Response to referee report on:
"A new projection algorithm for solving constrained equilibrium problems in Hilbert spaces"

by Nguyen The Vinh.

December 31, 2018

General comments to the editor and the reviewers: First of all, we would like to thank the editor and the reviewers for giving us the opportunity to revise the manuscript GOPT-2016-0486.R1 entitled "A new projection algorithm for solving constrained equilibrium problems in Hilbert spaces".

We are very grateful to your comments and useful suggestions, which are valuable in improving the quality of our manuscript. Based on your guidance, we have carefully revised the manuscript, and marked in red all the changes in the revised version. The revision also includes a number of positive changes.

Responses to the Associate Editor's comments:

Comment: There are only some very minor points to be corrected (see the two referee's reports). Once appropriately done, the paper will deserve publication.

Response: *Thank you for your supportive comments on my manuscript. Our point-by-point responses to the general and specific comments are presented below.* Based on your comments, we:

1. Added six references and removed the references [15] and [20] in the old version.

2. Revised lemmas 2.5 and 2.6 with the new citation.

3. Clarified some raised questions and conditions imposed on f.

4. Thanks to Lemma 2.5, the simplified proof of Case 1 of Theorem 3.5 is given.

5. Typos are fixed as well as grammar corrections.

We next detail our responses to the reviewer' concerns and comments.

Responses to the reviewer #1's comments:

Comment:

1) The numerical experiment could be a genuine CEP.

2) Could you provide a short remark on the choice of the parameters, in the numerical experiments, for MPA and Algorithm 1?

Response: *Thank you very much for evaluating my manuscript. We added a remark as suggested.*

Comment: In this revision, I found that the author improved the manuscript as commented by reviewers. However, there are some minor errors which can be done.

1. Some errors on the English usage still appear in the paper. Please check it throughout carefully.

2. Where is x1?

3. From Figures 1-3, the error plotting of the Algorithm involving inertial term seems do not stable. I suggest the author for giving more experiments and also adding some observations in remarks.

4. There have been some articles concerning this topic that can be considered to cite in Introduction:

   4.1 S. Suantai and P. Cholamjiak, Algorithms for solving generalized equilibrium problems and fixed points of nonexpansive semigroups in Hilbert spaces, Optimization, 63 (2014), 799-815.

   4.2 P. Cholamjiak and S. Suantai, Iterative methods for solving equilibrium problems, variational inequalities and fixed points of nonexpansive semigroups, J. Glob. Optim. 57 (2013), 1277-1297.

   4.3 S. Suantai, N. Pholasa and P. Cholamjiak, Relaxed CQ algorithms involving the inertial technique for multiple-sets split feasibility problems, RACSAM (2018). https://doi.org/10.1007/s13398-018-0535-7.

   4.4 S. Suantai, N. Pholasa and P. Cholamjiak, The modified inertial relaxed CQ algorithm for solving the split feasibility problems, J. Indust. Manag.Optim. (2018), doi: 10.3934/jimo.2018023.

   4.5 W. Cholamjiak, P. Cholamjiak and S. Suantai, An inertial forwardbackward splitting method for solving inclusion problems in Hilbert spaces, J. Fixed Point Theory Appl. (2018) 20: 42. https://doi.org/10.1007/s11784-018-0526-5.

After revising, I suggest it for publication in Optimization.

Response: Thank you very much for the useful remarks. Misprints and typos are fixed as well as grammar corrections. In the numerical experiment, we take $x^1 = x^0$ for our proposed algorithm. The error plotting of the Algorithm 3.1 involving inertial term seems do not stable. The reason is that we computed x^{k+1} by the formula

$$x^{k+1} = P_{D \cap H_k}(w^k),$$

where the convex set $D \cap H_k$ contains the solution set but $\{D \cap H_k\}$ is not necessary a decreasing nested sequence. We gave more experiments and also added some observations in remarks. New references (4.1-4.5) are added as suggested. Please check the change in the revised version.

Again, we highly appreciate all of the referee's insightful comments.