SUPPLEMENT TO "HETEROSKEDASTICITY ROBUST PANEL UNIT ROOT TESTS": ADDITIONAL DETAILS AND MONTE CARLO RESULTS

September 12, 2013

Abstract

In this supplement, we (i) study in detail the variance test statistics discussed the main paper, (ii) report some Monte Carlo evidence on the quasi-maximum likelihood (QML) group selection approach considered in that same paper, and (iii) provide some additional Monte Carlo results on the unit root testing problem.

1 The variance tests

In this section we begin by presenting the test statistics discussed in the main paper for testing the equality of the group-wise variances. The asymptotic distributions of these statistics are reported, which are evaluated by means of Monte Carlo simulation in Section 1.2.

1.1 The test statistics and their asymptotic distributions

As mentioned in the main paper, if there is uncertainty regarding the equality of the group-wise variances, then the appropriate approach will depend on the extent to which there is a priori knowledge regarding the grouping. In this section, we begin by considering the case when the researcher knows which units that belong to which group, and the question is if the variances of those groups are in fact equal. Let us therefore consider two such groups, m and n. A natural candidate for a test is the following t-statistic:

$$\tau_{mn,t} = \frac{\sqrt{N}(\hat{\sigma}^2_{m,t} - \hat{\sigma}^2_{n,t})}{\hat{\omega}_{mn,t}},$$

where

$$\hat{\omega}^2_{mn,t} = N\left(\frac{1}{(N_m - N_{m-1})} + \frac{1}{(N_n - N_{n-1})}\right)(\hat{\sigma}^2_{m,t} + \hat{\sigma}^2_{n,t})^2(\hat{\kappa}_t - 1),$$

with $\hat{\kappa}_t = \sum_{i=1}^{N} \hat{\epsilon}_{i,t}^4 / N$. While $\tau_{mn,t}$ is only point-wise in t, the point-wise statistics can be combined. The first combination statistic that we consider is simply the empirical rejection frequency of $\tau_{mn,t}$, and is given by

$$F_{mn} = \frac{1}{T} \sum_{t=p_i+2}^{T} 1(|\tau_{mn,t}| > c_\alpha),$$

where $1(A)$ and c_α denote is the indicator function for the event A, and the $(1-\alpha)$-quantile of the standard normal distribution, respectively. This statistic allows $\hat{\sigma}_{m,t}^2$ to deviate from $\hat{\sigma}_{n,t}^2$ for a pre-specified number of time points, as specified by α. A stronger test is to require $\hat{\sigma}_{m,t}^2$ not to deviate from $\hat{\sigma}_{n,t}^2$ by more than the sampling error at every t. The following test statistic is based on this stronger requirement:

$$M_{mn} = \max_{t=p_i+2,\dots,T} |\tau_{mn,t}|.$$

The relevant asymptotic results are reported in Proposition 1.

Proposition 1. *Under the conditions of Theorem 1 and the null hypothesis of $H_0 : \sigma_{m,t}^2 = \sigma_{n,t}^2 = \sigma_t^2$ for all t, given $\kappa \geq 0$, as N, $T \to \infty$ with $N/T \to 0$*

$$\begin{aligned} F_{mn} &\to 2\alpha, \\ P(M_{mn} < x) &\sim (2\Phi(x) - 1)^T, \end{aligned}$$

where $\sim$ and $\Phi(x)$ signify asymptotic equivalence and the standard normal cumulative distribution function, respectively.

Proof of Proposition 1.

