Appendix

A Measurement Equations

All the data I use in the estimation come from surveys. In virtually all cases, the question asked of the forecasters does not correspond exactly to a simple τ-month-ahead forecast, in the form of $\pi_t(\tau)$ for some $\tau > 0$, so I do some transformations as I explain in detail below. I convert all raw data to annualized percentage points to conform with the previous notation. In other words, I show how the "fixed event" forecasts in surveys can be converted to forecasts of "fixed horizons".

Unless otherwise noted, all data start in 1998. In all cases, the forecasters are asked to forecast the seasonally adjusted CPI inflation rate. Both the *SPF* and *Blue Chip* forecasts are released around the middle of the month, with the forecasts due a few days prior to the release.[A-1] I will thus consider both of the forecasts released in month t as forecasts made in month t.

As will be clear below, in some cases what is asked of the forecasters is a mixture of realized (past) inflation and a forecast of future inflation. Most of the realized inflation will be in the form of $\pi_{s\to r}$, where $s < r \leq t-2$ so that in period t the forecasts are able to observe the official data release before making their forecasts.[A-2] I use Archival Federal Reserve Economic Data (ALFRED) at the Federal Reserve Bank of St. Louis to obtain the exact inflation rate the forecasters would have observed in real time.[A-3] Furthermore, there will be instances in which I need $\pi_{t-2\to t-1}$, $\pi_{t-1\to t}$, or $\pi_{t-2\to t}$, none of which are observed by the time forecasters make their forecasts in period t. Since it is difficult to know explicitly what the forecasters knew when they sent their forecasts in month t about these inflation

[A-1] See Figure 1 in Stark (2010) that shows the timing of *SPF* forecasts. Similar information is confirmed for the *Blue Chip* forecasts.

[A-2] For example, $\pi_{t-3\to t-2}$ would involve P_{t-3} and P_{t-2}, and the latter is released (and perhaps the former is revised) in the second half of the month $t-1$. Remember that both the *SPF* and *Blue Chip* forecasts are made in the first half of month t, before P_{t-1} is released.

[A-3] The data are available at http://alfred.stlouisfed.org/series?seid=CPIAUCSL.

rates that are realized (but not yet released by statistical agencies), I assume that these expectations are equal to the longer horizon being forecast. Once I show the equations below, what I mean by this will be clear.

A.1 *Survey of Professional Forecasters*

The *SPF* is a quarterly survey that has been conducted by the FRBP since 1990. The forecasters are asked to make forecasts for a number of key macroeconomic indicators several quarters into the future, and in the case of CPI inflation, they are also asked to make 5-year and 10-year forecasts. I use the median of these forecasts.

A.1.1 *SPF* Quarterly Forecasts

The *SPF* reports six quarterly forecasts ranging from "minus 1 quarter" to "plus 4 quarters" from the current quarter. The forecasts labeled "3," "4," "5," and "6" are forecasts for one, two, three, and four quarters after the current quarter, respectively.[A-4] More specifically, the forecasters are asked to forecast the annualized percentage change in the quarterly average of the CPI price level. Using my notation, the "4" forecast made in period t is

$$\text{SPF-4}_t = 100\left[\left(\frac{\frac{P_{t+5}+P_{t+6}+P_{t+7}}{3}}{\frac{P_{t+2}+P_{t+3}+P_{t+4}}{3}}\right)^4 - 1\right],$$

where the numerator is the average CPI price level in the second quarter following the current one and the denominator is the average CPI price level for the next quarter. Using

[A-4]The "1" and "2" forecasts contain at least some realized inflation rates, and I do not use them since I want to focus as much as possible on pure forecasts.

continuous compounding and geometric averaging, this forecast can be written as[A-5]

$$
\begin{aligned}
\text{SPF-4}_t &\approx 400\left\{\log\left[\left(P_{t+5}P_{t+6}P_{t+7}\right)^{1/3}\right]-\log\left[\left(P_{t+2}P_{t+3}P_{t+4}\right)^{1/3}\right]\right\}\\
&= \frac{400}{3}\left\{\log\left[\left(P_{t+5}P_{t+6}P_{t+7}\right)\right]-\log\left[\left(P_{t+2}P_{t+3}P_{t+4}\right)\right]\right\}\\
&= \frac{400}{3}\left[\log\left(P_{t+5}\right)-\log\left(P_{t+2}\right)+\log\left(P_{t+6}\right)-\log\left(P_{t+3}\right)+\log\left(P_{t+7}\right)-\log\left(P_{t+4}\right)\right]\\
&= \frac{\pi_{t+2\to t+5}+\pi_{t+3\to t+6}+\pi_{t+4\to t+7}}{3},
\end{aligned}
$$

which is the arithmetic average of three quarterly inflation rates.

The SPF-3 forecast is special (as will be the other *SPF* forecasts I turn to next) in that a part of the object being forecast refers to the past and not to the future. Using similar derivations as above, the SPF-3 forecast in period t can be written as

$$
\begin{aligned}
\text{SPF-3}_t &= \frac{\pi_{t-1\to t+2}+\pi_{t\to t+3}+\pi_{t+1\to t+4}}{3}\\
&= \frac{\left(\frac{\pi_{t-1\to t}+2\pi_{t\to t+2}}{3}\right)+\pi_{t\to t+3}+\pi_{t+1\to t+4}}{3}\\
&= \frac{1}{9}\left(\pi_{t-1\to t}+2\pi_{t\to t+2}+3\pi_{t\to t+3}+3\pi_{t+1\to t+4}\right)\\
&= \frac{1}{8}\left(2\pi_{t\to t+2}+3\pi_{t\to t+3}+3\pi_{t+1\to t+4}\right),
\end{aligned}
$$

where in the last line I replace $\pi_{t-1\to t}$ with SPF-3$_t$. This is the assumption I will maintain whenever formulas call for $\pi_{t-2\to t-1}$, $\pi_{t-1\to t}$, or $\pi_{t-2\to t}$ – I will assume each of them are equal to the main object being forecast.[A-6]

Using similar derivations for the "5" and "6" forecasts, and using definitions $x_t^1 \equiv$SPF-3$_t$, $x_t^2 \equiv$SPF-4$_t$, $x_t^3 \equiv$SPF-5$_t$, and $x_t^4 \equiv$SPF-6$_t$, the measurement equations for the quarterly *SPF*

[A-5] The correlation of actual inflation computed using the exact formula and the approximation I use is 0.9993.

