## SUPPLEMENTARY INFORMATION

Table 1 provides Euler angles of all $\beta$ grains and $\mathrm{TJ} \alpha$ for each of the 5 triple junctions that have been analyzed. Table 2-6 contains the deviation from BOR of all 36 possible TJ $\alpha$ variants at each triple junction from each of the 2 adjacent grains to which they are nonburgers oriented. The methods used in these calculations are similar to those used in [1] and are not repeated here. Finally, Table 8 provides all combinations of $3 \beta$ grains at a triple junction related by a special misorientations given in Table 7 [2] that will allow TJ $\alpha$ to be BOR related to all 3 grains


Table 1: The Euler angles (Bunge notation) for the three $\beta$ grains constituting all TJ examined in this study. The disorientations between the 3 grains are also provided.

| Variants are in BOR with $\beta_{1}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{3}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $29.6{ }^{\circ}$ | 1 | $23.7^{\circ}$ | $26.7^{\circ}$ |
| 2 | $22.4{ }^{\circ}$ | 2 | $27.9^{\circ}$ | $25.1^{\circ}$ |
| 3 | $29.6{ }^{\circ}$ | 3 | $33.9{ }^{\circ}$ | $31.8{ }^{\circ}$ |
| 4 | $25.0^{\circ}$ | 4 | $38.3^{\circ}$ | $31.6^{\circ}$ |
| 5 | $29.0^{\circ}$ | 5 | $34.8{ }^{\circ}$ | $31.9^{\circ}$ |
| 6 | $21.5^{\circ}$ | 6 | $33.9{ }^{\circ}$ | $27.7^{\circ}$ |
| 7 | $29.6{ }^{\circ}$ | 7 | $43.1^{\circ}$ | $36.4{ }^{\circ}$ |
| 8 | $25.7^{\circ}$ | 8 | $43.6{ }^{\circ}$ | $34.7^{\circ}$ |
| 9 | $28.7^{\circ}$ | 9 | $20.2^{\circ}$ | $24.4{ }^{\circ}$ |
| 10 | $29.6{ }^{\circ}$ | 10 | $25.9{ }^{\circ}$ | $27.7^{\circ}$ |
| 11 | $29.6{ }^{\circ}$ | 11 | $32.1^{\circ}$ | $30.8{ }^{\circ}$ |
| 12 | $29.6{ }^{\circ}$ | 12 | $36.6{ }^{\circ}$ | $33.1^{\circ}$ |
| Variants are in BOR with $\beta_{2}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{3}$ | $\triangle \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $22.4{ }^{\circ}$ | 1 | $22.1^{\circ}$ | $22.2^{\circ}$ |
| 2 | $29.6{ }^{\circ}$ | 2 | $21.3^{\circ}$ | $25.5^{\circ}$ |
| 3 | $25.0^{\circ}$ | 3 | $32.2{ }^{\circ}$ | $28.6{ }^{\circ}$ |
| 4 | $29.6{ }^{\circ}$ | 4 | $28.5^{\circ}$ | $29.1^{\circ}$ |
| 5 | 21.5 ${ }^{\circ}$ | 5 | $42.0{ }^{\circ}$ | $31.8{ }^{\circ}$ |
| 6 | $29.0^{\circ}$ | 6 | $42.6{ }^{\circ}$ | $35.8{ }^{\circ}$ |
| 7 | $25.7^{\circ}$ | 7 | $28.5^{\circ}$ | $27.1^{\circ}$ |
| 8 | $29.6{ }^{\circ}$ | 8 | $27.2^{\circ}$ | $28.4{ }^{\circ}$ |
| 9 | $29.6{ }^{\circ}$ | 9 | $27.2^{\circ}$ | $28.4{ }^{\circ}$ |
| 10 | $28.7^{\circ}$ | 10 | $28.7^{\circ}$ | $28.7^{\circ}$ |
| 11 | $29.6{ }^{\circ}$ | 11 | $13.4{ }^{\circ}$ | $21.5^{\circ}$ |
| 12 | $29.6{ }^{\circ}$ | 12 | $19.6{ }^{\circ}$ | $24.6{ }^{\circ}$ |
| Variants are in BOR with $\beta_{3}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | $\triangle \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $34.2{ }^{\circ}$ | 1 | $19.6{ }^{\circ}$ | $26.9{ }^{\circ}$ |
| 2 | $32.1^{\circ}$ | 2 | $13.4{ }^{\circ}$ | $22.7^{\circ}$ |
| 3 | $33.9{ }^{\circ}$ | 3 | $21.3^{\circ}$ | $27.6^{\circ}$ |
| 4 | $34.8{ }^{\circ}$ | 4 | $22.1^{\circ}$ | $28.4{ }^{\circ}$ |
| 5 | $20.2^{\circ}$ | 5 | $35.8{ }^{\circ}$ | $28.0^{\circ}$ |
| 6 | $25.9{ }^{\circ}$ | 6 | $35.4{ }^{\circ}$ | $30.6{ }^{\circ}$ |
| 7 | $27.9^{\circ}$ | 7 | $27.2^{\circ}$ | $27.5^{\circ}$ |
| 8 | $23.7^{\circ}$ | 8 | $28.5{ }^{\circ}$ | $26.1^{\circ}$ |
| 9 | 38.3 | 9 | $28.5{ }^{\circ}$ | $33.4{ }^{\circ}$ |
| 10 | $33.9{ }^{\circ}$ | 10 | $32.2^{\circ}$ | $33.0{ }^{\circ}$ |
| 11 | $38.0^{\circ}$ | 11 | $28.7^{\circ}$ | $33.4{ }^{\circ}$ |
| 12 | $34.8{ }^{\circ}$ | 12 | $27.2^{\circ}$ | $31.0^{\circ}$ |

