Supplemental Material On

"Common Threshold in Quantile Regressions With An Application to Pricing for Reputation"

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In this supplement we first prove proposition 4.1 and then conduct robustness check for the empirical application in the above paper.

C Proof of Proposition 4.1

First, taking the first-order derivative of Π with respect to p yields the first-order condition (FOC):

$$\frac{\partial \Pi}{\partial p}(p;r) = (1-2\alpha p) - \alpha\beta f(1-\alpha p-\bar{r}+r) = 0. \tag{C.1}$$

It is worth noting that equation (C.1) implies, for any given r, that $\frac{\partial \Pi}{\partial p}(p;r) < 0$ if $p \geq \frac{1}{2\alpha} \equiv p^m$. Therefore, the optimal price in the model must entail a price cut from p^m if the concerns of goodwill matter.

Assumptions M1 and M2 together imply that there must exist two points E_1, $E_2 \in (0,\hat{e})$ such that $f'(E_1) = f'(E_2) = 2/(\alpha\beta)$. Without loss of generality, we assume that $E_1 < E_2$, which in turn implies that $f(E_1) < f(E_2)$ by M1. Define v_1 and v_2 such that

$$1-2\alpha v_1 - \beta\alpha f(E_1) = 0 \text{ and } 1-2\alpha v_2 - \beta\alpha f(E_2) = 0. \tag{C.2}$$

Then, we must have $v_1 > v_2$. Further, we define r_1 and r_2 such that

$$1-\alpha v_1 - \bar{r} + r_1 = E_1 \text{ and } 1-\alpha v_2 - \bar{r} + r_2 = E_2. \tag{C.3}$$

Note that

$$\begin{aligned} r_1 &= \bar{r} - 1 + \alpha v_1 + E_1 \\ &= \bar{r} - 1 + \alpha\frac{1-\beta\alpha f(E_1)}{2\alpha} + E_1 \\ &= \bar{r} - \frac{1}{2} - \frac{\beta\alpha f(E_1)}{2} + E_1. \end{aligned}$$

Analogously, $r_2 = \bar{r} - \frac{1}{2} - \frac{\beta\alpha f(E_2)}{2} + E_2$. Therefore, by the mean value theorem there exists $\ddot{e} \in (E_1, E_2)$ such that

$$\begin{aligned} r_1 - r_2 &= -\frac{\beta\alpha}{2}[f(E_1)-f(E_2)] + (E_1 - E_2) \\ &= (E_1 - E_2)\left[1 - \frac{\beta\alpha}{2} f'(\ddot{e})\right] > 0, \end{aligned}$$

where the last inequality follows from the fact that $f'(e) > 2/(\alpha\beta)$ for any $e \in (E_1, E_2)$ by M2. Consequently we have shown that $r_1 > r_2$. To understand the optimal pricing strategy in the model, we consider three cases: (1) $r \leq r_2$, (2) $r \geq r_1$, and (3) $r_2 < r < r_1$.

Case 1: $r \leq r_2$.