[A-6] This creates a small inconsistency across different forecasts when the same object, say $\pi_{t-2\to t-1}$, is set equal to different forecasts with different values. Given that these terms receive small weights and the absence of a clearly better alternative, I choose this route.

forecasts are

$$
\begin{aligned}
x_t^1 &= \frac{1}{8}\left(2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right) + \varepsilon_t^1 \\
x_t^2 &= \frac{1}{3}\left(\pi_{t+2\to t+5} + \pi_{t+3\to t+6} + \pi_{t+4\to t+7}\right) + \varepsilon_t^2 \\
x_t^3 &= \frac{1}{3}\left(\pi_{t+5\to t+8} + \pi_{t+6\to t+9} + \pi_{t+7\to t+10}\right) + \varepsilon_t^3 \\
x_t^4 &= \frac{1}{3}\left(\pi_{t+8\to t+11} + \pi_{t+9\to t+12} + \pi_{t+10\to t+13}\right) + \varepsilon_t^4.
\end{aligned}
$$

Once stated as combinations of $\pi_{t+\tau_1\to t+\tau_2}$, it is straightforward, though somewhat tedious, to write the full measurement equations for these forecasts using (5).[A-7]

A.1.2 *SPF* Annual Forecasts

The *SPF* provides three annual forecasts, one for the survey calendar year and one each for the next two calendar years. I use the latter two, the "B" forecast and the "C" forecast, since they are (mostly) pure forecasts into the future. The "C" forecast is available starting in 2005Q3. More specifically, in every quarter of the survey year, for the "B" forecast the forecasters are asked to forecast the change in average price level of the last quarter of the year after the survey year relative to the last quarter of the survey year. Similarly for the "C" forecast, they need to forecast the change in the average price level of the last quarter of the year that is two years after the survey year, relative to the last quarter of the year that is one year after the survey year. As such, as we progress further into the current year, the distance between the period being forecast and the point of forecast gets shorter. This requires me to define forecasts made in particular quarters as separate variables.[A-8]

[A-7]For example, the second measurement equation will be

$$
\begin{aligned}
x_t^2 &= L_t + \left[\frac{e^{-2\lambda} - e^{-5\lambda} + e^{-3\lambda} - e^{-6\lambda} + e^{-4\lambda} - e^{-7\lambda}}{9\lambda}\right](C_t - S_t) \\
&\quad + \left(\frac{2e^{-2\lambda} - 5e^{-5\lambda} + 3e^{-3\lambda} - 6e^{-6\lambda} + 4e^{-4\lambda} - 7e^{-7\lambda}}{9}\right) C_t + \varepsilon_t^2.
\end{aligned}
$$

[A-8]To be clear, I split each variable into four variables, each of which is observed only once a year.

The "B" forecast released in February (denoted by t) is thus

$$\text{SPF-B-Q1}_t = 100\left[\left(\frac{P_{t+20}+P_{t+21}+P_{t+22}}{P_{t+8}+P_{t+9}+P_{t+10}}\right)-1\right].$$

Using the same derivations as in $SPF4$, this simplifies to

$$\text{SPF-B-Q1}_t \approx \frac{\pi_{t+8\to t+20}+\pi_{t+9\to t+21}+\pi_{t+10\to t+22}}{3}.$$

Doing the same derivations for the other quarters 1 through 3, and using definitions $x_t^5 \equiv$SPF-B-Q1$_t$, $x_t^6 \equiv$SPF-B-Q2$_t$, and $x_t^7 \equiv$SPF-B-Q3$_t$, I get the measurement equations

$$\begin{aligned}
x_t^5 &= \frac{1}{3}\left(\pi_{t+8\to t+20}+\pi_{t+9\to t+21}+\pi_{t+10\to t+22}\right)+\varepsilon_t^5\\
x_t^6 &= \frac{1}{3}\left(\pi_{t+5\to t+17}+\pi_{t+6\to t+18}+\pi_{t+7\to t+19}\right)+\varepsilon_t^6\\
x_t^7 &= \frac{1}{3}\left(\pi_{t+2\to t+14}+\pi_{t+3\to t+15}+\pi_{t+4\to t+16}\right)+\varepsilon_t^7.
\end{aligned}$$

For the last quarter, I need to take into account that a small part of the object being forecast is realized by the time the forecast is made. In particular, the Q4 forecast is

$$\text{SPF-B-Q4}_t = \frac{1}{35}\left(11\pi_{t\to t+11}+12\pi_{t\to t+12}+12\pi_{t+1\to t+13}\right),$$

where, again, I replaced $\pi_{t-1\to t}$ with SPF-B-Q4$_t$. Defining $x_t^8 \equiv$SPF-B-Q4$_t$, I get the measurement equation

$$x_t^8 = \frac{1}{35}\left(11\pi_{t\to t+11}+12\pi_{t\to t+12}+12\pi_{t+1\to t+13}\right)+\varepsilon_t^8.$$

For the "C" forecast, in the first quarter of a year, the expression being forecast is

$$\text{SPF-C-Q1}_t = 100\left[\left(\frac{P_{t+32}+P_{t+33}+P_{t+34}}{P_{t+20}+P_{t+21}+P_{t+22}}\right)-1\right],$$

and defining $x_t^9 \equiv$SPF-C-Q1$_t$, $x_t^{10} \equiv$SPF-C-Q2$_t$, $x_t^{11} \equiv$SPF-C-Q3$_t$, $x_t^{12} \equiv$SPF-C-Q4$_t$, the measurement equations are

$$\begin{aligned}
x_t^9 &= \frac{1}{3}\left(\pi_{t+20\to t+32} + \pi_{t+21\to t+33} + \pi_{t+22\to t+34}\right) + \varepsilon_t^9 \\
x_t^{10} &= \frac{1}{3}\left(\pi_{t+17\to t+29} + \pi_{t+18\to t+30} + \pi_{t+19\to t+31}\right) + \varepsilon_t^{10} \\
x_t^{11} &= \frac{1}{3}\left(\pi_{t+14\to t+26} + \pi_{t+15\to t+27} + \pi_{t+16\to t+28}\right) + \varepsilon_t^{11} \\
x_t^{12} &= \frac{1}{3}\left(\pi_{t+11\to t+23} + \pi_{t+12\to t+24} + \pi_{t+13\to t+25}\right) + \varepsilon_t^{12}.
\end{aligned}$$

Given these expressions, I directly apply (5) to get the measurement equations.

A.1.3 *SPF* 5-Year and 10-Year Forecasts

Although much of the *SPF* contains short- to medium-term forecasts, the forecasters are asked to provide five-year and 10-year forecasts for inflation as well. In particular, they are asked to forecast five and 10 years into the future, starting from the last quarter of the previous year. In other words, as with the other forecasts, these forecasts compare the average price level over the last quarter of the previous year with the average price level over the last quarter of four or nine years following the current year. Similar to the annual forecasts, I divide the 10-year and the five-year forecasts into four separate variables, taking into account the different forecast horizons at each quarter. The 10-year forecast has been a part of the *SPF* since 1991Q4, and the five-year forecast was added in 2005Q3.

In all cases below, what is reported by the forecasters includes some inflation that is already realized. Denoting February with period t, the five-year forecast made in the first quarter is

$$
\begin{aligned}
\text{SPF-5YR-Q1}_t &= 100\left[\left(\frac{P_{t+56}+P_{t+57}+P_{t+58}}{P_{t-4}+P_{t-3}+P_{t-2}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{\pi_{t-4\to t+56}+\pi_{t-3\to t+57}+\pi_{t-2\to t+58}}{3} \\
&= \frac{1}{3}\left\{\left[\frac{4}{60}\pi_{t-4\to t}+\frac{56}{60}\pi_{t\to t+56}\right]+\left[\frac{3}{60}\pi_{t-3\to t}+\frac{57}{60}\pi_{t\to t+57}\right]\right. \\
&\quad \left.+\left[\frac{2}{60}\pi_{t-2\to t}+\frac{58}{60}\pi_{t\to t+58}\right]\right\} \\
&= \frac{1}{180}\left(\pi_{t-4\to t-3}+\pi_{t-3\to t-2}+2\pi_{t-2\to t}\right)+\frac{1}{180}\left(\pi_{t-3\to t-2}+2\pi_{t-2\to t}\right) \\
&\quad +\frac{1}{180}2\pi_{t-2\to t}+\frac{1}{180}\left(56\pi_{t\to t+56}+57\pi_{t\to t+57}+58\pi_{t\to t+58}\right) \\
&= \frac{1}{180}\left(\pi_{t-4\to t-3}+2\pi_{t-3\to t-2}+6\pi_{t-2\to t}\right)+\frac{1}{180}\left(56\pi_{t\to t+56}+57\pi_{t\to t+57}+58\pi_{t\to t+58}\right) \\
&= \frac{1}{174}\left(\pi_{t-4\to t-3}+2\pi_{t-3\to t-2}\right)+\frac{1}{174}\left(56\pi_{t\to t+56}+57\pi_{t\to t+57}+58\pi_{t\to t+58}\right),
\end{aligned}
$$

