**DETAILED SOLUTIONS FOR THE EXAMPLES**

**Case 1: Numerical Example for** $A\_{2}$ **Uniform**

$Let A\_{0}=\$1,600, A\_{1}=\$10,000 and A\_{2}∼uniform \left(a\_{2}=\$8,000, b\_{2}=\$12,000\right).$ Here $2.32×10^{7}\leq A\_{1}^{2}-4 A\_{0}A\_{2}\leq 4.88×10^{7}$ indicating there are two real roots with probability one.

***The IRR/First Root:*** Using Table 1, the valid range is $3.6302\leq i\leq 4.3080$. The PDF and the CDF expressions are $f\left(i\right)=0.8(i-2.125)$ and $F\left(i\right)=0.4(i-2.125)^{2}-0.91$.

***The IRR/Second Root:*** The valid range is $-0.0580\leq i\leq 0.6198$. The PDF and the CDF expressions are $f\left(i\right)=-0.8(i-2.125)$ and $F\left(i\right)=-0.4\left(i-2.125\right)^{2}+1.90625$.

If the MARR is 3.95 or 395%, $F\_{2}(3.95)-F\_{1}(3.95)$ = P($i\_{2}$) ≤ 3.95) – P($i\_{1}$) ≤ 3.95) =

1-(1-P($i\_{1}$) > 3.95) = 1 - $F\_{1}(3.95)$ = 1 – (0.4(3.95 – 2.125)2 – 0.91) = 0.578. Unlike the IRR, the ERR solution requires a MARR value as an input. Using the applicable ERR CDF formula for a MARR of 3.95 in Table 3, $F\left(ERR\right)=12.801-\frac{303.22}{\left(ERR+1\right)^{2}}$ in a valid range of 3.867 ≤ ERR ≤ 4.069 and P(ERR > 3.95) = 1 – F(3.95) = 0.574. Both the IRR and the ERR CDF expressions result in the same probability of project being acceptable.

If MARR is 0.25, $F\_{2}(0.25)-F\_{1}(0.25)$ = P($i\_{2}$) ≤ 0.25) – P($i\_{1} $≤ 0.25) = P($i\_{2}$ ≤ 0.25) - 0

 = $F\_{2}$(0.25) =$\left(-0.4\left(0.25-2.125\right)^{2}+1.90625\right)=0.50$. The decision maker can use this information to make a decision for the project for accepting, rejecting, or further study.

Using$ MARR=0.25 for the ERR, $the valid range is $0.1606\leq ERR\leq 0.3639$. The PDF and the CDF expressions are: $f\left(ERR\right)=\frac{9.7656}{(ERR+1)^{3}} and F\left(ERR\right)=3.625-\frac{4.8828}{\left(ERR+1\right)^{2}} .$

Using the CDF above we can also compute the probability the project is desirable, $P\left(ERR>MARR\right)=1-F\left(0.25\right)=1-\left(3.625-\frac{4.8828}{\left(0.25+1\right)^{2}}\right)=0.50$. This confirms the answer found above using the IRR approach.

**Case 2: Numerical Example for** $A\_{1}$ **Uniform**

$$Let A\_{0}=\$1,600 , A\_{1}∼uniform \left(a\_{1}=\$8,000,b\_{1}=\$12,000\right), and A\_{2}=\$10,000. $$

The term $A\_{1}^{2}-4 A\_{0}A\_{2}$ ranges in [0, 80000000] indicating two real roots.

***The IRR/First Root:*** Using Table 1, the valid range is $1.5\leq i\leq 5.5451$. The PDF and the CDF expressions are: $f\left(i\right)=\frac{-2.5}{(i+1)^{2}}+0.4 and F\left(i\right)=\frac{2.5}{i+1}+0.4\left(i+1\right)-2.$

***The IRR/Second Root:*** The valid range is $-0.0451\leq i\leq 1.5$. The PDF and the CDF expressions are $f\left(i\right)=\frac{2.5}{(i+1)^{2}}-0.4 and F\left(i\right)=\frac{-2.5}{i+1}-0.4\left(i+1\right)+3.$

If the MARR is 2.00, $F\_{2} $(2.00)- $F\_{1} $(2.00) = P ($i\_{2}\leq $2.00) – P($i\_{1}\leq $ 2.00) = 1 – $F\_{1} $(2.00) = 0.966. The applicable ERR CDF for a MARR of 2.00 is 0.226\*(ERR + 1)2 – 2 in 1.975 $\leq $ ERR $\leq $ 2.644 range. P (ERR > MARR) = 1 – F(2.00) = 0.966.