Table 2: Analysis of TJ 1. The deviation from the BOR with adjacent grains for 3 three sets of $12 \alpha$ variants that are oriented in the Burgers relationship with grains $\beta_{1}, \beta_{2}$ and $\beta_{2}$ respectively. The experimentally observed variant is in bold lettering.

| Variants are in BOR with $\beta_{1}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| variant | $\triangle \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | variant | $\triangle \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{3}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $16.2^{\circ}$ | 1 | $17.7^{\circ}$ | $16.9^{\circ}$ |
| 2 | $19.4{ }^{\circ}$ | 2 | $19.9{ }^{\circ}$ | $19.6{ }^{\circ}$ |
| 3 | $19.4{ }^{\circ}$ | 3 | $19.1^{\circ}$ | $19.2^{\circ}$ |
| 4 | $19.4{ }^{\circ}$ | 4 | $19.9^{\circ}$ | $19.6{ }^{\circ}$ |
| 5 | $19.4{ }^{\circ}$ | 5 | $9.8{ }^{\circ}$ | $14.6{ }^{\circ}$ |
| 6 | $19.4{ }^{\circ}$ | 6 | $19.9{ }^{\circ}$ | $19.6{ }^{\circ}$ |
| 7 | $10.5^{\circ}$ | 7 | $19.9{ }^{\circ}$ | $15.2^{\circ}$ |
| 8 | $19.4{ }^{\circ}$ | 8 | $19.9{ }^{\circ}$ | $19.6{ }^{\circ}$ |
| 9 | $18.3{ }^{\circ}$ | 9 | $19.9{ }^{\circ}$ | $19.1^{\circ}$ |
| 10 | $19.4{ }^{\circ}$ | 10 | $15.4{ }^{\circ}$ | $17.4{ }^{\circ}$ |
| 11 | $12.8{ }^{\circ}$ | 11 | $19.9{ }^{\circ}$ | $16.3^{\circ}$ |
| 12 | $19.4{ }^{\circ}$ | 12 | $17^{\circ}$ | $18.2^{\circ}$ |
| Variants are in BOR with $\beta_{2}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{3}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $19.4{ }^{\circ}$ | 1 | $32.6{ }^{\circ}$ | $26^{\circ}$ |
| 2 | $16.2^{\circ}$ | 2 | $32.6^{\circ}$ | $24.4{ }^{\circ}$ |
| 3 | $19.4{ }^{\circ}$ | 3 | $31.9^{\circ}$ | $25.7^{\circ}$ |
| 4 | $19.4{ }^{\circ}$ | 4 | $32.6{ }^{\circ}$ | $26^{\circ}$ |
| 5 | $19.4{ }^{\circ}$ | 5 | $25.6^{\circ}$ | $22.5{ }^{\circ}$ |
| 6 | $19.4{ }^{\circ}$ | 6 | $32.6^{\circ}$ | $26^{\circ}$ |
| 7 | $19.4{ }^{\circ}$ | 7 | $32.6^{\circ}$ | $26^{\circ}$ |
| 8 | $10.5^{\circ}$ | 8 | $27.2^{\circ}$ | $18.8{ }^{\circ}$ |
| 9 | $19.4{ }^{\circ}$ | 9 | $32.6{ }^{\circ}$ | $26^{\circ}$ |
| 10 | $18.3^{\circ}$ | 10 | $28.3^{\circ}$ | $23.3^{\circ}$ |
| 11 | $19.4{ }^{\circ}$ | 11 | $28^{\circ}$ | $23.7^{\circ}$ |
| 12 | $12.8{ }^{\circ}$ | 12 | $24.4{ }^{\circ}$ | $18.6^{\circ}$ |
| Variants are in BOR with $\beta_{3}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $19.1{ }^{\circ}$ | 1 | $31.9^{\circ}$ | $25.5^{\circ}$ |
| 2 | $19.9{ }^{\circ}$ | 2 | $32.6{ }^{\circ}$ | $26.2^{\circ}$ |
| 3 | $19.9{ }^{\circ}$ | 3 | $32.6{ }^{\circ}$ | $26.2^{\circ}$ |
| 4 | $17.7^{\circ}$ | 4 | $32.6^{\circ}$ | $25.1^{\circ}$ |
| 5 | $19.9{ }^{\circ}$ | 5 | $32.6^{\circ}$ | $26.2^{\circ}$ |
| 6 | $15.4{ }^{\circ}$ | 6 | $28.3^{\circ}$ | $21.8{ }^{\circ}$ |
| 7 | $17^{\circ}$ | 7 | $24.4{ }^{\circ}$ | $20.7^{\circ}$ |
| 8 | $19.9{ }^{\circ}$ | 8 | $28^{\circ}$ | $24^{\circ}$ |
| 9 | $19.9{ }^{\circ}$ | 9 | $27.2^{\circ}$ | $23.5^{\circ}$ |
| 10 | $19.9{ }^{\circ}$ | 10 | $32.6{ }^{\circ}$ | $26.2^{\circ}$ |
| 11 | $9.8{ }^{\circ}$ | 11 | $25.6{ }^{\circ}$ | $17.7^{\circ}$ |
| 12 | $19.9{ }^{\circ}$ | 12 | $32.6^{\circ}$ | $26.2^{\circ}$ |