At r_2, the point $p = v_2$ makes the FOC in (C.1) hold by construction. Further, $\frac{\partial^2 \Pi}{\partial p^2}(v_2; r_2) = -2\alpha + \alpha^2 \beta f'(E_2) = 0$ and $p = v_2$ is an inflexion point on the graph $\Pi(\cdot; r_2)$. Define $p_1(r_2) = v_2 + \frac{E_2 - E_1}{\alpha}$. Using (C.2), (C.3), and the fact that $f'(E_1) = 2/(\alpha\beta)$, we can readily verify that $\frac{\partial \Pi}{\partial p}(p_1(r_2); r_2) = 1 - 2\alpha p_1(r_2) - \alpha\beta f(E_1) = 2(r_1 - r_2) > 0$ and $\frac{\partial^2 \Pi}{\partial p^2}(p_1(r_2); r_2) = -2\alpha + \alpha^2 \beta f'(E_1) = 0$. Therefore, $p_1(r_2)$ corresponds to a local maximum on the graph of $\frac{\partial \Pi}{\partial p}(\cdot; r_2)$ and an inflexion point on the $\Pi(\cdot; r_2)$ graph by M2. As $\frac{\partial \Pi}{\partial p}(p^m; r_2) < 0$, there must exist: $p_1^* \in (p_1(r_2), p^m)$ such that $\frac{\partial \Pi}{\partial p}(p_1^*; r_2) = 0$. Moreover, p_1^* is the unique maximum. (Refer to Figure 2.) Extending to the case of $r < r_2$, define two local extremes, $v_2(r)$ and $p_1(r)$ on the function $\frac{\partial \Pi}{\partial p}(\cdot; r)$ with $v_2(r) < p_1(r)$. By definition, $\frac{\partial^2 \Pi}{\partial p^2}(v_2(r); r) = \frac{\partial^2 \Pi}{\partial p^2}(p_1(r); r) = 0$. It is easily verified that $\frac{\partial \Pi}{\partial p}(v_2; r) > 0$, $\frac{\partial \Pi}{\partial p}(p_1(r_2); r) > \frac{\partial \Pi}{\partial p}(p_1(r_2); r_2) > 0$, $\frac{\partial^2 \Pi}{\partial p^2}(p_1(r_2); r) < 0$, $\frac{\partial \Pi}{\partial p}(p_1^*; r) > 0$, and $\frac{\partial \Pi}{\partial p}(v_2(r); r) > \frac{\partial \Pi}{\partial p}(v_2(r); r_2) > 0 \; \forall \; r < r_2$. The first three inequalities imply that $\forall \; r < r_2$, the graph of $\frac{\partial \Pi}{\partial p}(\cdot; r)$ can be obtained by shifting that of $\frac{\partial \Pi}{\partial p}(\cdot; r_2)$ to the upper left, and the last two, in conjunction with the fact that $\frac{\partial \Pi}{\partial p}(p^m; r) < 0$, imply the existence of a unique local maximum $p_1^*(r) \in (p_1^*, p^m) \;\; \forall \; r \leq r_2$. By the FOC $\frac{\partial \Pi}{\partial p}(p_1^*(r); r) = 0$ and the implicit function theorem, we have

$$\frac{\partial p_1^*(r)}{\partial r} = -\frac{\frac{\partial^2 \Pi}{\partial p \partial r}(p_1^*(r); r)}{\frac{\partial^2 \Pi}{\partial p^2}(p_1^*(r); r)} = \frac{\alpha\beta f'(1 - \alpha p_1^*(r) - \bar{r} + r)}{\frac{\partial^2 \Pi}{\partial p^2}(p_1^*(r); r)} < 0$$

because $f'(e) > 0$ for any $e < \hat{e}$, $1 - \alpha p_1^*(r) - \bar{r} + r < 1 - \alpha p_1^* - \bar{r} + r_2 < \hat{e}$, and $\frac{\partial^2 \Pi}{\partial p^2}(p_1^*(r); r) < 0$. That is, $p_1^*(r)$ is **decreasing** in r.

Case 2: $r \geq r_1$**.**

Note again that at r_1, the point $p = v_1$ is an inflexion point on the graph of $\Pi(\cdot; r_1)$. Similar to the arguments in Case (1), define $p_2(r_1) = v_1 - \frac{E_2 - E_1}{\alpha}$. Because $\frac{\partial \Pi}{\partial p}(p_2(r_1); r_1) < 0$ and $\lim_{p \to 0} \frac{\partial \Pi}{\partial p}(p; r_1) > 0$ by M1, there exists a $p_2^* \in (0, p_2(r_1))$ such that $\Pi(p; r_1)$ achieves a local maximum. (Refer to Figure 1.)

To extend to the case of $r > r_1$, let $p_2(r)$ denote the local minimum point on the function $\frac{\partial \Pi}{\partial p}(\cdot; r)$. We can apply arguments analogous to the case of $r < r_2$ to show that the graph of $\frac{\partial \Pi}{\partial p}(\cdot; r)$ can be obtained by shifting that of $\frac{\partial \Pi}{\partial p}(\cdot; r_2)$ down and right, and there exists a unique $p_2^*(r) \in (0, \; p_2(r)) \; \forall \; r > r_1$ that maximizes profits. However, noting that $1 - \alpha p_2^*(r) - \bar{r} + r$ can be either larger or smaller than $\hat{e}$, $f'(1 - \alpha p_2^*(r) - \bar{r} + r)$ can take positive, negative, or zero values, which implies that $p_2^*(r)$ may be **either increasing or decreasing** when $r > r_1$.[Note that $1 - \alpha p_2^*(r) - \bar{r} + r > 1 - \alpha p^* - \bar{r} + r_1$, and nothing ensures that $1 - \alpha p_2^*(r) - \bar{r} + r < \hat{e}$ as $r > r_1$.]

