Appendix A. Mathematical details

A.1. Notation

This paper uses the decomposition of the Total Factor Productivity (TFP), as proposed by Ferreira and Marques (2016), in order to evaluate the relative performance of a set of four different groups of councils. For the sake of simplicity and without loss of generality, let us consider two groups, A and B, with sizes N_A and N_B, respectively. Each group is characterized by an efficient frontier, which is composed of the efficient entities in the Pareto-Koopmans sense. We use the following notation:

- $\Omega_A = \{1, \dots, j_A, \dots, n_A\}$ is the set of decision making units (councils) composing the group A;
- $\left(X_{ij_A}, Y_{rj_A} | Z_{pj_A}\right) \in \mathbb{R}_+^{(m+s)\times 1} \times \mathbb{R}^{q\times 1}$ the set of m resources (X) consumed by the j_Ath council from A in order to deliver s different types of services (Y), and subjected to q characteristics, Z, external to the production process;
- $\partial\Omega_A$ is the efficient frontier (also called *technology*) of groups A;
- $\left(x_{0,i}, y_{0,r} | z_{0,p}\right) \in \Psi_A$ is the council under evaluation, also denoted by council$_0$;
- $\left(x_{0,i,\vec{d}}^{\star B}, y_{0,r,\vec{d}}^{\star B}\right) \in \mathbb{R}_+^{(m+s)\times 1}$ is the set of m+s targets of council$_0$ with respect to the frontier of group B, $\partial\Omega_B$, and $\vec{d} = (\overrightarrow{d_X}, \overrightarrow{d_Y})$ is a directional vector controlling for the direction in which council $\left(x_{0,i}, y_{0,r} | z_{0,p}\right)$ is projected on $\partial\Omega_*$.

A.2. Assessing economies of scope

Definition 1 deals with how the TFP between two councils can be assessed. Together with Hypothesis 1, it will be employed henceforth. From Definition 1, if council$_2$ is less productive than council$_1$, then $TFP_{AB}(x_1, y_1; x_2, y_2) < 1$. Furthermore, this TFP formulation is decomposable into several terms. With respect to economies of scope, the most important is the technological gap, as described in Definition 2.

Definition 1 (TFP): TFP between two councils, $(x_1, y_1|z_1)$ and $(x_2, y_2|z_2)$ is defined by $TFP(x_1, y_1; x_2, y_2) = \frac{\mathcal{A}(y_2)}{\mathcal{A}(y_1)} \Big/ \frac{\mathcal{D}(x_2)}{\mathcal{D}(x_1)}$, with $\mathcal{A}: \mathbb{R}^s \to \mathbb{R}$ and $\mathcal{D}: \mathbb{R}^m \to \mathbb{R}$.■

Hypothesis 1 (TFP): If both $\mathcal{A}$ and $\mathcal{D}$ functions are the geometric mean of their own arguments (vectors), then the measure of TFP obeys/verifies the axioms of positivity, continuity, monotonicity, homogeneity, identity, commensurability and reversal property (O'Donnell, 2012): that is, it is a multiplicatively complete index (Ferreira and Marques, 2016). The TFP can be computed as in equation (1).

$$TFP(x_1, y_1; x_2, y_2) = \frac{\mathcal{A}(y_2)}{\mathcal{A}(y_1)} \Big/ \frac{\mathcal{D}(x_2)}{\mathcal{D}(x_1)} = \left(\prod_{r=1}^{s} \frac{y_{2,r}}{y_{1,r}}\right)^{\frac{1}{s}} \Big/ \left(\prod_{i=1}^{m} \frac{x_{2,i}}{x_{1,i}}\right)^{\frac{1}{m}} \quad (1)$$

■

Definition 2 (Technological gap): There is a technological gap (TG) between two technologies, $\partial\Omega_A$ and $\partial\Omega_B$, if one of them can deliver more services (outputs) with fewer resources than the other. Considering the councils $(x_1, y_1|z_1) \in \Omega_A$ and

$(x_2, y_2|z_2) \in \Omega_B$, council$_1$ lies in a region of Ω_A more productive than Ω_B if: its benchmarks consume fewer resources than the ones in $\partial\Omega_B$, *i.e.* $\delta I_{AB}(x_1, y_1; x_2, y_2) < 1$, and/or those benchmarks produce more goods/services than their counterparts from Ω_A, *i.e.* $\delta O_{AB}(x_1, y_1; x_2, y_2) < 1$. Additionally, $\phi_{AB}(*) = \delta O_{AB}(*) \cdot \delta I_{AB}(*)$. Thus, $\phi_{AB}(x_1, y_1; x_2, y_2) < 1$ when $\partial\Omega_A$ is more productive than $\partial\Omega_B$ in the production regions of COUNCIL$_1$ and council$_2$, respectively, (Portela and Thanassoulis, 2006; Ferreira and Marques, 2016) being:

$$\begin{cases} \delta I_{AB}(x_1, y_1; x_2, y_2) = \sqrt[2\cdot m]{\prod_{i=1}^{m} \dfrac{x_{2,i,\vec{d}}^{\star A} \cdot x_{1,i,\vec{d}}^{\star A}}{x_{2,i,\vec{d}}^{\star B} \cdot x_{1,i,\vec{d}}^{\star B}}} \\ \delta O_{AB}(x_1, y_1; x_2, y_2) = \sqrt[2\cdot s]{\prod_{r=1}^{s} \dfrac{y_{2,r,\vec{d}}^{\star B} \cdot y_{1,r,\vec{d}}^{\star B}}{y_{2,r,\vec{d}}^{\star A} \cdot y_{1,r,\vec{d}}^{\star A}}} \end{cases} \tag{2}$$

■

Definition 3 (Economies of scope): A council exploits economies of scope when it can produce two goods by consuming fewer resources than it would occur under the separate production of those outputs (Baumol *et al.*, 1988).