Table 3: Analysis of TJ 2. The deviation from the BOR with adjacent grains for 3 three sets of $12 \alpha$ variants that are oriented in the Burgers relationship with grains $\beta_{1}, \beta_{2}$ and $\beta_{2}$ respectively. The experimentally observed variant is in bold lettering.

| Variants are in BOR with $\beta_{1}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| variant | $\triangle \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | variant | $\triangle \mathrm{BOR}_{\text {th }}$ with respect to $\qquad$ | $\Delta \mathrm{BOR}_{\text {thav }}$ |
| 1 | $27.8^{\circ}$ | 1 | $27.4^{\circ}$ | $27.6^{\circ}$ |
| 2 | $24.2^{\circ}$ | 2 | $26.3^{\circ}$ | $25.3^{\circ}$ |
| 3 | $11.1^{\circ}$ |  | $27.4^{\circ}$ | $19.2^{\circ}$ |
| 4 | $14.7^{\circ}$ | 4 | $27.4^{\circ}$ | $21.0^{\circ}$ |
| 5 | $11.3^{\circ}$ | 5 | $22.9^{\circ}$ | $17.1^{\circ}$ |
| 6 | $11.1^{\circ}$ | 6 | $27.4^{\circ}$ | $19.2^{\circ}$ |
| 7 | $33.3^{\circ}$ | 7 | $27.4^{\circ}$ | $30.3^{\circ}$ |
| 8 | $31.0^{\circ}$ | 8 | $20.5^{\circ}$ | $25.7^{\circ}$ |
| 9 | $18.9^{\circ}$ | 9 | $27.4^{\circ}$ | $23.1^{\circ}$ |
| 10 | $11.1^{\circ}$ | 10 | $24.2^{\circ}$ | $17.6^{\circ}$ |
| 11 | $38.8{ }^{\circ}$ | 11 | $27.4^{\circ}$ | $33.1^{\circ}$ |
| 12 | $34.6^{\circ}$ | 12 | $18.9^{\circ}$ | $26.7^{\circ}$ |
| Variants are in BOR with $\beta_{2}$ |  |  |  |  |
| variant | $\triangle \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\triangle \mathrm{BOR}_{\mathrm{th}}$ with respect to $\beta_{3}$ | $\Delta \mathrm{BOR}_{\text {thav }}$ |
| 1 | $18.9^{\circ}$ | 1 | $15.1^{\circ}$ | $17.0^{\circ}$ |
| 2 | $11.1^{\circ}$ | 2 | $15.1^{\circ}$ | $13.1^{\circ}$ |
| 3 | $31.0^{\circ}$ | 3 | $22.7^{\circ}$ | $26.8^{\circ}$ |
| 4 | $34.9{ }^{\circ}$ | 4 | $18.2^{\circ}$ | $26.6^{\circ}$ |