This proof is almost an immediate consequence of Lemma A.1 in the main paper. Note first that by that lemma,

$$\hat{\sigma}_{m,t}^2 = \sigma_{m,t}^2 + \overline{C}_{0m,t} = \sigma_{m,t}^2 + \overline{c}_{0m,t} + O_p\left(\frac{1}{C_{2NT}}\right).$$

where $C_{2NT} = \min\{N, \sqrt{T}\}$ and $\overline{c}_{0m,t} = \sum_{i=N_{m-1}+1}^{N_m} (\epsilon_{i,t}^2 - \sigma_{m,t}^2)/(N_m - N_{m-1})$. Consider $\overline{c}_{0m,t}$. Clearly, $E(\overline{c}_{0m,t}) = 0$ and

$$\begin{aligned} E[(\sqrt{N_m - N_{m-1}}\overline{c}_{0m,t})^2] &= \sigma_{m,t}^4 \frac{1}{(N_m - N_{m-1})} \sum_{i=N_{m-1}+1}^{N_m} E[(\varepsilon_{i,t}^2 - 1)^2] \\ &= \sigma_{m,t}^4 \frac{1}{(N_m - N_{m-1})} \sum_{i=N_{m-1}+1}^{N_m} (E(\varepsilon_{i,t}^4) - 1) \\ &= \sigma_{m,t}^4 (\kappa_t - 1). \end{aligned}$$

Therefore, by a central limit theorem,

$$\sqrt{N_m - N_{m-1}}\bar{c}_{0m,t} \to_d N(0, \sigma_{m,t}^4(\kappa_t - 1)) \tag{1}$$

as N, $T \to \infty$. But we also have, for $m \neq n$,

$$\begin{aligned}
&(N_n - N_{n-1})(N_m - N_{m-1})E(\bar{c}_{0n,t}\bar{c}_{0m,t}) \\
&= \sigma_{n,t}^4\sigma_{m,t}^4 \sum_{i=N_{n-1}+1}^{N_n} \sum_{k=N_{m-1}+1}^{N_m} E[(\varepsilon_{i,t}^2 - 1)(\varepsilon_{k,t}^2 - 1)] \\
&= \sigma_{n,t}^4\sigma_{m,t}^4 \sum_{i=N_{n-1}+1}^{N_n} \sum_{k=N_{m-1}+1}^{N_m} [E(\varepsilon_{i,t}^2)E(\varepsilon_{k,t}^2) - E(\varepsilon_{i,t}^2) - E(\varepsilon_{k,t}^2) + 1] = 0,
\end{aligned}$$

where the second equality holds because of cross-section independence. These results imply that under the null hypothesis of $H_0 : \sigma_{m,t}^2 = \sigma_{n,t}^2 = \sigma_t^2$,

$$\begin{aligned}
\sqrt{N}(\hat{\sigma}_{m,t}^2 - \hat{\sigma}_{n,t}^2) &= \sqrt{N}(\sigma_{m,t}^2 + \bar{c}_{0m,t}) - \sqrt{N}(\sigma_{n,t}^2 + \bar{c}_{0n,t}) + O_p\left(\frac{\sqrt{N}}{C_{2NT}}\right) \\
&= \sqrt{N}\bar{c}_{0m,t} - \sqrt{N}\bar{c}_{0n,t} + O_p\left(\frac{\sqrt{N}}{C_{2NT}}\right) \\
&= \frac{\sqrt{N_m - N_{m-1}}}{\sqrt{\lambda_m - \lambda_{m-1}}}\bar{c}_{0m,t} - \frac{\sqrt{N_n - N_{n-1}}}{\sqrt{\lambda_n - \lambda_{n-1}}}\bar{c}_{0n,t} + O_p\left(\frac{\sqrt{N}}{C_{2NT}}\right) \\
&\to_d N(0, \omega_{mn,t}^2) \qquad (2)
\end{aligned}$$

as N, $T \to \infty$ with $N/T \to 0$, where

$$\omega_{mn,t}^2 = \left(\frac{1}{(\lambda_m - \lambda_{m-1})} + \frac{1}{(\lambda_n - \lambda_{n-1})}\right)\sigma_t^4(\kappa_t - 1).$$