Case 3: $r_2 < r < r_1$**.**

There exist two local maxima, $p_1^*(r) \in (p_1(r), p^m)$ and $p_2^*(r) \in (0, p_2(r))$. (Refer to Figure 3.) Let $\triangle(r) = \Pi(p_1^*(r); r) - \Pi(p_2^*(r); r)$. By the envelope theorem and FOC,

$$\begin{aligned}\frac{\partial \triangle (r)}{\partial r} &= \beta f(1 - \alpha p_1^*(r) - \bar{r} + r) - \beta f(1 - \alpha p_2^*(r) - \bar{r} + r) \\ &= \frac{1 - 2\alpha p_1^*(r)}{\alpha} - \frac{1 - 2\alpha p_2^*(r)}{\alpha} \\ &= 2\left[p_2^*(r) - p_1^*(r)\right] < 0.\end{aligned}$$

Moreover, noting that $\triangle(r_2) > 0$ and $\triangle(r_1) < 0$, there must exist a unique point $\gamma_0 \in (r_2, r_1)$ such that $\triangle(\gamma_0) = 0$. It follows that the seller should adopt $p_1^*(r)$ if $r \leq \gamma_0$ and $p_2^*(r)$ otherwise, and the desired optimal pricing strategy holds. ■

Remark on the intuition.

Intuitively, the discontinuous pricing strategy occurs as follows. The restrictions in Assumptions M1 and M2 produce a peculiar shape of $\Pi'(\cdot; r)$. Along with the increase in p, $\Pi'(\cdot; r)$ is initially downward sloping and convex, then becomes positive sloping and concave, and then eventually slopes downwards again. Thus, in order for the FOC in (C.1) to hold, there are three possible ways that $\Pi'(\cdot; r)$ intersects the horizontal p-axis:

Case 1: the intersection occurs in the concave region alone (refer to Figure 2);
Case 2: the intersection occurs in the convex region alone (refer to Figure 1); and
Case 3: the intersection occurs in both regions (refer to Figure 3).

In the proof, we show that the pricing scheme in Cases 1 and 2 are associated with small and large values of r, respectively, in Case 3 there exists a threshold value γ_0 such that the seller will switch between the two pricing schemes when r increases from a number below γ_0 to one above γ_0. It is the presence of a positively sloped segment of $\Pi'(\cdot; r)$ that makes the profit function $\Pi(\cdot; r)$ exhibit a bimodal shape, which in turn induces discontinuity in the optimal pricing. If it were not the case, the profit function would be globally concave and a change in pricing scheme may not occur.

We first take a close look at Case 1 by considering a slight change in p. When r is small, the marginal profit in the current monopoly pricing always dominates the marginal cost of losing the potential benefit of goodwill. Therefore, the unique maximum of Π occurs in the concave region of $\Pi'(\cdot; r)$ in this case. Parallel to the first case, we next consider a change in p in Case 2. The loss of marginal profit in the current monopoly pricing may now be compensated for by the potential gain from future goodwill. Therefore, the unique maximum in this case occurs in the convex region of $\Pi'(\cdot; r)$.

In the pricing situation in Case 3, the seller needs to choose between two local maxima, $p_1^*(r)$ and $p_2^*(r)$, as illustrated in Figure 3. Switching from $p_1^*(r)$ to $p_2^*(r)$ induces a trade-off between the two areas in the region, $(p_2^*(r), p_1^*(r))$. The size of the gain is represented by the area below the horizontal axis, whereas the magnitude of loss is shown by the area above the horizontal axis. Consider a seller with an r close to r_2. As the gain from changing is not significant enough to compensate for the loss, the seller will continue