where I use the properties of continuous compounding to simplify the expressions, and the last line uses $\pi_{t-2\to t}$ =SPF-5YR-Q1$_t$. Note that this forecast contains two realized inflation terms and three terms that are forecasts from period t onward. It is important to emphasize that this correction causes sizable differences between what the SPF reports and what enters the estimation. Figure A1 demonstrates this. It shows the raw SPF 10-year data as downloaded from FRBP, SPF-5YR-Q1$_t$, the adjusted version – for example for Q1 this is SPF-5YR-Q1$_t-\frac{1}{174}\left(\pi_{t-4\to t-3}+2\pi_{t-3\to t-2}\right)$ – and the filtered counterpart of the adjusted version from the model – this will be $\frac{1}{174}\left(56\pi_{t\to t+56}+57\pi_{t\to t+57}+58\pi_{t\to t+58}\right)$. This adjustment is particularly visible as we progress in the year and more of the raw data contains inflation that is already realized. For example looking at the Q4 panel, there are a large number of 2.5% entries in the raw data but these get adjust down to as low as 2.1%.

Turning to the second-quarter forecast, where period t now denotes May, I have

$$\begin{aligned}
\text{SPF-5YR-Q2}_t &= 100\left[\left(\frac{P_{t+53}+P_{t+54}+P_{t+55}}{P_{t-7}+P_{t-6}+P_{t-5}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{1}{174}\left(\pi_{t-7\to t-6}+2\pi_{t-6\to t-5}+9\pi_{t-5\to t-2}\right) \\
&\quad +\frac{1}{174}\left(53\pi_{t\to t+53}+54\pi_{t\to t+54}+55\pi_{t\to t+55}\right).
\end{aligned}$$

The third-quarter forecast, where period t now denotes August, is given by

$$\begin{aligned}
\text{SPF-5YR-Q3}_t &= 100\left[\left(\frac{P_{t+50}+P_{t+51}+P_{t+52}}{P_{t-10}+P_{t-9}+P_{t-8}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{1}{174}\left(\pi_{t-10\to t-9}+2\pi_{t-9\to t-8}+18\pi_{t-8\to t-2}\right) \\
&\quad +\frac{1}{174}\left(50\pi_{t\to t+50}+51\pi_{t\to t+51}+52\pi_{t\to t+52}\right).
\end{aligned}$$

Finally, the fourth-quarter forecast, with t denoting November, is given by

$$\begin{aligned}
\text{SPF-5YR-Q4}_t &= 100\left[\left(\frac{P_{t+47}+P_{t+48}+P_{t+49}}{P_{t-13}+P_{t-12}+P_{t-11}}\right)^{\frac{1}{5}}-1\right] \\
&\approx \frac{1}{174}\left(\pi_{t-13\to t-12}+2\pi_{t-12\to t-11}+27\pi_{t-11\to t-2}\right) \\
&\quad +\frac{1}{174}\left(47\pi_{t\to t+47}+48\pi_{t\to t+48}+49\pi_{t\to t+49}\right).
\end{aligned}$$

Using the definitions

$$\begin{aligned}
x_t^{13} &\equiv \text{SPF-5YR-Q1}_t-\frac{1}{174}\left(\pi_{t-4\to t-3}+2\pi_{t-3\to t-2}\right) \\
x_t^{14} &\equiv \text{SPF-5YR-Q2}_t-\frac{1}{174}\left(\pi_{t-7\to t-6}+2\pi_{t-6\to t-5}+9\pi_{t-5\to t-2}\right) \\
x_t^{15} &\equiv \text{SPF-5YR-Q3}_t-\frac{1}{174}\left(\pi_{t-10\to t-9}+2\pi_{t-9\to t-8}+18\pi_{t-8\to t-2}\right) \\
x_t^{16} &\equiv \text{SPF-5YR-Q4}_t-\frac{1}{174}\left(\pi_{t-13\to t-12}+2\pi_{t-12\to t-11}+27\pi_{t-11\to t-2}\right).
\end{aligned}$$

The measurement equations for the 5-year forecasts are

$$
\begin{aligned}
x_t^{13} &= \frac{1}{174}\left(56\pi_{t\to t+56} + 57\pi_{t\to t+57} + 58\pi_{t\to t+58}\right) + \varepsilon_t^{13} \\
x_t^{14} &= \frac{1}{174}\left(53\pi_{t\to t+53} + 54\pi_{t\to t+54} + 55\pi_{t\to t+55}\right) + \varepsilon_t^{14} \\
x_t^{15} &= \frac{1}{174}\left(50\pi_{t\to t+50} + 51\pi_{t\to t+51} + 52\pi_{t\to t+52}\right) + \varepsilon_t^{15} \\
x_t^{16} &= \frac{1}{174}\left(47\pi_{t\to t+47} + 48\pi_{t\to t+48} + 49\pi_{t\to t+49}\right) + \varepsilon_t^{16}.
\end{aligned}
$$

Applying the same idea to 10-year forecasts, I define

$$
\begin{aligned}
x_t^{17} &\equiv \text{SPF-10YR-Q1}_t - \frac{1}{354}\left(\pi_{t-4\to t-3} + 2\pi_{t-3\to t-2}\right) \\
x_t^{18} &\equiv \text{SPF-10YR-Q2}_t - \frac{1}{354}\left(\pi_{t-7\to t-6} + 2\pi_{t-6\to t-5} + 9\pi_{t-5\to t-2}\right) \\
x_t^{19} &\equiv \text{SPF-10YR-Q3}_t - \frac{1}{354}\left(\pi_{t-10\to t-9} + 2\pi_{t-9\to t-8} + 18\pi_{t-8\to t-2}\right) \\
x_t^{20} &\equiv \text{SPF-10YR-Q4}_t - \frac{1}{354}\left(\pi_{t-13\to t-12} + 2\pi_{t-12\to t-11} + 27\pi_{t-11\to t-2}\right),
\end{aligned}
$$

and the remaining measurement equations are

$$
\begin{aligned}
x_t^{17} &= \frac{1}{354}\left(116\pi_{t\to t+116} + 117\pi_{t\to t+117} + 118\pi_{t\to t+118}\right) + \varepsilon_t^{17} \\
x_t^{18} &= \frac{1}{354}\left(113\pi_{t\to t+113} + 114\pi_{t\to t+114} + 115\pi_{t\to t+115}\right) + \varepsilon_t^{18} \\
x_t^{19} &= \frac{1}{354}\left(110\pi_{t\to t+110} + 111\pi_{t\to t+111} + 112\pi_{t\to t+112}\right) + \varepsilon_t^{19} \\
x_t^{20} &= \frac{1}{354}\left(107\pi_{t\to t+107} + 108\pi_{t\to t+108} + 109\pi_{t\to t+109}\right) + \varepsilon_t^{20}.
\end{aligned}
$$

The full measurement equations follow from applying (5) to the right-hand sides of these equations.

