# Appendices

**Appendix A: Decentralized Supply chain**

In the decentralized setting, we first solve the retailer’s problem.

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Now, the first order conditions are given by

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| --- | --- |
|  |  |
|  |  |

The second order conditions are also given by

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| --- | --- |
|  |  |

So, the determinant of the Hessian matrix is. Therefore, the Hessian matrix is negative definite. Thus, the profit function of the retailer is a joint concave in 

Solve the equations  and  for  and , we get,

|  |  |
| --- | --- |
|  |  |
|  |  |

Using  and , the order quantity  becomes

|  |  |
| --- | --- |
|  |  |

Therefore, the manufacturer problem becomes

|  |  |  |
| --- | --- | --- |
|  |  |  |

The first order conditions are given by

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|  | | |
|  | |  |  |

The second order conditions are also given by

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, .

Now, we check the concavity of the manufacturer’s profit function.

The first principal minor of the Hessian matrix is given by

|  |  |
| --- | --- |
| , since  according to our assumption. |  |

The second principal minor of the Hessian matrix is given by

|  |  |
| --- | --- |
| , by assumption. |  |

The determinant of the Hessian matrix is given by

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | | |  | |
|  | , | | | |
|  | , | according to our assumption. | |  |

From equations, we can say that the Hessian matrix is negative definite and hence the manufacturer’s profit function is a joint concave function in.

Now, we solve the equations . The optimal values of the manufacturer’s decisions are given by

|  |  |
| --- | --- |
|  |  |

Using , the equations become,

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| --- | --- |
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|  |  |

**Appendix B: Centralized Supply chain**

In the centralized supply chain, the total supply chain profits are given by

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| --- | --- | --- |
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Now, the first order conditions are given by

|  |  |  |  |
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|  | |  | |

The second order conditions are also given by

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| --- | --- |
|  |  |

, , ,

, , .

Now, we check the concavity of the centralized profit function.

The first order principal minor of the Hessian matrix is given by

|  |  |
| --- | --- |
|  |  |

The second order principal minor of the Hessian matrix is given by

|  |  |
| --- | --- |
| , according to our assumption. |  |

The third order principal minor of the Hessian matrix is given by

|  |  |
| --- | --- |
|  |  |
| , by assumption. |  |

The determinant of the Hessian matrix is given by

|  |  |  |
| --- | --- | --- |
|  | | |
|  |  |  |

From equationswe can say that the Hessian matrix is negative definite and hence the manufacturer’s profit function is a joint concave function in

Now, we solve the equations . The optimal values of the decision variables are given by

|  |  |
| --- | --- |
|  |  |
|  |  |