| 5 | $11.1{ }^{\circ}$ | 5 | $16.6^{\circ}$ | $13.8{ }^{\circ}$ |
| 6 | $11.3^{\circ}$ | 6 | $24.9^{\circ}$ | $18.1^{\circ}$ |
| 7 | $24.2^{\circ}$ | 7 | $38.7^{\circ}$ | $31.5^{\circ}$ |
| 8 | $27.8^{\circ}$ | 8 | $33.9^{\circ}$ | $30.9^{\circ}$ |
| 9 | $33.3^{\circ}$ | 9 | $31.0^{\circ}$ | $32.1^{\circ}$ |
| 10 | $33.3^{\circ}$ | 10 | $30.3^{\circ}$ | $31.8^{\circ}$ |
| 11 | $11.1^{\circ}$ | 11 | $16.6^{\circ}$ | $13.8{ }^{\circ}$ |
| 12 | $14.7^{\circ}$ | 12 | $17.7^{\circ}$ | $16.2^{\circ}$ |
| Variants are in BOR with $\beta_{3}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\mathrm{th}}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\mathrm{th}}$ with respect to $\beta_{2}$ | $\triangle \mathrm{BOR}_{\text {thav }}$ |
| 1 | $26.3^{\circ}$ | 1 | $18.2^{\circ}$ | $22.2^{\circ}$ |
| 2 | $27.4^{\circ}$ | 2 | $22.7^{\circ}$ | $25.0^{\circ}$ |
| 3 | $27.4^{\circ}$ | 3 | $16.6^{\circ}$ | $22.0^{\circ}$ |
| 4 | $27.4^{\circ}$ | 4 | $17.7^{\circ}$ | $22.5{ }^{\circ}$ |
| 5 | $27.4^{\circ}$ | 5 | $24.9^{\circ}$ | $26.2^{\circ}$ |
| 6 | $22.9^{\circ}$ | 6 | $16.6^{\circ}$ | $19.8{ }^{\circ}$ |
| 7 | $20.5^{\circ}$ | 7 | $37.1^{\circ}$ | $28.8{ }^{\circ}$ |
| 8 | $27.4^{\circ}$ | 8 | $42.0^{\circ}$ | $34.7^{\circ}$ |
| 9 | $24.2^{\circ}$ | 9 | $15.1^{\circ}$ | $19.6{ }^{\circ}$ |
| 10 | $27.4{ }^{\circ}$ | 10 | $15.1^{\circ}$ | $21.2^{\circ}$ |
| 11 | $18.9^{\circ}$ | 11 | $30.3^{\circ}$ | $24.6{ }^{\circ}$ |
| 12 | $27.4^{\circ}$ | 12 | $31.0^{\circ}$ | $29.2^{\circ}$ |

Table 4: Analysis of TJ 3. The deviation from the BOR with adjacent grains for 3 three sets of $12 \alpha$ variants that are oriented in the Burgers relationship with grains $\beta_{1}, \beta_{2}$ and $\beta_{3}$ respectively. The experimentally observed variant is in bold lettering.