Using a MARR of 0.25, $F\_{2}$ (0.25)- $F\_{1}$ (0.25) = P($i\_{2}$ $\leq $ 0.25) – 0 = $F\_{2}$ (0.25) = 0.50.

$For the$ ERR, $let MARR=0.25$. Using Table 3, the valid range is $0.1180\leq ERR\leq 0.3693$. The PDF and the CDF expressions are: $f\left(ERR\right)=3.2\left(ERR+1\right) and F\left(ERR\right)=1.6(ERR+1)^{2}-2.$

Here, $P\left(ERR>MARR\right)=1-F\left(MARR\right)=1-\left(1.6\left(MARR+1\right)^{2}-2\right)=.50$, which is the same as the value found using the IRR CDF above.

**Case 3: Numerical Example for** $A\_{2}$ **Exponential**

$Let A\_{0}=\$1,600, A\_{1}=\$10,000 and A\_{2} ∼Exponential \left(λ=1/\$10,000\right)$.

The term $A\_{1}^{2}-4 A\_{0}A\_{2}$ ranges in [- ∞ , 100000000] indicating there are not always roots. The roots that exist happen when A2 $\leq $ $15,625 which happens with a probability of 0.7904.

***The IRR/First Root:*** Using Table 2, the valid range is $2.125\leq i\leq 5.250$. The conditional PDF and the CDF expressions are:

$$f\left(i\right)=0.32\left(i-2.125\right)\frac{e^{0.16\left(i-2.125\right)^{2}-1.5625}}{1-e^{-1.5625}} and F\left(i\right)=1-\frac{1-e^{0.16\left(i-2.125\right)^{2}-1.5625}}{1-e^{-1.5625}}.$$

***The IRR/Second Root:*** The valid range is $-1\leq i\leq 2.125$. The PDF and the CDF expressions are:$f\left(i\right)=-0.32\left(i-2.125\right)\frac{e^{0.16\left(i-2.125\right)^{2}-1.5625}}{1-e^{-1.5625}} and F\left(i\right)=\frac{1-e^{0.16\left(i-2.125\right)^{2}-1.5625}}{1-e^{-1.5625}}.$

If the MARR = 4.5, for the project to be favorable, the MARR must be between the roots.

 $F\_{2}$(4.5) – $F\_{1}$(4.5) = 1 – $F\_{1}$(4.5) = 0.611. Since $P(A\_{1}^{2}-4A\_{0}A\_{2}\geq 0)=P(10^{8}\geq 6400A\_{2}) $= $1-e^{-1.5625}=0.7904,$ the actual probability the project is desirable is (0.611 \* 0.7904) = 0.483. Using Table 4, the applicable ERR CDF for a MARR of 4.5 is$F\left(ERR\right)=e^{-\frac{166.375}{\left(ERR+1\right)^{2}}+4.84}$ in

 -1 $\leq $ ERR $\leq $ 4.86 range. Then, P (ERR > MARR) = 1-$ F(4.5)$ = 0.483 which is the same as the IRR solution. Using a MARR of 0.25, $F\_{2}$(0.25) – $F\_{1}$(0.25) = P($i\_{2}$ $\leq $ 0.25) – 0 = $F\_{2}\left(0.25\right)$. $F\_{2}\left(MARR\right)=\frac{1-e^{0.16\left(MARR-2.125\right)^{2}-1.5625}}{1-e^{-1.5625}} =0.7998$. As noted earlier, this is a conditional probability. Since $P(A\_{1}^{2}-4A\_{0}A\_{2}\geq 0)=P(10^{8}\geq 6400A\_{2}) $= $1-e^{-1.5625}=0.7904. $ The actual probability the project is acceptable is (0.7998 \* 0.7904) = 0.6321. Letting $MARR=0.25$ and using Table 4, the valid range for the ERR is $-1\leq ERR\leq $ $1.7951$. The PDF and the CDF expressions are: $f\left(ERR\right)=\frac{3.9062}{(ERR+1)^{3}}e^{-\frac{1.9531}{\left(ERR+1\right)^{2}}+0.25} and F\left(ERR\right)=e^{-\frac{1.9531}{\left(ERR+1\right)^{2}}+0.25}.$

Figure 1 shows that simulated PDF of the ERR resembles to the analytical one shown in Figure 8

in the article. Figure 2 shows the analytical CDF of the ERR for this case.