Definitions 2 and 3 complement each other. Economies of scope occur whenever the non-externalizing councils outperform the externalizing ones. Mathematically, this occurs when $\phi_{E,NE}(x_1, y_1; x_2, y_2) < 1$, with $(x_1, y_1|z_1) \in \Omega_E$ and $(x_2, y_2|z_2) \in \Omega_{NE}$. It is promoted by $\delta I_{E,NE} < 1$ (for the output levels close to y_1 and y_2, *NE* consumes less resources than *E*) and/or $\delta O_{E,NE} < 1$ (for the input levels close to x_1 and x_2, *E* produces less goods/services than *NE*). We should note that the values of $\phi_{E,NE}(x_1, y_1; x_2, y_2)$ are dependent on the input/output relationship, since it is not usual that a frontier, $\partial\Omega_E$ or $\partial\Omega_{NE}$, dominates the other over the whole range of inputs and outputs. In other words, after a certain threshold, we should be able to determine (dis)economies of scope.

A.3. Targets computation

As we can see from equation (2), it is defined by means of both input and output targets, with respect to two different frontiers and according to a pre-specified directional vector. Hypothesis 2 and hypothesis 3 are valid hereinafter:

Hypothesis 2 (Targets): Targets are assessed through a semi-parametric tool (directional order-α, cf. *e.g.* Aragon *et al.*, 2005; Daraio and Simar, 2014) for such a purpose. ■

The directional order-α (DO-α) method is less sensitive to outliers, extreme data and the curse of dimensionality (high number of variables and low number of councils). DO-α allows for both input contraction and output expansion to reach the efficient frontier. DO-α empirically determines that frontier after defining the probability of observing councils above the frontier, 1 – α, (Daraio and Simar, 2007), and estimates a radial distance measure, $\hat{\mathscr{b}}$, according to the direction defined by $\vec{d} = \left(\overrightarrow{d_X}, \overrightarrow{d_Y}\right) \neq \vec{0}$.

Hypothesis 3 (Parameters): In this study, we select α=0.99, *i.e.* assuming the existence of 1% of potential outliers. Furthermore, $\vec{d} = \left(\overrightarrow{d_X}, \overrightarrow{d_Y}\right) = (\vec{1}, \vec{1})$ is assumed for targets assessment. This choice of directional vector assumes that both the input contraction and output expansion occur at the same rate. ■

Definition 4. (DO-α). Consider the transformation $\left(X'_{ij_A}, Y'_{rj_A}\right) = \left(\exp\left\{\frac{X_{ij_A}}{d_X}\right\}, \exp\left\{\frac{Y_{rj_A}}{d_Y}\right\}\right), j_A = 1, \dots, N_A$ (Daraio and Simar, 2014), and the equation:

$$\mathfrak{F}^A\left(x_{0,i}, y_{0,r}\right) = \min\left\{\min_{i=1,\dots,m}\left(\frac{x'_{0,i}}{X'_{ij_A}}\right), \min_{r=1,\dots,s}\left(\frac{Y'_{rj_A}}{y'_{0,r}}\right)\right\} \tag{3}$$

where $\mathbb{I}$ is the indicator function. Let us denote by $\mathfrak{F}^{xy}_{(\mathcal{L})}$ the $\mathcal{L}$th order statistic of the N_A councils, such that $\mathfrak{F}^A_{(1)} \leq \cdots \leq \mathfrak{F}^A_{(\mathcal{L})} \leq \cdots \leq \mathfrak{F}^A_{(N_A)}$. The radial order-α based distance is given as follows:

$$\hat{\mathscr{b}}\left(x_{0,i}, y_{0,r}\right) = \log\begin{cases} \mathfrak{F}^A_{(\alpha n_A)} & \text{if } \alpha n_A \text{ is an integer} \\ \mathfrak{F}^A_{([\alpha n_A]+1)} & \text{otherwise} \end{cases} \tag{4}$$

Council $\left(x_{0,i}, y_{0,r}\right)$ is technically efficient regarding the α-level frontier of Ω_A, say $\partial\Omega_A^{(\alpha)}$, if $\hat{\mathscr{b}} = 0$. It is technically inefficient if $\hat{\mathscr{b}} > 0$ and super-efficient if $\hat{\mathscr{b}} < 0$.

Targets of council $(x_{0,i}, y_{0,r})$ are, then, computed through equation (5) [because of Hypothesis 3].

$$\begin{cases} x^{\star A}_{0,i,\overrightarrow{\delta_I}} = x_{0,i} - \hat{\mathscr{b}} \cdot \vec{d} = x_{0,i} - \vec{\hat{\mathscr{b}}} \\ y^{\star A}_{0,r,\overrightarrow{\delta_O}} = y_{0,r} + \hat{\mathscr{b}} \cdot \vec{d} = y_{0,r} + \vec{\hat{\mathscr{b}}} \end{cases} \tag{5}$$

Mutatis mutandis, it is easy to obtain the remaining targets required for equations (2). ■

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