Note how the second equality here follows from imposing H_0, while the third follows from the definition of N_m as $N_m = \lfloor \lambda_m N \rfloor$. Now, according to Lemma A.1, we have that $\hat{\sigma}_{m,t}^2 = \sigma_{m,t}^2 + O_p(1/\sqrt{N})$ and, similarly, $\hat{\kappa}_t = \kappa_t + O_p(1/\sqrt{N})$. This, together with Taylor expansion of the inverse square root, implies

$$\tau_{mn,t} = \frac{\sqrt{N}(\hat{\sigma}_{m,t}^2 - \hat{\sigma}_{n,t}^2)}{\hat{\omega}_{mn,t}} = \frac{\sqrt{N}(\hat{\sigma}_{m,t}^2 - \hat{\sigma}_{n,t}^2)}{\omega_{mn,t}} + O_p\left(\frac{1}{\sqrt{N}}\right) \to_d N(0,1) \tag{3}$$

as N, $T \to \infty$ with $N/T \to 0$. Hence, denoting by c_α the $(1-\alpha)$-quantile of $N(0,1)$, we have $P(|\tau_{mn,t}| > c_\alpha) \to 2\alpha$, suggesting that, by a law of large numbers,

$$F_{mn} = \frac{1}{T}\sum_{t=p_i+2}^{T} 1(|\tau_{mn,t}| > c_\alpha) \to 2\alpha, \tag{4}$$

where $1(A)$ is the indicator function for the event A. This establishes the first result.

As for the second result, note that since $\epsilon_{i,t}$ is serially uncorrelated, $\overline{c}_{0m,t}$ is serially correlated too. Hence, $\tau_{mn,t}$ is asymptotically uncorrelated across t, and hence independent by normality. Thus, letting $M_{mn} = \max_{t=p_i+2,...,T} |\tau_{mn,t}|$, we can show that

$$P(M_{mn} < x) = P\left(\max_{t=p_i+2,...,T} |\tau_{mn,t}| < x\right) \sim \prod_{t=p_i+2}^{T} P(|\tau_{mn,t}| < x) = (2\Phi(x) - 1)^T, \quad (5)$$

where $\sim$ signifies asymptotic equivalence and $\Phi(x)$ is the cumulative distribution function of $N(0,1)$. This establishes the second result, and hence the proof of the proposition is complete. ■

According to Proposition 1, the appropriate α-level critical value for use with M_{mn}, c_α, is defined by $\Phi(c_\alpha) = (1 + (1 - c_\alpha)^{1/T})/2$, which is increasing in T. This is illustrated in Table 1, which report the critical values for some selected values of T. If M_{mn} accepts, then we conclude that groups m and n have the same variance, which means that for the purpose of estimation of the group-specific variances, they can be treated as one. On the other hand, if the test rejects, then the groups should not be merged. In contrast to M_{mn}, F_{mn} is not a statistical test but rather a point estimator of the theoretical rejection frequency. If H_0 is true, then F_{mn} should be close to 2α, whereas if H_0 is false, then F_{mn} should be larger than 2α.

Table 1: Critical values for M_{mn}.

α	$T = 20$	$T = 50$	$T = 100$	$T = 200$	$T = 400$
0.01	3.479	3.718	3.889	4.054	4.214
0.05	3.016	3.283	3.474	3.656	3.830
0.1	2.791	3.075	3.276	3.467	3.649

Notes: α refers to the significance level.

