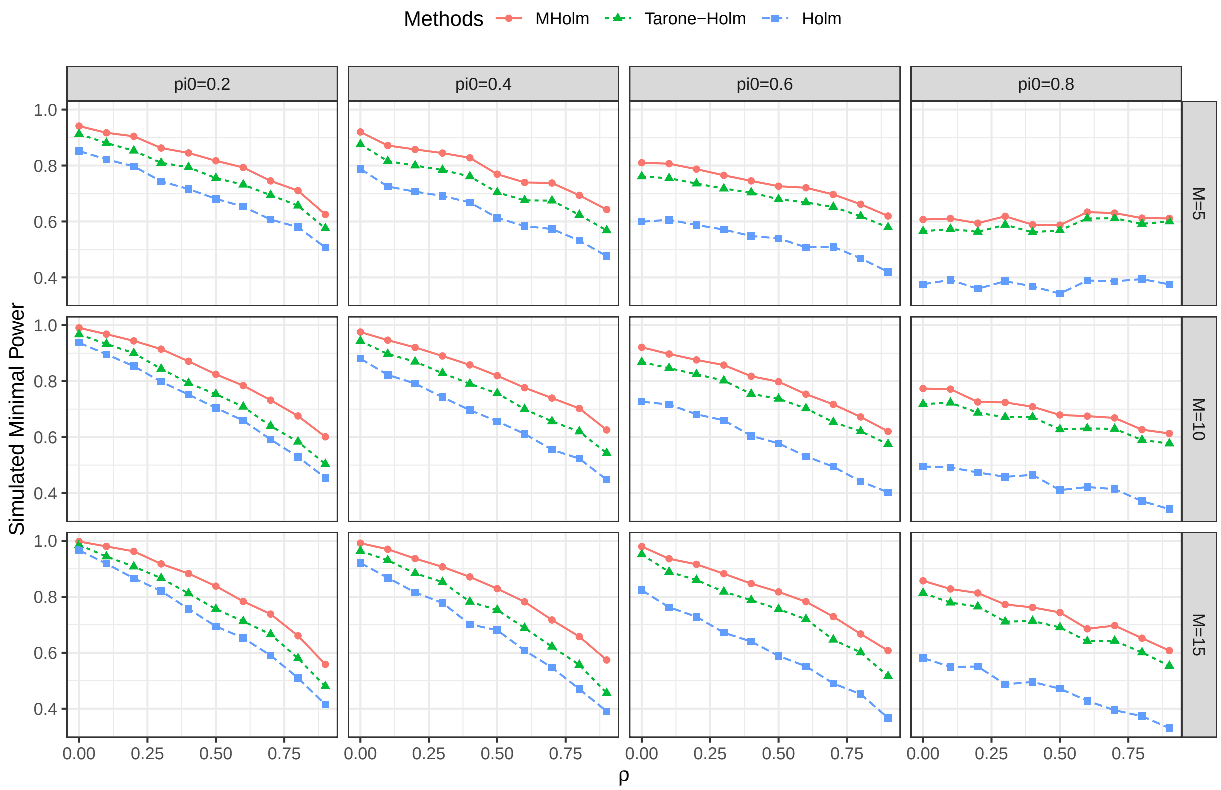

Figure S6: Simulated minimal power comparisons for different step-down procedures based on the blocking dependent BET, including Procedure 3.2 (MHolm), Procedure 2.3 (Tarone-Holm), and the conventional Holm procedure (Holm).

References

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[3] Zhu, Y. and Guo, W. (2018). *MHTdiscrete: Multiple Hypotheses Testing for Discrete Data*. R package version 1.0.1.