A.2 *Blue Chip* Quarterly Forecasts

I use the quarterly forecasts published in the *Blue Chip Economic Indicators.* Every month, the forecasters are asked to make between five and nine short-term forecasts. The table below summarizes the availability of forecasts for the year 2016 as an example in which "X" denotes a forecast, "-" reflects the absence of a forecast, and "*" next to an "X" shows a forecast that I use in this paper. I use the median of the individual forecasts.

	2015Q4	2016Q1	2016Q2	2016Q3	2016Q4	2017Q1	2017Q2	2017Q3	2017Q4
January	X	X	X*	X*	X*	X*	X*	X	X
February	-	X	X*	X*	X*	X*	X*	X	X
March	-	X	X*	X*	X*	X*	X*	X	X
April	-	X	X	X*	X*	X*	X*	X*	X
May	-	-	X	X*	X*	X*	X*	X*	X
June	-	-	X	X*	X*	X*	X*	X*	X
July	-	-	X	X	X*	X*	X*	X*	X*
August	-	-	-	X	X*	X*	X*	X*	X*
September	-	-	-	X	X*	X*	X*	X*	X*
October	-	-	-	X	X	X*	X*	X*	X*
November	-	-	-	-	X	X*	X*	X*	X*
December	-	-	-	-	X	X*	X*	X*	X*

The table shows that in most months I use five forecasts, each of which reflects the change in the average level of CPI over a quarter, relative to the previous quarter. The first quarter forecast I use is the one that follows the month the forecast is made – for example, in months that are in the first quarter, I use forecasts that are about the second, third, and fourth quarters of the current year and the first and second quarters of the following year. In the fourth quarter, I use only four forecasts. These choices follow from the timing and the unbalanced structure of the survey.

The first four of the forecasts I use follow the "3," "4," "5," and "6" forecasts of the *SPF*. Since the *SPF* forecasts were done in the second month of a quarter, I treated them identically. However, *Blue Chip* forecasts are made monthly and thus in different months of a quarter. This means I need to create three different versions of each *Blue Chip* forecast, depending on which month of the quarter it is made.

Starting with those made in the second month of a quarter, the expressions exactly mimic those from the *SPF* – for example the next-quarter forecast in February, May, August or

November will be

$$\text{BC-1Q-M2}_t = \frac{1}{8}\left(2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right),$$

where the notation is BC-XQ-MY means the forecast made in the Y^{th} month of a quarter covering the quarter that is X quarters after the current one. And the BC-5Q-M2 forecast, which was not available in the *SPF*, is given by

$$\text{BC-5Q-M2}_t = \frac{1}{3}\left(\pi_{t+11\to t+14} + \pi_{t+12\to t+15} + \pi_{t+13\to t+16}\right).$$

A forecast made in the first month of a quarter for the next quarter is

$$\begin{aligned}
\text{BC-1Q-M1}_t &= 100\left[\left(\frac{P_{t+3} + P_{t+4} + P_{t+5}}{P_t + P_{t+1} + P_{t+2}}\right)^4 - 1\right] \\
&\approx \frac{\pi_{t\to t+3} + \pi_{t+1\to t+4} + \pi_{t+2\to t+5}}{3},
\end{aligned}$$

and others are obtained as

$$\begin{aligned}
\text{BC-2Q-M1}_t &= \frac{\pi_{t+3\to t+6} + \pi_{t+4\to t+7} + \pi_{t+5\to t+8}}{3} \\
\text{BC-3Q-M1}_t &= \frac{\pi_{t+6\to t+9} + \pi_{t+7\to t+10} + \pi_{t+8\to t+11}}{3} \\
\text{BC-4Q-M1}_t &= \frac{\pi_{t+9\to t+12} + \pi_{t+10\to t+13} + \pi_{t+11\to t+14}}{3} \\
\text{BC-5Q-M1}_t &= \frac{\pi_{t+12\to t+15} + \pi_{t+13\to t+16} + \pi_{t+14\to t+17}}{3}.
\end{aligned}$$

Finally, a forecast made in the last month of a quarter for the next quarter is

$$\begin{aligned}
\text{BC-1Q-M3}_t &= 100\left[\left(\frac{P_{t+1} + P_{t+2} + P_{t+3}}{P_{t-2} + P_{t-1} + P_t}\right)^4 - 1\right] \\
&\approx \frac{1}{6}\left(\pi_{t\to t+1} + 2\pi_{t\to t+2} + 3\pi_{t\to t+3}\right),
\end{aligned}$$

where in the last line I used $2\pi_{t-2\to t} + \pi_{t-1\to t} = 3\text{BC-1Q-M3}_t$.

Other forecasts follow from

$$\begin{aligned}
\text{BC-2Q-M3}_t &= \frac{\pi_{t+1\to t+4} + \pi_{t+2\to t+5} + \pi_{t+3\to t+6}}{3} \\
\text{BC-3Q-M3}_t &= \frac{\pi_{t+4\to t+7} + \pi_{t+5\to t+8} + \pi_{t+6\to t+9}}{3} \\
\text{BC-4Q-M3}_t &= \frac{\pi_{t+7\to t+10} + \pi_{t+8\to t+11} + \pi_{t+9\to t+12}}{3} \\
\text{BC-5Q-M3}_t &= \frac{\pi_{t+10\to t+13} + \pi_{t+11\to t+14} + \pi_{t+12\to t+15}}{3}.
\end{aligned}$$

Collecting all these, I define

$$\begin{aligned}
x_t^{21} &\equiv \text{BC-1Q-M1}_t,\ x_t^{22} \equiv \text{BC-2Q-M1}_t,\ x_t^{23} \equiv \text{BC-3Q-M1}_t,\ x_t^{24} \equiv \text{BC-4Q-M1}_t, \\
x_t^{25} &\equiv \text{BC-5Q-M1}_t,\ x_t^{26} \equiv \text{BC-1Q-M2}_t,\ x_t^{27} \equiv \text{BC-2Q-M2}_t,\ x_t^{28} \equiv \text{BC-3Q-M2}_t, \\
x_t^{29} &\equiv \text{BC-4Q-M2}_t,\ x_t^{30} \equiv \text{BC-5Q-M2}_t,\ x_t^{31} \equiv \text{BC-1Q-M3}_t,\ x_t^{32} \equiv \text{BC-2Q-M3}_t, \\
x_t^{33} &\equiv \text{BC-3Q-M3}_t,\ x_t^{34} \equiv \text{BC-4Q-M3}_t,\ x_t^{35} \equiv \text{BC-5Q-M3}_t.
\end{aligned}$$