| Variants are in BOR with $\beta_{1}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{3}$ | $\triangle \mathrm{BOR}^{\text {av }}$ |
| 1 | $25.1^{\circ}$ | 1 | 7.2 | 16.2 |
| 2 | $25.9^{\circ}$ | 2 | 7.2 | 16.5 |
| 3 | $22.1{ }^{\circ}$ | 3 | 25.2 | 23.7 |
| 4 | $25.9{ }^{\circ}$ | 4 | 25.2 | 25.6 |
| 5 | $15.9^{\circ}$ | 5 | 12.7 | 14.3 |
| 6 | $25.2^{\circ}$ | 6 | 22.6 | 23.9 |
| 7 | $25.9{ }^{\circ}$ | 7 | 38.2 | 32.0 |
| 8 | $25.9{ }^{\circ}$ | 8 | 37.2 | 31.6 |
| 9 | $25.9{ }^{\circ}$ | 9 | 24.2 | 25.1 |
| 10 | $23.3^{\circ}$ | 10 | 24.2 | 23.8 |
| 11 | $25.9^{\circ}$ | 11 | 12.7 | 19.3 |
| 12 | $20.0^{\circ}$ | 12 | 15.9 | 17.9 |
| Variants are in BOR with $\beta_{2}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ (in degrees) | variant | $\Delta \mathrm{BOR}_{\mathrm{th}}$ with respect to $\beta_{3}$ | $\Delta \mathrm{BOR}_{\text {tav }}$ |
| 1 | $25.9^{\circ}$ | 1 | $19.2^{\circ}$ | $22.5^{\circ}$ |
| 2 | $25.1^{\circ}$ | 2 | $18.2^{\circ}$ | $21.7^{\circ}$ |
| 3 | $25.9^{\circ}$ | 3 | $19.5{ }^{\circ}$ | $22.7{ }^{\circ}$ |
| 4 | $22.1{ }^{\circ}$ | 4 | $19.5{ }^{\circ}$ | $20.8^{\circ}$ |
| 5 | $25.2^{\circ}$ | 5 | $12.4{ }^{\circ}$ | $18.8{ }^{\circ}$ |
| 6 | $15.9^{\circ}$ | 6 | $12.4{ }^{\circ}$ | $14.1^{\circ}$ |
| 7 | $25.9{ }^{\circ}$ | 7 | $19.8{ }^{\circ}$ | $22.8{ }^{\circ}$ |
| 8 | $25.9^{\circ}$ | 8 | $19.8{ }^{\circ}$ | $22.8{ }^{\circ}$ |
| 9 | $23.3^{\circ}$ | 9 | $42.2^{\circ}$ | $32.7^{\circ}$ |
| 10 | $25.9^{\circ}$ | 10 | $40.7^{\circ}$ | $33.3^{\circ}$ |
| 11 | $20.0^{\circ}$ | 11 | $18.2^{\circ}$ | $19.1{ }^{\circ}$ |
| 12 | $25.9^{\circ}$ | 12 | $22.5{ }^{\circ}$ | $24.2{ }^{\circ}$ |
| Variants are in BOR with $\beta_{3}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\mathrm{th}}$ with respect to $\beta_{2}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $7.2^{\circ}$ | 1 | $18.2^{\circ}$ | $12.7^{\circ}$ |
| 2 | $7.2^{\circ}$ | 2 | $19.2{ }^{\circ}$ | $13.2{ }^{\circ}$ |
| 3 | $24.2^{\circ}$ | 3 | $19.8{ }^{\circ}$ | $22.0^{\circ}$ |
| 4 | $24.2^{\circ}$ | 4 | $19.8{ }^{\circ}$ | $22.0{ }^{\circ}$ |
| 5 | $25.2^{\circ}$ | 5 | $42.1{ }^{\circ}$ | $33.7{ }^{\circ 0}$ |
| 6 | $25.2^{\circ}$ | 6 | $40.5^{\circ}$ | $32.9{ }^{\circ}$ |
| 7 | $15.9{ }^{\circ}$ | 7 | $22.5^{\circ}$ | $19.2^{\circ}$ |
| 8 | $12.7^{\circ}$ | 8 | $18.2^{\circ}$ | $15.4{ }^{\circ}$ |
| 9 | $12.7^{\circ}$ | 9 | $12.4{ }^{\circ}$ | $12.5^{\circ}$ |
| 10 | $22.6^{\circ}$ | 10 | $12.4{ }^{\circ}$ | $17.5^{\circ}$ |
| 11 | $35.8{ }^{\circ}$ | 11 | $19.5{ }^{\circ}$ | $27.7^{\circ}$ |
| 12 | $37.5^{\circ}$ | 12 | $19.5{ }^{\circ}$ | $28.5^{\circ}$ |

