**DETAILED SOLUTIONS FOR THE EXAMPLES**

**Case 1: Numerical Example for Uniform**

Here indicating there are two real roots with probability one.

***The IRR/First Root:*** Using Table 1, the valid range is . The PDF and the CDF expressions are and .

***The IRR/Second Root:*** The valid range is . The PDF and the CDF expressions are and .

If the MARR is 3.95 or 395%, = P() ≤ 3.95) – P() ≤ 3.95) =

1-(1-P() > 3.95) = 1 - = 1 – (0.4(3.95 – 2.125)2 – 0.91) = 0.578. Unlike the IRR, the ERR solution requires a MARR value as an input. Using the applicable ERR CDF formula for a MARR of 3.95 in Table 3, in a valid range of 3.867 ≤ ERR ≤ 4.069 and P(ERR > 3.95) = 1 – F(3.95) = 0.574. Both the IRR and the ERR CDF expressions result in the same probability of project being acceptable.

If MARR is 0.25, = P() ≤ 0.25) – P(≤ 0.25) = P( ≤ 0.25) - 0

= (0.25) =. The decision maker can use this information to make a decision for the project for accepting, rejecting, or further study.

Usingthe valid range is . The PDF and the CDF expressions are:

Using the CDF above we can also compute the probability the project is desirable, . This confirms the answer found above using the IRR approach.

**Case 2: Numerical Example for Uniform**

The term ranges in [0, 80000000] indicating two real roots.

***The IRR/First Root:*** Using Table 1, the valid range is . The PDF and the CDF expressions are:

***The IRR/Second Root:*** The valid range is . The PDF and the CDF expressions are

If the MARR is 2.00, (2.00)- (2.00) = P (2.00) – P( 2.00) = 1 – (2.00) = 0.966. The applicable ERR CDF for a MARR of 2.00 is 0.226\*(ERR + 1)2 – 2 in 1.975 ERR 2.644 range. P (ERR > MARR) = 1 – F(2.00) = 0.966.

Using a MARR of 0.25, (0.25)- (0.25) = P( 0.25) – 0 = (0.25) = 0.50.

ERR, . Using Table 3, the valid range is . The PDF and the CDF expressions are:

Here, , which is the same as the value found using the IRR CDF above.

**Case 3: Numerical Example for Exponential**

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The term ranges in [- ∞ , 100000000] indicating there are not always roots. The roots that exist happen when A2 $15,625 which happens with a probability of 0.7904.

***The IRR/First Root:*** Using Table 2, the valid range is . The conditional PDF and the CDF expressions are:

***The IRR/Second Root:*** The valid range is . The PDF and the CDF expressions are:

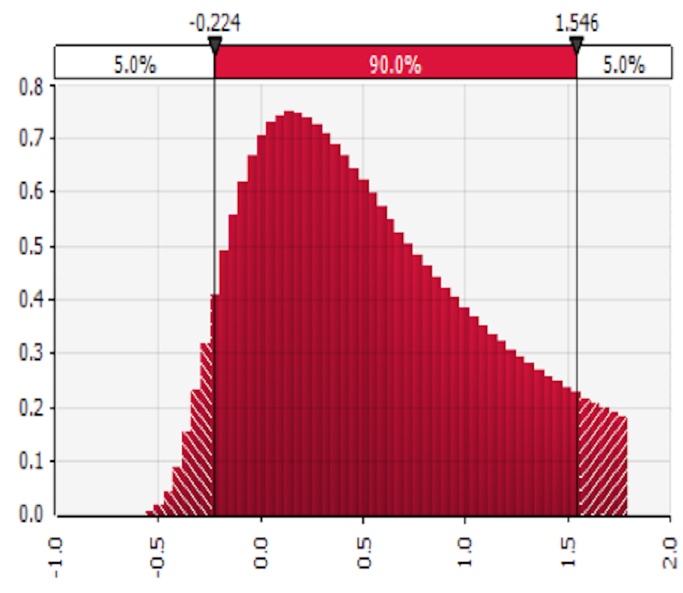
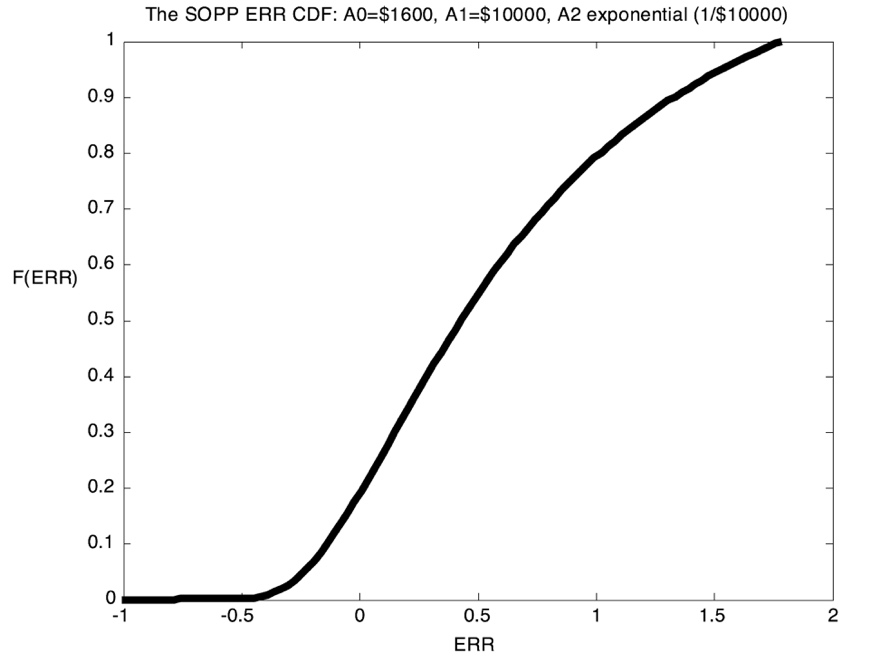
If the MARR = 4.5, for the project to be favorable, the MARR must be between the roots.