The measurement equations are

$$\begin{aligned}
x_t^{21} &= \frac{1}{3}\left(\pi_{t\to t+3} + \pi_{t+1\to t+4} + \pi_{t+2\to t+5}\right) + \varepsilon_t^{21} \\
x_t^{22} &= \frac{1}{3}\left(\pi_{t+3\to t+6} + \pi_{t+4\to t+7} + \pi_{t+5\to t+8}\right) + \varepsilon_t^{22} \\
x_t^{23} &= \frac{1}{3}\left(\pi_{t+6\to t+9} + \pi_{t+7\to t+10} + \pi_{t+8\to t+11}\right) + \varepsilon_t^{23} \\
x_t^{24} &= \frac{1}{3}\left(\pi_{t+9\to t+12} + \pi_{t+10\to t+13} + \pi_{t+11\to t+14}\right) + \varepsilon_t^{24} \\
x_t^{25} &= \frac{1}{3}\left(\pi_{t+12\to t+15} + \pi_{t+13\to t+16} + \pi_{t+14\to t+17}\right) + \varepsilon_t^{25}
\end{aligned}$$

$$\begin{aligned}
x_t^{26} &= \frac{1}{8}\left(2\pi_{t\to t+2} + 3\pi_{t\to t+3} + 3\pi_{t+1\to t+4}\right) + \varepsilon_t^{26} \\
x_t^{27} &= \frac{1}{3}\left(\pi_{t+2\to t+5} + \pi_{t+3\to t+6} + \pi_{t+4\to t+7}\right) + \varepsilon_t^{27} \\
x_t^{28} &= \frac{1}{3}\left(\pi_{t+5\to t+8} + \pi_{t+6\to t+9} + \pi_{t+7\to t+10}\right) + \varepsilon_t^{28} \\
x_t^{29} &= \frac{1}{3}\left(\pi_{t+8\to t+11} + \pi_{t+9\to t+12} + \pi_{t+10\to t+13}\right) + \varepsilon_t^{29} \\
x_t^{30} &= \frac{1}{3}\left(\pi_{t+11\to t+14} + \pi_{t+12\to t+15} + \pi_{t+13\to t+16}\right) + \varepsilon_t^{30}
\end{aligned}$$

$$
\begin{aligned}
x_t^{31} &= \frac{1}{6}\left(\pi_{t\to t+1} + 2\pi_{t\to t+2} + 3\pi_{t\to t+3}\right) + \varepsilon_t^{31} \\
x_t^{32} &= \frac{1}{3}\left(\pi_{t+1\to t+4} + \pi_{t+2\to t+5} + \pi_{t+3\to t+6}\right) + \varepsilon_t^{32} \\
x_t^{33} &= \frac{1}{3}\left(\pi_{t+4\to t+7} + \pi_{t+5\to t+8} + \pi_{t+6\to t+9}\right) + \varepsilon_t^{33} \\
x_t^{34} &= \frac{1}{3}\left(\pi_{t+7\to t+10} + \pi_{t+8\to t+11} + \pi_{t+9\to t+12}\right) + \varepsilon_t^{34} \\
x_t^{35} &= \frac{1}{3}\left(\pi_{t+10\to t+13} + \pi_{t+11\to t+14} + \pi_{t+12\to t+15}\right) + \varepsilon_t^{35}.
\end{aligned}
$$

A.3 *Blue Chip* Long-Range Forecasts

In the March and October issues of the *Blue Chip Economic Indicators* and the June and December issues of the *Blue Chip Financial Forecasts*, the forecasters are asked about their long-term forecasts. They are asked to make six forecasts of long-range inflation: five annual forecasts, each covering one calendar year, and one five-year forecast covering the five years following the five years in the last forecast. The annual forecasts are labeled as "year-over-year" forecasts, which means they are the percentage change in the average price level across years. More specifically, since October 2008, both of these publications ask the forecasters to forecast five years following the next year – for 2008 this would be years 2010, 2011, 2012, 2013, and 2014 – as well as the five-year forward forecast of 2015-2019. Prior to October 2008, in most years the format remained the same, but in some years the horizon shifted earlier by one year. To keep variables consistent throughout the sample, I use the format since 2008, and in years in which there is a shift, I use missing observations where appropriate.

In March of a year, the first object being forecast is defined as

$$
\begin{aligned}
\text{BCLR-2Y-M}_t &= 100\left[\left(\frac{\sum_{s=22}^{33} P_{t+s}}{\sum_{s=10}^{21} P_{t+s}}\right) - 1\right] \\
&\approx \frac{1}{12}\sum_{s=10}^{21} \pi_{t+s\to t+s+12},
\end{aligned}
$$

which I label as a two-year forecast simply because the forecast window is in the calendar year following the next. I will continue using the same notation for the rest of the three months in which these publications are released to keep things simple even though the forecasting window moves and it approaches the period in which the forecast is made. The "M" at the end of the variable name reflects the "March" forecast, and I use "J," "O," and "D" to represent June, October, and December, respectively, below. The first annual forecast made in June, October, and December refer to the year that is 19, 15, and 13 months following the month the forecast is made, respectively.

Defining

$$
\begin{aligned}
x_t^{36} &\equiv \text{BCLR-2Y-M}_t,\ x_t^{37} \equiv \text{BCLR-3Y-M}_t,\ x_t^{38} \equiv \text{BCLR-4Y-M}_t,\ x_t^{39} \equiv \text{BCLR-5Y-M}_t \\
x_t^{40} &\equiv \text{BCLR-6Y-M}_t,\ x_t^{41} \equiv \text{BCLR-2Y-J}_t,\ x_t^{42} \equiv \text{BCLR-3Y-J}_t,\ x_t^{43} \equiv \text{BCLR-4Y-J}_t \\
x_t^{44} &\equiv \text{BCLR-5Y-J}_t,\ x_t^{45} \equiv \text{BCLR-6Y-J}_t,\ x_t^{46} \equiv \text{BCLR-2Y-O}_t,\ x_t^{47} \equiv \text{BCLR-3Y-O}_t \\
x_t^{48} &\equiv \text{BCLR-4Y-O}_t,\ x_t^{49} \equiv \text{BCLR-5Y-O}_t,\ x_t^{50} \equiv \text{BCLR-6Y-O}_t,\ x_t^{51} \equiv \text{BCLR-2Y-D}_t \\
x_t^{52} &\equiv \text{BCLR-3Y-D}_t,\ x_t^{53} \equiv \text{BCLR-4Y-D}_t,\ x_t^{54} \equiv \text{BCLR-5Y-D}_t,\ x_t^{55} \equiv \text{BCLR-6Y-D}_t,
\end{aligned}
$$

the measurement equations for March are

$$
\begin{aligned}
x_t^{36} &= \frac{1}{12}\sum_{s=10}^{21} \pi_{t+s\to t+s+12} + \varepsilon_t^{36} \\
x_t^{37} &= \frac{1}{12}\sum_{s=22}^{33} \pi_{t+s\to t+s+12} + \varepsilon_t^{37} \\
x_t^{38} &= \frac{1}{12}\sum_{s=34}^{45} \pi_{t+s\to t+s+12} + \varepsilon_t^{38} \\
x_t^{39} &= \frac{1}{12}\sum_{s=46}^{57} \pi_{t+s\to t+s+12} + \varepsilon_t^{39} \\
x_t^{40} &= \frac{1}{12}\sum_{s=58}^{69} \pi_{t+s\to t+s+12} + \varepsilon_t^{40}.
\end{aligned}
$$