Table 5: Analysis of TJ 4. The deviation from the BOR with adjacent grains for 3 three sets of $12 \alpha$ variants that are oriented in the Burgers relationship with grains $\beta_{1}, \beta_{2}$ and $\beta_{3}$ respectively. The experimentally observed variant is in bold lettering.

| Variants are in BOR with $\beta_{1}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | variant | $\Delta \mathrm{BOR}_{\mathrm{th}}$ with respect to $\beta_{3}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $19.7^{\circ}$ | 1 | 18.6 | 19.1 |
| 2 | $27.9^{\circ}$ | 2 | 18.6 | 23.2 |
| 3 | $34.3{ }^{\circ}$ | 3 | 18.6 | 26.5 |
| 4 | $31.6^{\circ}$ | 4 | 9.0 | 20.3 |
| 5 | $23.7^{\circ}$ | 5 | 18.6 | 21.2 |
| 6 | $22.0^{\circ}$ | 6 | 15.4 | 18.7 |
| 7 | $9.9^{\circ}$ | 7 | 18.6 | 14.2 |
| 8 | $11.5^{\circ}$ | 8 | 18.6 | 15.0 |
| 9 | $19.7^{\circ}$ | 9 | 18.6 | 19.1 |
| 10 | $24.2^{\circ}$ | 10 | 17.3 | 20.7 |
| 11 | $28.9^{\circ}$ | 11 | 13.2 | 21.1 |
| 12 | $28.9^{\circ}$ | 12 | 18.6 | 23.8 |
| Variants are in BOR with $\beta_{2}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{3}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $28.9^{\circ}$ | 1 | $30.6^{\circ}$ | $29.7^{\circ}$ |
| 2 | $28.9{ }^{\circ}$ | 2 | $31.1^{\circ}$ | $30.0^{\circ}$ |
| 3 | $11.5^{\circ}$ | 3 | $14.6{ }^{\circ}$ | $13.0{ }^{\circ}$ |
| 4 | $9.9{ }^{\circ}$ | 4 | $9.0{ }^{\circ}$ | $9.4{ }^{\circ}$ |
| 5 | 27.9 | 5 | $29.0^{\circ}$ | 28.4 |
| 6 | $19.7^{\circ}$ | 6 | $23.8{ }^{\circ}$ | $21.7^{\circ}$ |
| 7 | $38.2^{\circ}$ | 7 | $24.3^{\circ}$ | $31.2^{\circ}$ |
| 8 | $38.2^{\circ}$ | 8 | $25.9^{\circ}$ | $32.0{ }^{\circ}$ |
| 9 | $24.2^{\circ}$ | 9 | $31.1^{\circ}$ | $27.6^{\circ}$ |
| 10 | $19.7{ }^{\circ}$ | 10 | $23.8{ }^{\circ}$ | $21.7^{\circ}$ |
| 11 | $23.7^{\circ}$ | 11 | $37.2^{\circ}$ | $30.5^{\circ}$ |
| 12 | $22.0^{\circ}$ | 12 | $37.2^{\circ}$ | $29.6{ }^{\circ}$ |
| Variants are in BOR with $\beta_{3}$ |  |  |  |  |
| variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{1}$ | variant | $\Delta \mathrm{BOR}_{\text {th }}$ with respect to $\beta_{2}$ | $\Delta \mathrm{BOR}_{\text {th-av }}$ |
| 1 | $18.6^{\circ}$ | 1 | $23.8{ }^{\circ}$ | $21.2^{\circ}$ |
| 2 | $18.6^{\circ}$ | 2 | $29.0^{\circ}$ | $23.8{ }^{\circ}$ |
| 3 | $9.0^{\circ}$ | 3 | $31.1^{\circ}$ | $20.0^{\circ}$ |
| 4 | $18.6^{\circ}$ | 4 | $31.1^{\circ}$ | $24.9^{\circ}$ |
| 5 | $15.4{ }^{\circ}$ | 5 | $24.3{ }^{\circ}$ | $19.8{ }^{\circ}$ |
| 6 | $18.6^{\circ}$ | 6 | $25.9^{\circ}$ | $22.2^{\circ}$ |
| 7 | $18.6^{\circ}$ | 7 | $9.0^{\circ}$ | $13.8{ }^{\circ}$ |
| 8 | $18.6^{\circ}$ | 8 | $14.6{ }^{\circ}$ | $16.6^{\circ}$ |
| 9 | $17.3^{\circ}$ | 9 | $23.8{ }^{\circ}$ | $20.5^{\circ}$ |
| 10 | $18.6^{\circ}$ | 10 | $31.1^{\circ}$ | $24.9{ }^{\circ}$ |
| 11 | $18.6^{\circ}$ | 11 | $32.9{ }^{\circ}$ | $25.7^{\circ}$ |
| 12 | $13.2^{\circ}$ | 12 | $30.6^{\circ}$ | $21.9^{\circ}$ |