(4.5) – (4.5) = 1 – (4.5) = 0.611. Since = the actual probability the project is desirable is (0.611 \* 0.7904) = 0.483. Using Table 4, the applicable ERR CDF for a MARR of 4.5 is in

-1 ERR 4.86 range. Then, P (ERR > MARR) = 1- = 0.483 which is the same as the IRR solution. Using a MARR of 0.25, (0.25) – (0.25) = P( 0.25) – 0 = . . As noted earlier, this is a conditional probability. Since = The actual probability the project is acceptable is (0.7998 \* 0.7904) = 0.6321. Letting and using Table 4, the valid range for the ERR is . The PDF and the CDF expressions are:

Figure 1 shows that simulated PDF of the ERR resembles to the analytical one shown in Figure 8

in the article. Figure 2 shows the analytical CDF of the ERR for this case.

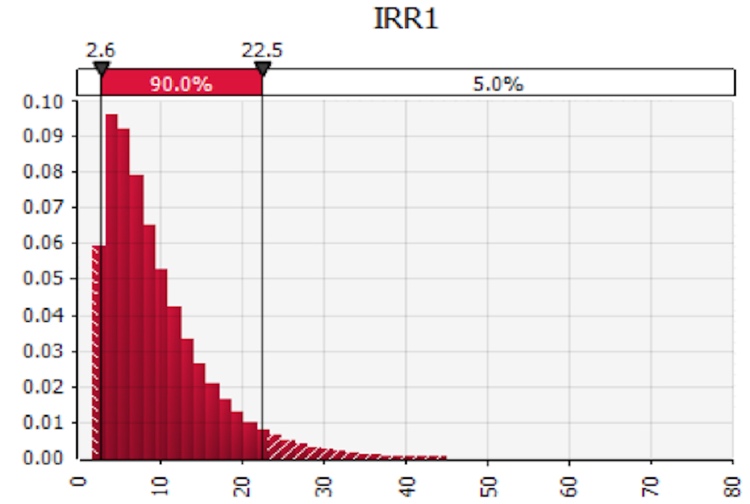
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Figure 1. Simulated PDF of the ERR for (MARR = 0.25)  
  
****Figure 2. The CDF of the ERR for (MARR = 0.25)

Again, the decision maker would accept the project if the probability of the ERR exceeding a given MARR is high enough. , then which is the same as the result found with the IRR approach.

**Case 4: Numerical Example for Exponential**

The term ranges in [-64000000, ∞] indicating there are not real always roots. The real roots happen when A1 ≥ $8,000 which happens with a probability of 0.4493.

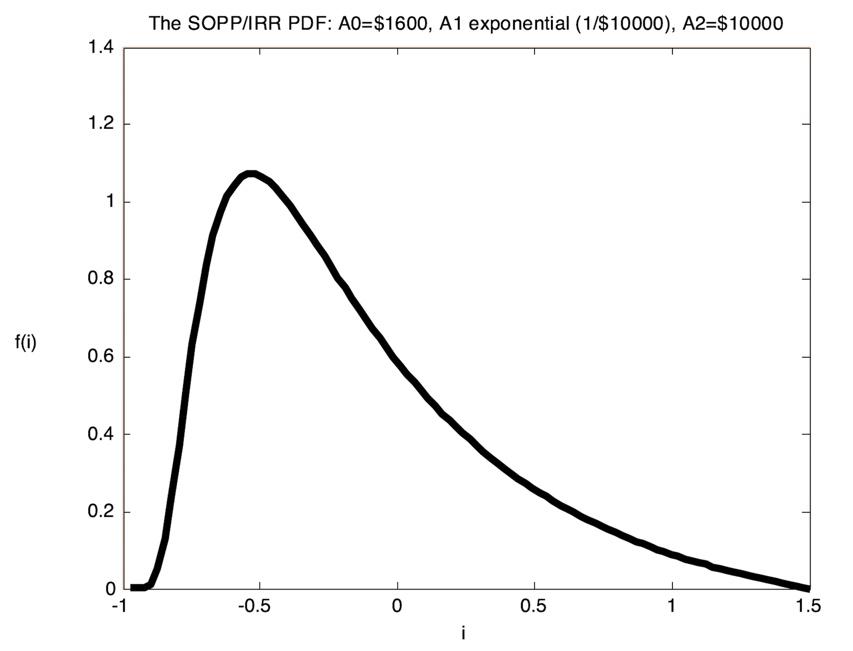
***The IRR/First Root:*** Using Table 2, the valid range is . The PDF and the CDF are , and.   
Figure 3 shows the simulated conditional PDF of the first IRR root which resembles to Figure 9 in the article.