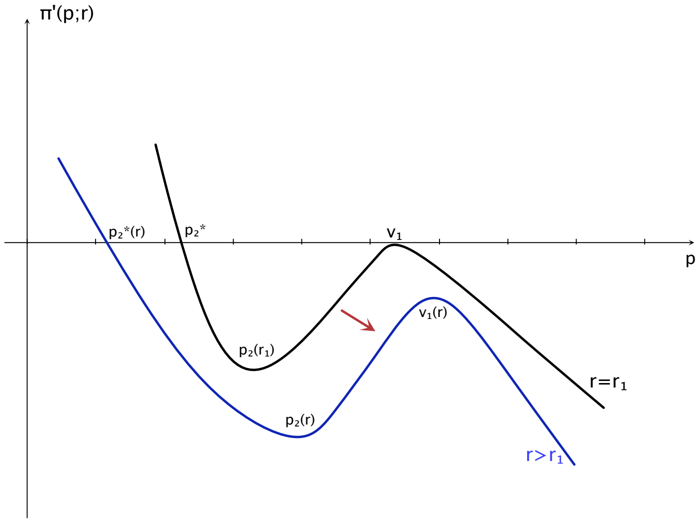

Figure 1: Pricing strategy when $r \geq r_1$

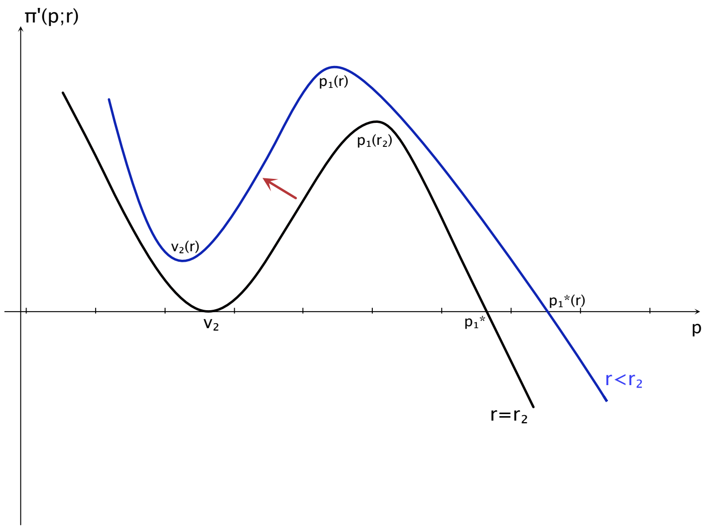

Figure 2: Pricing strategy when $r \leq r_2$

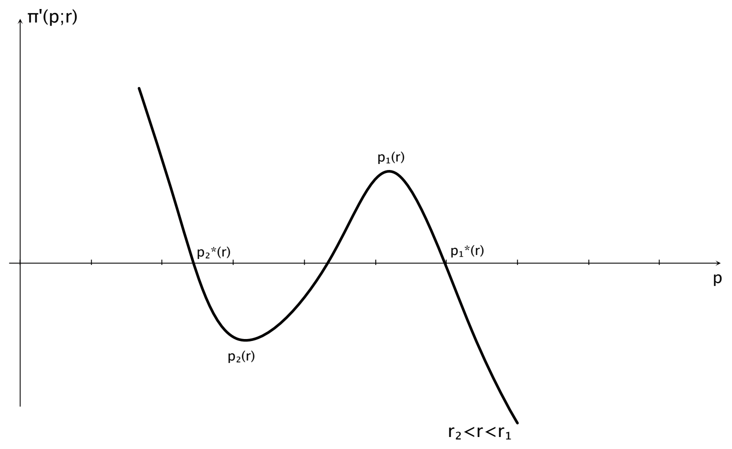

Figure 3: Pricing strategy when $r \in (r_2, r_1)$

to charge $p_1^*(r)$. However, along with the increase in r, there must exist a γ_0 that makes it worthwhile for the seller to switch to the pricing regime $p_2^*(r)$.

In Case 1 where the value of r is small, the tail of f, the distribution of not recruiting good reviews, is relevant. As a matter of fact, being to the left of the mode implies that the optimal prices will decrease with r. Such a decreasing pricing pattern simply reflects the fact that the potential benefit of goodwill becomes more significant as r increases. However, in Case 2, as r is sufficiently close to $\bar{r}$, the pricing decision may face e on either side of $\hat{e}$. Therefore, the pricing pattern in r results in an ambiguous sign.

D Robustness check

In our sample, we observe posting prices (price), the reputation score and category of the seller at the time of posting (reputation score), whether postage is included in the posted prices (postage), the total number of items sold by the seller (total items), the sales volume last recorded per posting item (sales volume), the rate of good reviews obtained by the seller (rate of good reviews), and the seller's location (area code). We also observe the actual transaction prices. However, these prices have a great deal of noise, due to the options of an additional set menu at each seller's store. We therefore decide to focus on the posting price in our empirical analysis.