Table 6: Analysis of TJ 5. The deviation from the BOR with adjacent grains for 3 three sets of $12 \alpha$ variants that are oriented in the Burgers relationship with grains $\beta_{1}, \beta_{2}$ and $\beta_{3}$ respectively. The experimentally observed variant is in bold lettering.

The 4 special misorientations between $\beta$ grains that allow $\mathrm{GB} \alpha$ to be BOR related to both pairs of grains are given in Table 7 [2]

| 1. | $[011] / 60^{0}$ |
| :---: | :---: |
| 2. | $[011] / 49.47^{0}$ |
| 3. | $[011] / 10.52^{0}$ |
| 4. | $[111] / 60^{\circ}$ |

Table 7: Special misorientations between $\beta$ grains [2]

A brute force method has been used to examine the misorientations between $3 \beta$ grains at a triple junction that will allow an $\alpha$ variant at the triple junction to be Burgers related to all 3 grains. The method consists of selecting a given [110] axis in grain 1 and establishing the orientation of grain 2 when rotated by a Type 1 misorientation as in Table 7 above. Then the orientation of grain 3 is established from grain 2 by rotation about all possible $\langle 110\rangle$ and $<111>$ axes of grain 2 (including positive and negative rotations) by the angles associated with Type 1-4 misorientations of Table 7. Finally, the disorientation between grain 1 and grain 3 is then examined for all these possibilities. The combinations that lead to special orientations between grain 1 and grain 3 as well are highlighted in bold in Table 8. These again reduce to 4 distinct cases of special misorientations at triple junctions that will allow TJ $\alpha$ to be BOR related to all 3 grains of triple junction.

| Sl. No | $\beta_{12}$ axis | $\beta_{12}\left(^{\circ}\right)$ | $\beta_{23}$ axis | $\beta_{23}\left({ }^{\circ}\right)$ | $\beta_{13}$ axis | $\boldsymbol{\beta}_{13}\left({ }^{\circ}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | [110] | 60 | [-1-10] | 60 | [110] | 60 |
| 2 | [110] | 60 | [1-10] | 60 | [-0.7 0.40 .7 0.7] | 31.1 |
| 3 | [110] | 60 | [-110] | 60 | [ 0.4-0.7-0.7] | 31.1 |
| 4 | [110] | 60 | [101] | 60 | [ 0.1-0.6-0.8] | 45.1 |
| 5 | [110] | 60 | [-10-1] | 60 | [ 0.40 .80 .80 .4 ] | 18.2 |
| 6 | [110] | 60 | [10-1] | 60 | [ 0.6 0.8-0.1] | 45.1 |
| 7 | [110] | 60 | [-101] | 60 | [-0.3-0.0 1.0] | 31.1 |
| 8 | [110] | 60 | [011] | 60 | [ 0.80 .60 .1 ] | 45.1 |
| 9 | [110] | 60 | [0-1-1] | 60 | [-0.0-0.3-1.0] | 31.1 |
| 10 | [110] | 60 | [01-1] | 60 | [-0.6 0.1 0.8 ] | 45.1 |
| 11 | [110] | 60 | [0-11] | 60 | [ 0.8 0.4-0.4] | 18.2 |
| 12 | [110] | -60 | [110] | 60 | [-1-10] | 60 |
|  |  |  |  |  |  |  |
| 1 | [110] | 60 | [110] | 49.5 | [-1-10] | 10.5 |
| 2 | [110] | 60 | [-1-10] | 49.5 | [-111] | 60 |
| 3 | [110] | 60 | [1-10] | 49.5 | [-0.7 0.6 0.4] | 30.6 |
| 4 | [110] | 60 | [-110] | 49.5 | [ 0.6-0.7-0.4] | 30.6 |
| 5 | [110] | 60 | [101] | 49.5 | [-0.4-0.5 0.8] | 54 |
| 6 | [110] | 60 | [-10-1] | 49.5 | [ 0.30 .80 .6 0.6] | 27.8 |
| 7 | [110] | 60 | [10-1] | 49.5 | [ 0.40 .9 0.0.2] | 42.8 |
| 8 | [110] | 60 | [-101] | 49.5 | [-0.0-0.2 1.0] | 30.6 |
| 9 | [110] | 60 | [011] | 49.5 | [ 0.90 .40 .2 ] | 42.8 |
| 10 | [110] | 60 | [0-1-1] | 49.5 | [-0.2-0.0-1.0] | 30.6 |
| 11 | [110] | 60 | [01-1] | 49.5 | [-0.5-0.4-0.8] | 54 |
| 12 | [110] | 60 | [0-11] | 49.5 | [ 0.8 0.3-0.6] | 27.8 |
|  |  |  |  |  |  |  |
| 1 | [110] | 60 | [110] | 10.5 | [-1-10] | 49.5 |
| 2 | [110] | 60 | [-1-10] | 10.5 | [-11-1] | 60 |
| 3 | [110] | 60 | [1-10] | 10.5 | [-0.6 0.7-0.4] | 52.1 |
| 4 | [110] | 60 | [-110] | 10.5 | [ 0.7-0.6 0.4] | 52.1 |