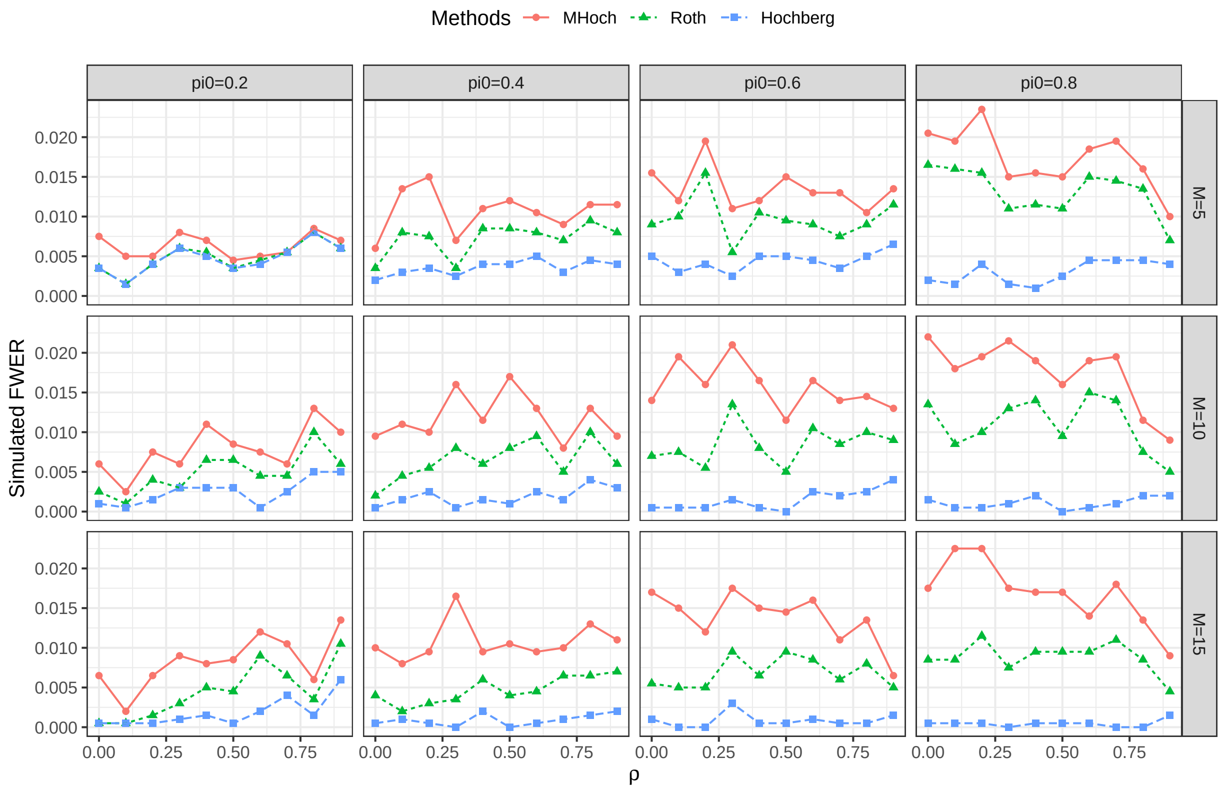

Figure S7: Simulated FWER comparisons for different step-up procedures based on the blocking dependent BET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).

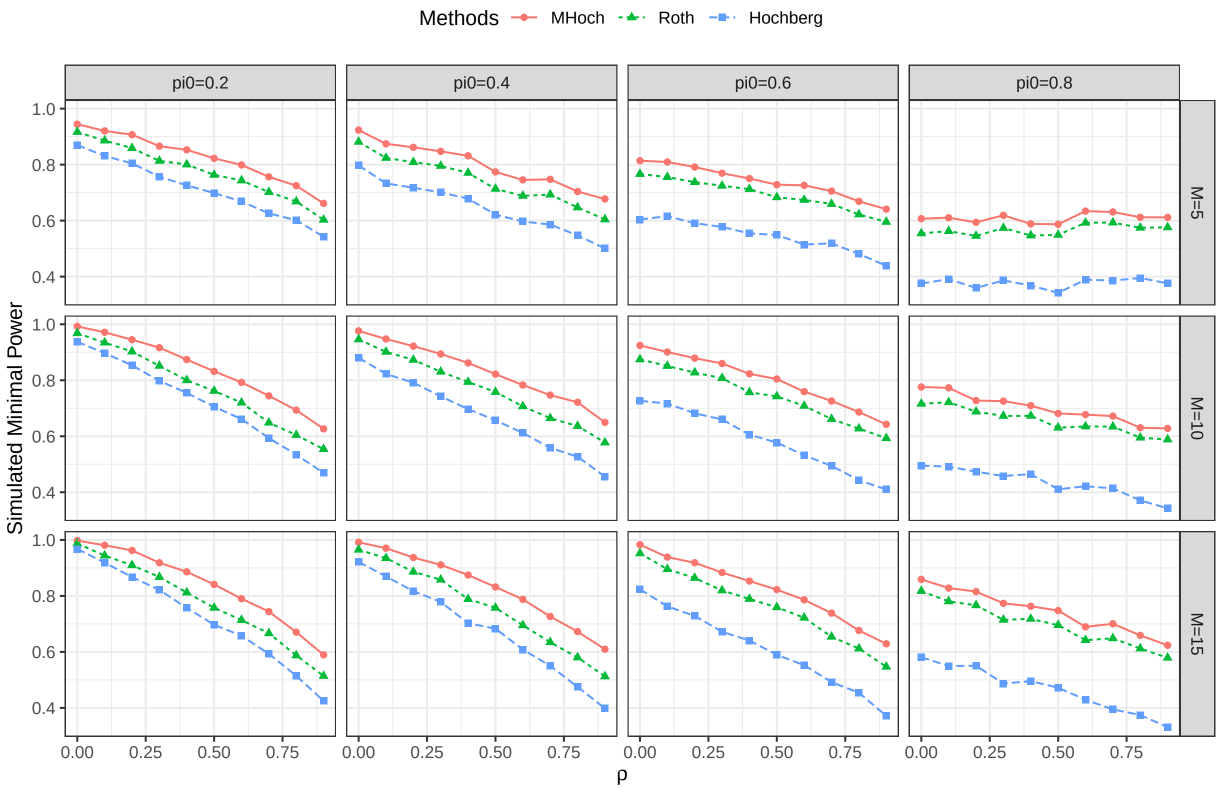

Figure S8: Simulated minimal power comparisons for different step-up procedures based on the blocking dependent BET, including Procedure 3.3 (MHoch), the Roth procedure (Roth), and the conventional Hochberg procedure (Hochberg).