Table A.1 lists the basic summary statistics for our data. We observe a substantial amount of variation in prices, which touches on the core of our study, that is, whether reputation contributes to providing a causal term for such rich variation in price. We observe only limited information on sellers in the dataset, among which "total items" is the most important. It represents the total number of items for sale in a particular online store. We view this variable as a proxy for a seller's scale and specialization. The significant variation observed in total items may reflect the fact that sellers' heterogeneity is at work. The sales volume variable exhibits much less variation. Lastly, the variation in the rate of good reviews indicates that it is less likely for sellers to get a bad or neutral review than a good one. This is consistent with other empirical findings that only reviewers who provide good reviews tend to break the silence. See, e.g., Dellarocas and Wood (2008). This pattern partially validates our theoretical model, in which a distribution that does not elicit good reviews plays a central role in equilibrium pricing. Our focus on the left tail of the distribution becomes more relevant.

Although the reputation pricing pattern is found in the data, there remain critical issues in the foregoing empirical analysis. For example, we did not consider the possible impact of the observed covariates on the posting prices. As a robustness check, we repeat the previous empirical exercises by including all observed variables. We thus augment our TQR model in Section 4 only by the covariates listed in Table A.1.

Following the testing approach suggested in Section 2.2, we detect the existence of change points in the data for all of the quantiles between 0.1 and 0.9. We then estimate the model for quantiles $\tau = 0.1, ..., 0.9$. The estimation and inference results for the threshold parameters are reported in Table A.2. From these estimates, we suspect quantiles $0.4 - 0.7$ may share a common threshold. Therefore, we did the test for common threshold among these quantiles, whose results strongly supports the null of common threshold. Then, the estimated common threshold parameter is 3252, with a confidence interval of $[3246, 3260]$.

The jump sizes are evaluated at the mean values of each covariate in the quantile regressions. It is observed that the price-cut pattern occurs for all $\tau \in [0.2, 0.9]$, and the jump sizes are much smaller than those unconditional on the covariates. Moreover, roughly speaking, the larger the τ, the higher the reputation level at which the jump occurs (that is the closer to the exogenous cutoff of reputation). Although jumps are identified in the quantiles of $\tau = 0.8, 0.9$, they are smaller in magnitude, relative to other quantiles. What can be concluded is that for the sellers in most of the quantiles, a price-cut strategy may be useful when their reputation scores are in the range of 3200 to 3400.

A jump-up occurs at the quantile regression of $\tau = 0.1$. Recall that the slope estimate before the jump for $\tau = 0.1$ is statistically significant and positive in Table 6. These inconsistent findings may indicate that sellers posting extremely low prices may possibly have objectives other than an enhanced reputation. If this is indeed the case, then our model cannot, in general, explain the pricing behavior of these sellers.

Our choice of the iPod Nano for this study stemmed from our concern with product homogeneity. An additional concern is whether a seller would choose this product to accrue good reviews to obtain the goodwill benefit. To address this issue, we repeat the testing and estimation procedure with a much more restrictive sample, that is, the sellers with fewer than 100 items in total to sell. These sellers are smaller in scale and possibly more specialized in selling electronic items. Our major findings on the pricing patterns remain valid with this restrictive sample. However, we also acknowledge that this issue may be significantly more complicated. In particular, consumers' willingness to provide positive reviews in exchange for lower prices may be dependent on product-specific characteristics. The issue of consumer responsiveness to this type of product is beyond the scope of this project and is therefore left for future research.

Table A.1: Summary statistics

Variables	Min	Max	Mean	Median	Std dev
price	508	1400	995.10	1050	171.43
reputation score	501	4995	1875.48	1392	1225.58
total items	7	36811	645.30	288	1787.90
postage	0	100	11.40	10	10.42
sales volume	0	300	3.84	1	12.86
rate of good reviews	0	100	89.06	99.83	30.79

NOTE: The sample includes only sellers that belong to Categories 7 to 9. The total number of observations is 1903.

Table A.2: Estimation and inference on the threshold parameter

τ	Jump Size	γ	95% lower bound	95% upper bound
0.1	277.68	1984	1975	2018
0.2	-77.83	3264	3231	3271
0.3	-125.60	2979	2975	3002
0.4	-101.01	3252	3232	3272
0.5	-52.38	3252	3247	3261
0.6	-87.07	3252	3232	3272
0.7	-64.23	3252	3247	3261
0.8	-28.00	3364	3355	3367
0.9	-27.64	3849	3750	3947
0.4-0.7	–	3252	3246	3260

References

Dellarocas, C., Wood, C. A. (2008). The sound of silence in online feedback: estimating trading risks in the presence of reporting bias. *Management Science* 54: 460-476.