| 5 | $[110]$ | 60 | $[101]$ | 10.5 | $[-0.7-0.70 .2]$ | 55.4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | $[110]$ | 60 | $[-10-1]$ | 10.5 | $[0.6-0.60 .4]$ | 60 |
| 7 | $[110]$ | 60 | $[10-1]$ | 10.5 | $[-0.40 .8-0.5]$ | 54 |
| 8 | $[110]$ | 60 | $[-101]$ | 10.5 | $[0.7-0.50 .6]$ | 52.1 |
| 9 | $[110]$ | 60 | $[011]$ | 10.5 | $[0.8-0.40 .5]$ | 54 |
| 10 | $[110]$ | 60 | $[0-1-1]$ | 10.5 | $[-0.50 .7-0.6]$ | 52.1 |
| 11 | $[110]$ | 60 | $[01-1]$ | 10.5 | $[-0.7-0.7-0.2]$ | 55.4 |
| 12 | $[110]$ | 60 | $[0-11]$ | 10.5 | $[-0.60 .6-0.4]$ | 60 |
|  |  |  | $[111]$ | 60 | $[-0.50 .20 .8]$ | 34.8 |
| 1 | $[110]$ | 60 | $[-1-1-1]$ | 60 | $[-0.50 .20 .8]$ | 34.8 |
| 2 | $[110]$ | 60 | $[11-1]$ | 60 | $[0.2-0.5-0.8]$ | 34.8 |
| 3 | $[110]$ | 60 | $[-1-11]$ | 60 | $[0.2-0.5-0.8]$ | 34.8 |
| 4 | $[110]$ | 60 | $[-11]$ | $\mathbf{6 0}$ | $[\mathbf{1 1 0}]$ | $\mathbf{1 0 . 5}$ |
| $\mathbf{5}$ | $[\mathbf{1 1 0}]$ | $\mathbf{6 0}$ | $[\mathbf{1 - 1} \mathbf{- 1}]$ | $\mathbf{6 0}$ | $[\mathbf{1 1 0}]$ | $\mathbf{1 0 . 5}$ |
| $\mathbf{6}$ | $[\mathbf{1 1 0}]$ | $\mathbf{6 0}$ | $[\mathbf{1 - 1 1}]$ | $\mathbf{6 0}$ | $[\mathbf{1 1 0}]$ | $\mathbf{4 9 . 5}$ |
| $\mathbf{7}$ | $[\mathbf{1 1 0}]$ | $\mathbf{6 0}$ | $[\mathbf{- 1 1 - 1 ]}$ | $\mathbf{6 0}$ | $[\mathbf{1 1 0}]$ | $\mathbf{4 9 . 5}$ |
| $\mathbf{8}$ | $[\mathbf{1 1 0}]$ | $\mathbf{6 0}$ |  |  |  |  |

Table 8: The table lists all possible combinations of special misorientation relationships (see Table 1 of the main paper) between adjacent grains $\beta_{1} / \beta_{2}$ and $\beta_{2} / \beta_{3}$ at a triple junction. Those in bold lettering also allow a special misorientation relationship between $\beta_{2} / \beta_{3}$
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