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Figure 1. Simulated PDF of the ERR for $A\_{2} Exponential$ (MARR = 0.25)

****Figure 2. The CDF of the ERR for $A\_{2} Exponential$ (MARR = 0.25)

Again, the decision maker would accept the project if the probability of the ERR exceeding a given MARR is high enough. $P\left(ERR>MARR\right)=1-F\left(MARR\right)=1-e^{-\frac{1.9531}{\left(MARR+1\right)^{2}}+0.25}.$ $Let MARR=0.25$, then $P\left(ERR>MARR\right)=1-F\left(MARR\right)=0.6321$ which is the same as the result found with the IRR approach.

**Case 4: Numerical Example for** $A\_{1}$ **Exponential**

$Let A\_{0}=\$1,600, A\_{1} ∼Exponential \left(λ=^{1}/\_{\$10,000}\right), and A\_{2}=\$10,000.$

The term $A\_{1}^{2}-4 A\_{0}A\_{2}$ ranges in [-64000000, ∞] indicating there are not real always roots. The real roots happen when A1 ≥ $8,000 which happens with a probability of 0.4493.

***The IRR/First Root:*** Using Table 2, the valid range is $1.5\leq i\leq \infty $. The PDF and the CDF are$ f\left(i\right)=\left(0.16-\frac{1}{\left(i+1\right)^{2}}\right)\left(e^{-0.16\left(i+1\right)-\frac{1}{\left(i+1\right)}+0.8}\right)$ , and$ F\left(i\right)=1-e^{-0.16\left(i+1\right)-\frac{1}{\left(i+1\right)}+0.8}$.
Figure 3 shows the simulated conditional PDF of the first IRR root which resembles to Figure 9 in the article.


Figure 3. Simulated Conditional PDF of the IRR – First Root for $A\_{1} Exponential$

***The IRR/Second Root:*** The valid range is $-1\leq i\leq 1.5$. The PDF and the CDF expressions are:

$f\left(i\right)=\left(-0.16+\frac{1}{\left(i+1\right)^{2}}\right)\left(e^{-0.16\left(i+1\right)-\frac{1}{\left(i+1\right)}+0.8}\right)and F\left(i\right)=e^{-0.16\left(i+1\right)-\frac{1}{\left(i+1\right)}+0.8}.$

Using MARR = 2.45, $F\_{2}$(2.45) – $F\_{1}$(2.45) = 1 – $F\_{1}$(2.45) = 0.959. The actual probability that the project is desirable is (0.959 \* 0.4493) = 0.430. The applicable ERR CDF is $1 – e^{-0.07073 \left(ERR + 1\right)^{2}} $and P (ERR > MARR) = 1 – F(2.45) = 0.4309. .

Using a MARR of 0.25, $F\_{1}\left(0.25\right)=0 $and Equation 10 becomes $F\_{2}\left(0.25\right) only$. $F\_{2}\left(MARR\right)=e^{-0.16\left(MARR+1\right)-\frac{1}{\left(MARR+1\right)}+0.8}$ = 0.8187. Again, this is a conditional probability. Here the IRR only exists if $A\_{1}\geq 8000$, which has a probability of 0.4493 so the actual probability that the project is desirable is (0.8187)\*(0.4493) = 0.3679. The analytical IRR PDF and CDF graphs for the second root are shown in Figures 4 and 5. Simulated conditional PDF plot for the second IRR is shown in Figure 6.


Figure 4. The Conditional PDF of the IRR – Second Root for $A\_{1} Exponential$


Figure 5. The Conditional CDF of the IRR - Second Root for $A\_{1} Exponential$

Figure 6. Simulated Conditional PDF of the IRR - Second Root for $A\_{1} Exponential$

For the ERR, $let MARR=0.25. $Using Table 4, the valid range is $-1\leq i\leq \infty $. The PDF and the CDF expressions are: $f\left(ERR\right)=1.28\left(ERR+1\right)e^{-0.64\left(ERR+1\right)^{2}} $$and F\left(ERR\right)=1-e^{-0.64\left(ERR+1\right)^{2}}.$

$P\left(ERR>MARR\right)=1-F\left(MARR\right)=1-(1-e^{-0.64\left(MARR+1\right)^{2}})$. $Let MARR=0.25$, then $P\left(ERR>MARR\right)=0.3679$ and is the same as the value found above using the IRR CDF.