Supplemental material for “A Central Limit Theorem for Correlated Limited Normal or Gamma Random Variables”

Part 1.0 of Supplement

The first portion of this section will demonstrate that the two matrices below are inverses of one another (compare with in expression (2.2) on page 5 of main body of the paper, when )

, 

To begin, write and 

The first observation is:  (S.1)

If , then (S.1) reduces to which itself reduces to if , or 0 if .

Similarly, if , then (S.1) becomes 

Finally, if , then (S.1) is equal to:



Consequently, (S.1) is equal to whenever and 0 otherwise; it follows that and are inverses of one another.

Part 1.1 of Supplement

The goal of this section of the supplement is to verify expression (2.5) on page 6 of the paper, which is:



Recall that the matrix , where .

First note,  (S.2)

In the interest of simplifying notation, let . To evaluate , write , then

 and, from part 1.1 of this supplement,



To analyze the matrix product, consider three cases:  In the first case,



In the second case, :



And finally, :



Consequently,

 (i.e., only the first column is non-zero)

A completely analogous computation shows that

 (i.e., only the last column is non-zero)

It follows that the product of matrices in expression (S.2) is equal to:

= (S.3)

That is, the only non-zero entries are along the main diagonal, in column  on or below the main diagonal, and in column +1, on or above the main diagonal. Therefore, to verify expression (7) of the main paper, i.e.,



it remains to show that the by matrix , defined by



has the property

 (S.4)

That is, multiplication by M eliminates the off-diagonal terms in expression (S.3) above. As a first step, recall:



Since has only two non-zero rows and two non-zero columns, to compute  it is necessary to only check four cases, the first of which is .



When we have:



Similarly, implies:



And finally, yields:



This completes the argument and verifies expression (S.4) and demonstrates that



Part 2.0 of Supplement (details supporting Appendix A)

The second section of this supplement will demonstrate that, if  is defined by





then the first derivative at zero, is equal to

> 0 (S.2)

Following Royden (1968, p.-), since the integrands in each of the two terms are bounded, the first derivative of is









Evaluating yields expression (S.2) above.

Part 3.0 of Supplement: An Example

Let denote an atomic limited normal random variable with distribution function, G, given by:



 (S.3)

Then , the expected value of , is equal to zero and a numeric computation shows that = , its variance, lies between 1.0 and 1.001.

Similarly, let denote a set of independent, non-atomic random variables that are also independent of and have distribution functions given by:

 (S.4)



( is defined in the paragraph below)

Then the expected value,  , and a numeric computation shows that the variance , ranges between 0.01 and 8.05 for .

Finally, let and define . Then the expected value, ( and the variance of each is , by the independence of and . So, to ensure that the variance of all the are all equal, select so that . It is straight forward to show that is the correlation between and . Finally, summing and dividing by the product of and the number of terms yields the following expression:

= (S.5)

As n increases, the first term on the left of (S.5) approaches zero and the Central Limit Theorem implies that the distribution of expression (S.5) approaches the standard normal, .