Then the measurement equations for June are

$$\begin{aligned}
x_t^{41} &= \frac{1}{12}\sum_{s=7}^{18}\pi_{t+s\to t+s+12}+\varepsilon_t^{41}\\
x_t^{42} &= \frac{1}{12}\sum_{s=19}^{30}\pi_{t+s\to t+s+12}+\varepsilon_t^{42}\\
x_t^{43} &= \frac{1}{12}\sum_{s=31}^{42}\pi_{t+s\to t+s+12}+\varepsilon_t^{43}\\
x_t^{44} &= \frac{1}{12}\sum_{s=43}^{54}\pi_{t+s\to t+s+12}+\varepsilon_t^{44}\\
x_t^{45} &= \frac{1}{12}\sum_{s=55}^{66}\pi_{t+s\to t+s+12}+\varepsilon_t^{45}.
\end{aligned}$$

And for October the measurement equations are

$$\begin{aligned}
x_t^{46} &= \frac{1}{12}\sum_{s=3}^{14}\pi_{t+s\to t+s+12}+\varepsilon_t^{46}\\
x_t^{47} &= \frac{1}{12}\sum_{s=15}^{26}\pi_{t+s\to t+s+12}+\varepsilon_t^{47}\\
x_t^{48} &= \frac{1}{12}\sum_{s=27}^{38}\pi_{t+s\to t+s+12}+\varepsilon_t^{48}\\
x_t^{49} &= \frac{1}{12}\sum_{s=39}^{50}\pi_{t+s\to t+s+12}+\varepsilon_t^{49}\\
x_t^{50} &= \frac{1}{12}\sum_{s=51}^{62}\pi_{t+s\to t+s+12}+\varepsilon_t^{50}.
\end{aligned}$$

And finally December forecasts use

$$
\begin{aligned}
x_t^{51} &= \frac{1}{12}\sum_{s=1}^{12} \pi_{t+s\rightarrow t+s+12} + \varepsilon_t^{51} \\
x_t^{52} &= \frac{1}{12}\sum_{s=13}^{24} \pi_{t+s\rightarrow t+s+12} + \varepsilon_t^{52} \\
x_t^{53} &= \frac{1}{12}\sum_{s=25}^{36} \pi_{t+s\rightarrow t+s+12} + \varepsilon_t^{53} \\
x_t^{54} &= \frac{1}{12}\sum_{s=37}^{48} \pi_{t+s\rightarrow t+s+12} + \varepsilon_t^{54} \\
x_t^{55} &= \frac{1}{12}\sum_{s=49}^{60} \pi_{t+s\rightarrow t+s+12} + \varepsilon_t^{55}.
\end{aligned}
$$

These publications contain two more forecasts. Once again using the October 2008 issue as an example, there are forecasts for "2010-2014" and "2015-2019." The former is an arithmetic average of the five annual forecasts I use and thus is not independently useful. In order to use the latter, I take its simple average with the former and label this the forecast for the 10-year period of 2010-2019. This forecast is defined as the average of the 10 annual price changes, each of which is in the format I use above – annual change in the average price level between two years. The March forecast can be written as

$$
\begin{aligned}
\text{BCLR-10Y-M}_t &= \frac{1}{10}\left\{\frac{1}{12}\left[\sum_{s=10}^{21} \pi_{t+s\rightarrow t+s+12} + \sum_{s=22}^{33} \pi_{t+s\rightarrow t+s+12} + \sum_{s=34}^{45} \pi_{t+s\rightarrow t+s+12}\right.\right. \\
&\quad + \sum_{s=46}^{57} \pi_{t+s\rightarrow t+s+12} + \sum_{s=58}^{69} \pi_{t+s\rightarrow t+s+12} + \sum_{s=70}^{81} \pi_{t+s\rightarrow t+s+12} \sum_{s=82}^{93} \pi_{t+s\rightarrow t+s+12} \\
&\quad \left.\left. + \sum_{s=94}^{105} \pi_{t+s\rightarrow t+s+12} + \sum_{s=106}^{117} \pi_{t+s\rightarrow t+s+12} + \sum_{s=118}^{129} \pi_{t+s\rightarrow t+s+12}\right]\right\} \\
&= \frac{1}{12}\sum_{s=10}^{21} \pi_{t+s\rightarrow t+s+120},
\end{aligned}
$$

where the last equality follows from the properties of continuous compounding.[A-9] Denoting x_t^{56} ≡BCLR-10Y-M$_t$, x_t^{57} ≡BCLR-10Y-J$_t$, x_t^{58} ≡BCLR-10Y-O$_t$, and x_t^{59} ≡BCLR-10Y-D$_t$, the measurement equations are

$$\begin{aligned}
x_t^{56} &= \frac{1}{12}\sum_{s=10}^{21} \pi_{t+s\to t+s+120} + \varepsilon_t^{56} \\
x_t^{57} &= \frac{1}{12}\sum_{s=7}^{18} \pi_{t+s\to t+s+120} + \varepsilon_t^{57} \\
x_t^{58} &= \frac{1}{12}\sum_{s=3}^{14} \pi_{t+s\to t+s+120} + \varepsilon_t^{58} \\
x_t^{59} &= \frac{1}{12}\sum_{s=1}^{12} \pi_{t+s\to t+s+120} + \varepsilon_t^{59}.
\end{aligned}$$

The measurement equations are directly obtained from (5) with measurement errors ε_t^i.

B State Space Model

The preceding section show the measurement equations of the observable variables I use in my analysis – all in all, 59 variables. Combining all the measurement equations and the transition equation for the three factors, I obtain a state-space system

$$\mathbf{x}_t = \mathbf{Z}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t \tag{A-1}$$

$$(\boldsymbol{\alpha}_t - \boldsymbol{\mu}) = \mathbf{T}(\boldsymbol{\alpha}_{t-1} - \boldsymbol{\mu}) + \boldsymbol{\eta}_t \tag{A-2}$$

[A-9] To see this, consider the simplified example

$$\begin{aligned}
\frac{1}{2}\left(\frac{1}{2}\sum_{s=13}^{14}\pi_{t+s\to t+s+2} + \frac{1}{2}\sum_{s=15}^{16}\pi_{t+s\to t+s+2}\right) &= \frac{1}{4}\left(\pi_{t+13\to t+15} + \pi_{t+14\to t+16} + \pi_{t+15\to t+17} + \pi_{t+16\to t+18}\right) \\
&= \frac{1}{4}\left(\pi_{t+13\to t+15} + \pi_{t+15\to t+17} + \pi_{t+14\to t+16} + \pi_{t+16\to t+18}\right) \\
&= \frac{1}{2}\left(\pi_{t+13\to t+17} + \pi_{t+14\to t+18}\right) \\
&= \frac{1}{2}\sum_{s=13}^{14}\pi_{t+s\to t+s+4}.
\end{aligned}$$

with

$$\boldsymbol{\varepsilon}_t \sim N(0, \mathbf{H}) \text{ and } \boldsymbol{\eta}_t \sim N(0, \mathbf{Q}), \tag{A-3}$$

where the notation follows the standard notation in Durbin and Koopman (2012). The vector $\mathbf{x}_t$ is a 59×1 vector containing all observed variables in period t, and $\boldsymbol{\alpha}_t$ is a 9×1 vector that collects the three inflation expectation factors and their two lags in period t:

$$\boldsymbol{\alpha}_t = \begin{bmatrix} L_t & S_t & C_t & L_{t-1} & S_{t-1} & C_{t-1} & L_{t-2} & S_{t-2} & C_{t-2} \end{bmatrix}'. \tag{A-4}$$

and the constant $\boldsymbol{\mu}$ is given by

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_L & \mu_S & \mu_C & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'. \tag{A-5}$$