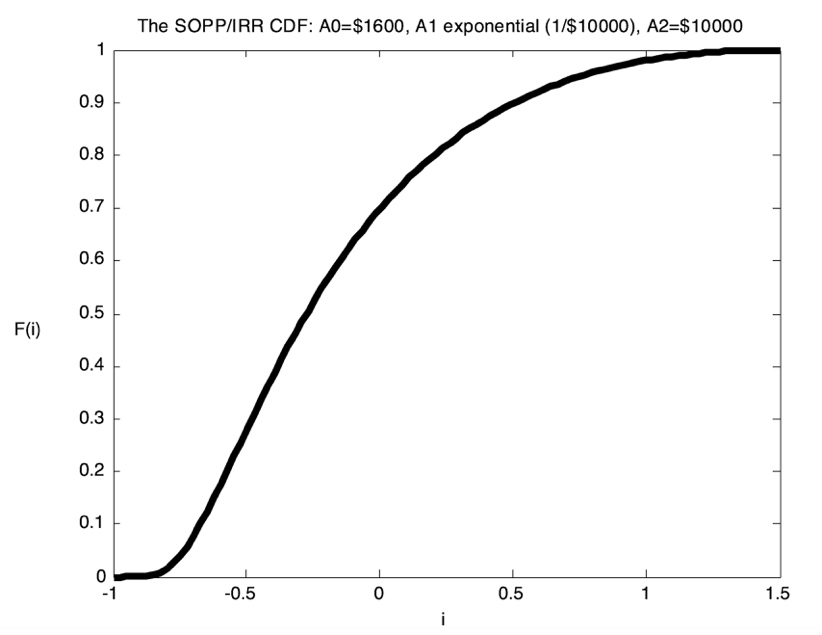
  
Figure 3. Simulated Conditional PDF of the IRR – First Root for

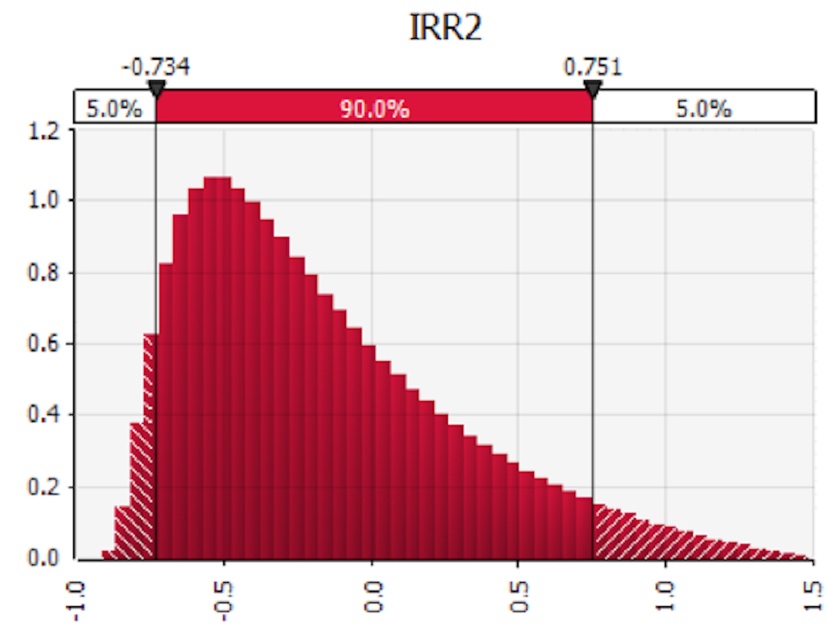
***The IRR/Second Root:*** The valid range is . The PDF and the CDF expressions are:

Using MARR = 2.45, (2.45) – (2.45) = 1 – (2.45) = 0.959. The actual probability that the project is desirable is (0.959 \* 0.4493) = 0.430. The applicable ERR CDF is and P (ERR > MARR) = 1 – F(2.45) = 0.4309. .

Using a MARR of 0.25, and Equation 10 becomes . = 0.8187. Again, this is a conditional probability. Here the IRR only exists if , which has a probability of 0.4493 so the actual probability that the project is desirable is (0.8187)\*(0.4493) = 0.3679. The analytical IRR PDF and CDF graphs for the second root are shown in Figures 4 and 5. Simulated conditional PDF plot for the second IRR is shown in Figure 6.

  
Figure 4. The Conditional PDF of the IRR – Second Root for

  
Figure 5. The Conditional CDF of the IRR - Second Root for

Figure 6. Simulated Conditional PDF of the IRR - Second Root for

For the ERR, Using Table 4, the valid range is . The PDF and the CDF expressions are:

. , then and is the same as the value found above using the IRR CDF.