The vector $\boldsymbol{\varepsilon}_t$ contains the measurement errors, and thus $\mathbf{H}$ is a diagonal matrix with

$$\mathbf{H} = diag\left(\sigma_1^2, \sigma_2^2, ..., \sigma_{35}^2\right). \tag{A-6}$$

The measurement matrix $\mathbf{Z}$ collects the factor loadings described in the previous section and is given by

$$\mathbf{Z} = \begin{bmatrix} f_L^1 & f_S^1 & f_C^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ f_L^2 & f_S^2 & f_C^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_L^{59} & f_S^{59} & f_C^{59} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A-7}$$

The transition matrix $\mathbf{T}$ takes the form

$$\mathbf{T} = \begin{bmatrix} \rho_{11} & 0 & 0 & \rho_{12} & 0 & 0 & \rho_{13} & 0 & 0 \\ 0 & \rho_{21} & 0 & 0 & \rho_{22} & 0 & 0 & \rho_{23} & 0 \\ 0 & 0 & \rho_{31} & 0 & 0 & \rho_{32} & 0 & 0 & \rho_{33} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \tag{A-8}$$

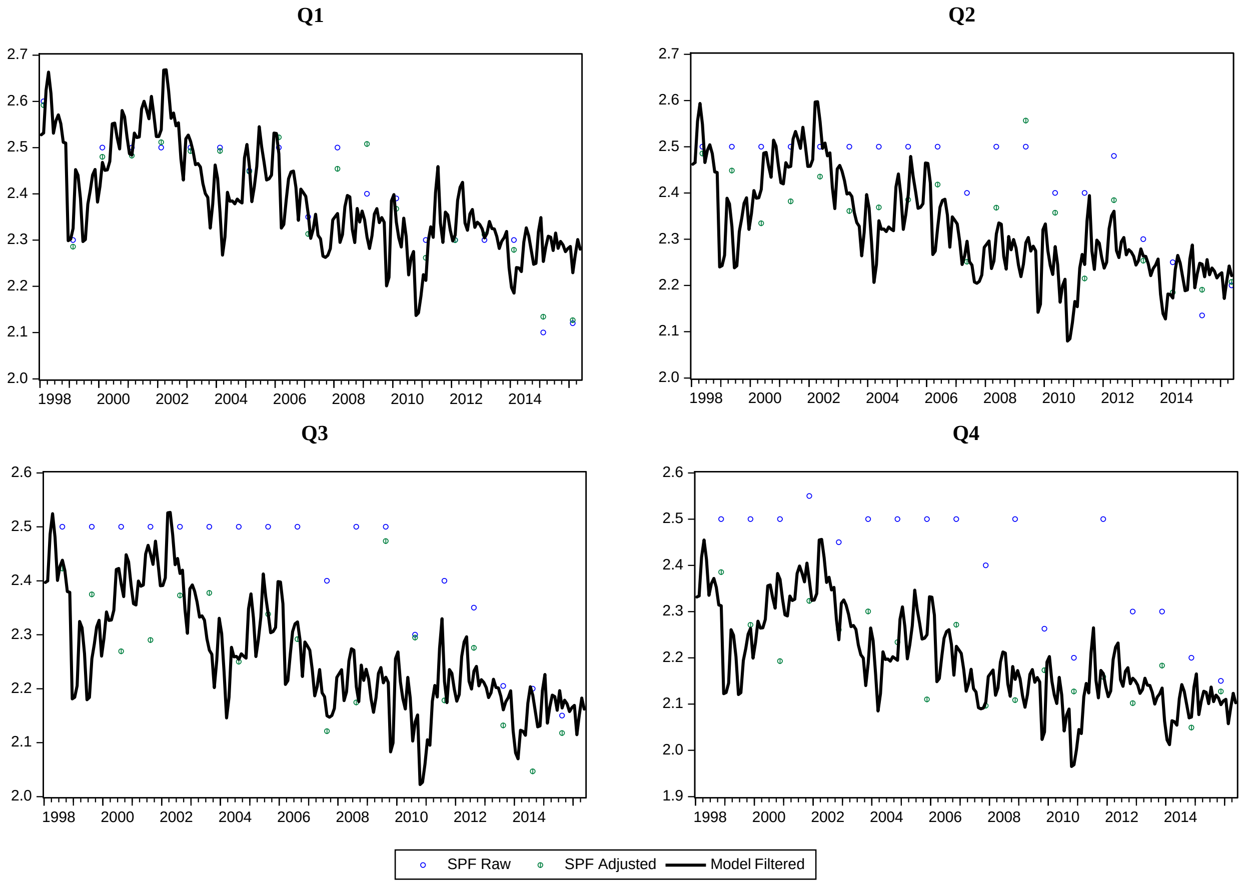
Finally, $\mathbf{Q}$ is a diagonal matrix with

$$\mathbf{Q} = diag\left(\eta_t^L, \eta_t^S, \eta_t^C, 0, 0, 0, 0, 0, 0\right). \tag{A-9}$$

References

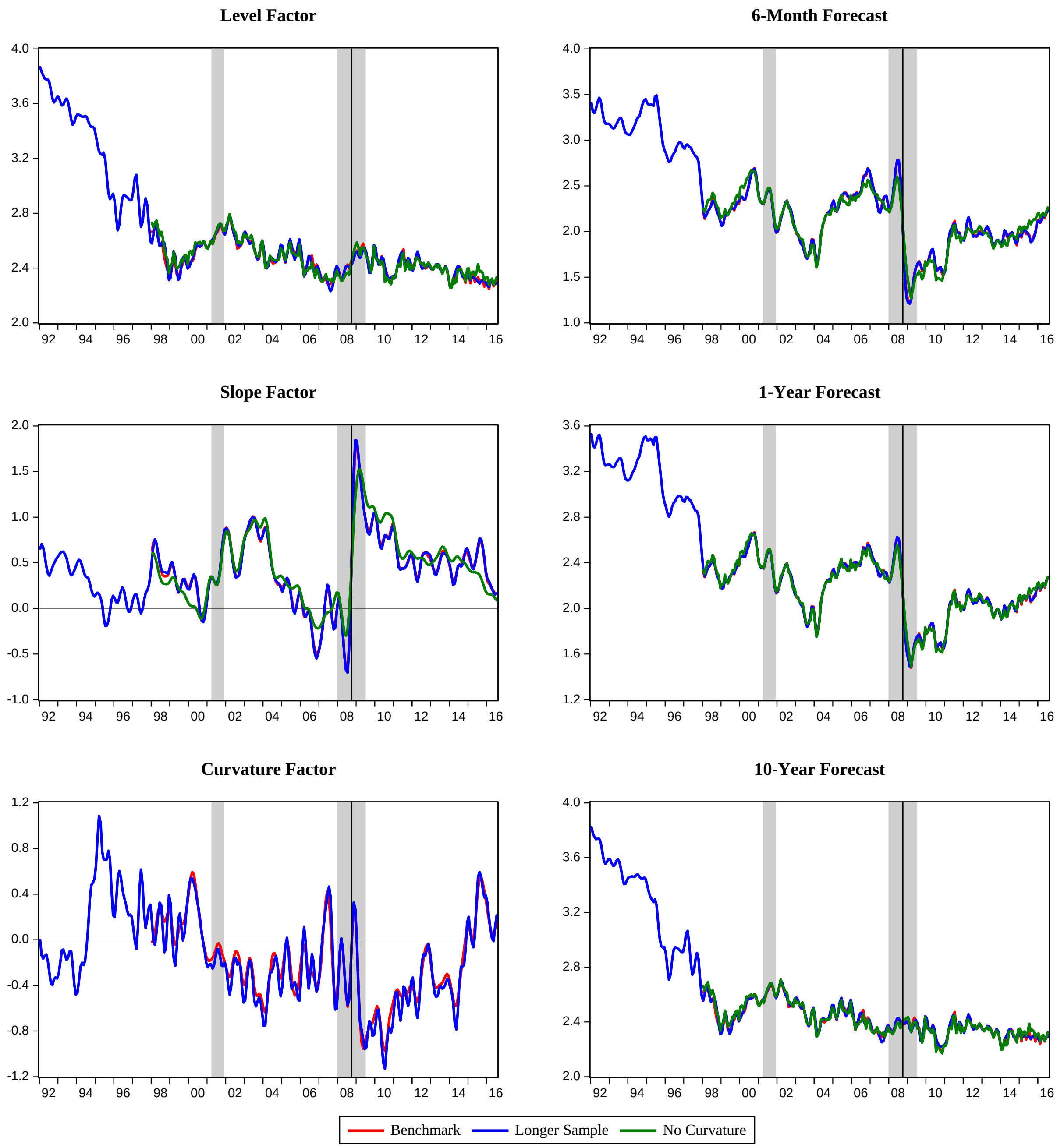
[1] Stark, T. (2010), "Realistic Evaluation of Real-Time Forecasts in the Survey of Professional Forecasters," Federal Reserve Bank of Philadelphia *Research Rap* Special Report.

Figure A1: Adjustment of the Raw SPF 10-Year Data



Notes: This figure shows how the adjustments explained in Appendix A.1 change the raw data for the SPF 10-year forecast.

Figure A2: Comparison of Factors and Forecasts in Alternative Specifications



Notes: The gray bars denote NBER recessions. The vertical line denotes September 2008.

Figure A3: Comparison of Factors and Forecasts in Alternative Methods for Extracting Factors

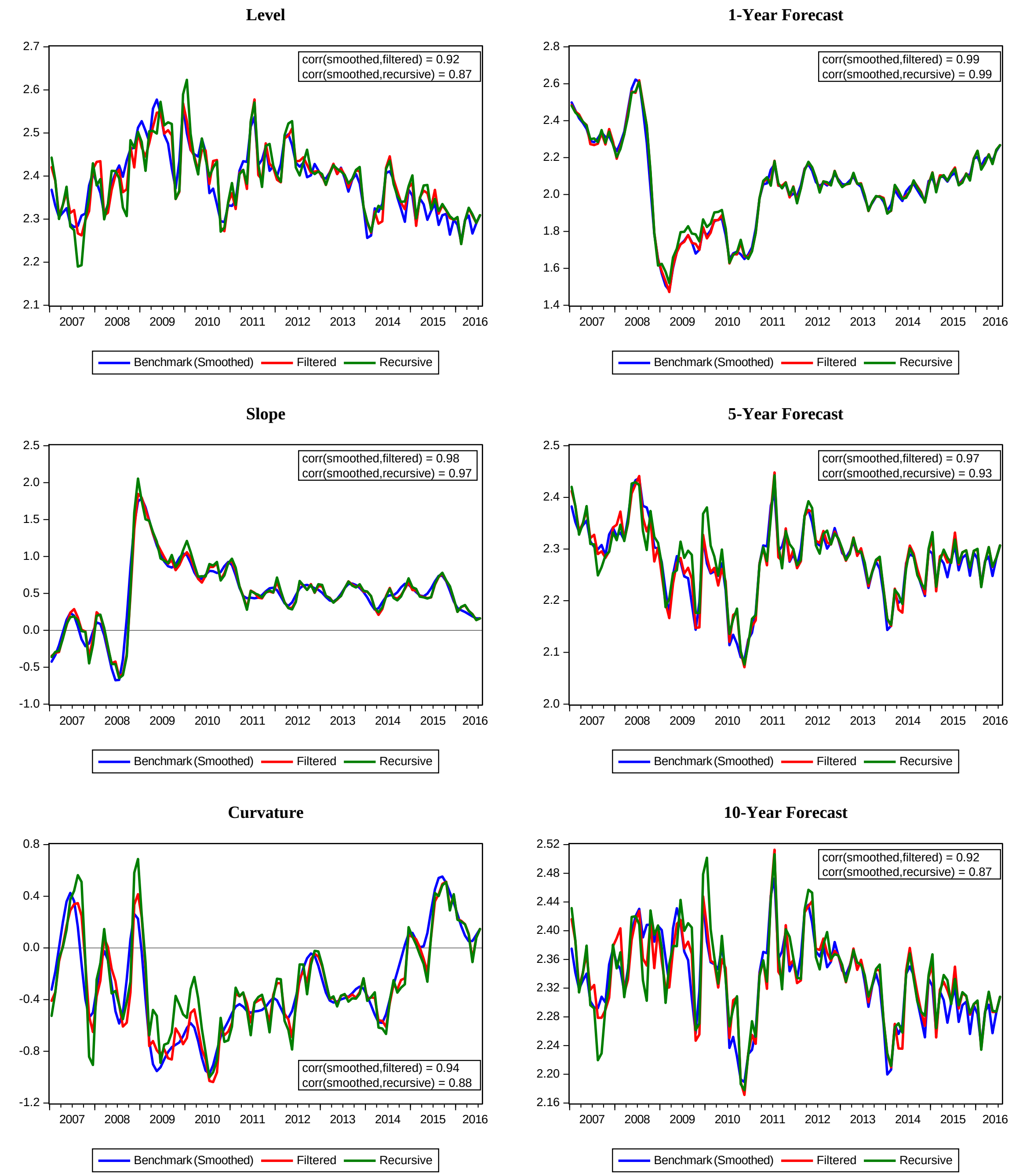


Table A1: Parameter Estimates for the UCSV Model

	Prior	5%	**Median**	95%
φ	U[0,1]	0.08	0.14	0.20
σ	IG(3,5)	1.25	1.39	1.65
ρ_η	N(0.9,5)	-0.40	0.33	0.80
ρ_ϵ	N(0.9 5)	0.85	0.93	0.97
σ_{ν_η}	IG(3,0.1)	0.14	0.24	0.44
σ_{ν_ϵ}	IG(3,0.1)	0.51	0.65	0.90

Notes: The first column shows the marginal prior distribution for each parameter where $U[a,b]$ means the uniform distribution between a and b, $N(a,b)$ means the normal distribution with mean a and variance b, and IG means the inverse gamma distribution $IG(a,b)$, which is parameterized as $p_{IG}(\sigma \mid a,b) \propto \sigma^{-a-1}\exp(b/\sigma)$. The priors for ρ_η and ρ_ϵ are truncated to ensure stationarity. The remaining columns show the given percentiles of the posterior distribution. Estimation uses data from January 1984 to December 2015.