The above tests suppose that the researcher knows the group identity of all the units of the panel. If this is not the case, then F_{mn} and M_{mn} may be applied in a sequential fashion to determine the groups. In the main paper an approach based on QML is proposed. A key step in that approach is to make use of the time series dimension to order the cross-section, after which the problem of group selection becomes a similar to that of breakpoint estimation. Specifically, the cross-section is ordered according to $\hat{\sigma}_i^2 = \sum_{t=p_i+2}^{T} \hat{\epsilon}_{i,t}^2 / T$. This makes it possible to use the following sequential test approach (see, for example, Bai, 1997; 1999, for similar proposals in the case of break estimation). We begin by testing $H_0 : \sigma_{1,t}^2 =$

$... = \sigma^2_{M+1,t} = \sigma^2_t$ for all t. The appropriate version of $\tau_{mn,t}$ to use in this case, denoted $\tau_{n,t}$, is the same as before but with $\hat{\sigma}^2_{m,t}$ and $\hat{\omega}^2_{mn,t}$ replaced by $\hat{\sigma}^2_t = \sum_{i=1}^N \hat{\epsilon}^2_{i,t}/N$ and

$$\hat{\omega}^2_{n,t} = N\left(\frac{1}{(N_n - N_{n-1})} - 1\right)\hat{\sigma}^4_t(\hat{\kappa}_t - 1),$$

respectively. The resulting combination statistics are written in an obvious notation as F_n and M_n.[1] If the null is not rejected by these test statistics, there is no heteroskedasticity, and so the testing is stopped. If, however, the null is rejected, then the testing continues by using F_{mn} and M_{mn} to test the equality of potential group candidates, which may be selected by looking for obvious breaks in the ordered $\hat{\sigma}^2_i$. The appropriate group to use in the first step (when testing $H_0 : \sigma^2_{1,t} = ... = \sigma^2_{M+1,t} = \sigma^2_t$) can be selected in the same way. Because in this case only one group is needed, one may focus on the group where the difference to the other units is most obvious.

The steps used for evaluating $\tau_{n,t}$ are exactly the same as in the proof of Proposition 1. The only difference is that now

$$\begin{aligned}
&\sqrt{N}(\hat{\sigma}^2_t - \hat{\sigma}^2_{n,t}) \\
&= \sum_{m=1}^{M+1}(\lambda_m - \lambda_{m-1})\sqrt{N}(\sigma^2_{m,t} + \bar{c}_{0m,t}) - \sqrt{N}(\sigma^2_{n,t} + \bar{c}_{0n,t}) + O_p\left(\frac{\sqrt{N}}{C_{2NT}}\right) \\
&= \sum_{m=1}^{M+1}(\lambda_m - \lambda_{m-1})\sqrt{N}\bar{c}_{0m,t} - \sqrt{N}\bar{c}_{0n,t} + O_p\left(\frac{\sqrt{N}}{C_{2NT}}\right) \\
&= \sum_{m=1}^{M+1}(\lambda_m - \lambda_{m-1})\frac{\sqrt{N_m - N_{m-1}}}{\sqrt{\lambda_m - \lambda_{m-1}}}\bar{c}_{0m,t} - \frac{\sqrt{N_n - N_{n-1}}}{\sqrt{\lambda_n - \lambda_{n-1}}}\bar{c}_{0n,t} + O_p\left(\frac{\sqrt{N}}{C_{2NT}}\right) \\
&= \sum_{m\neq n}^{M+1}(\lambda_m - \lambda_{m-1})\frac{\sqrt{N_m - N_{m-1}}}{\sqrt{\lambda_m - \lambda_{m-1}}}\bar{c}_{0m,t} \\
&+ (\lambda_n - \lambda_{n-1} - 1)\frac{\sqrt{N_n - N_{n-1}}}{\sqrt{\lambda_n - \lambda_{n-1}}}\bar{c}_{0n,t} + O_p\left(\frac{\sqrt{N}}{C_{2NT}}\right) \to_d N(0, \omega^2_{n,t}) \qquad (6)
\end{aligned}$$

as N, $T \to \infty$ with $N/T \to 0$, where

$$\begin{aligned}
\omega^2_{n,t} &= \left(\sum_{m\neq n}^{M+1}(\lambda_m - \lambda_{m-1}) + \frac{(\lambda_n - \lambda_{n-1} - 1)^2}{(\lambda_n - \lambda_{n-1})}\right)\sigma^4_t(\kappa_t - 1) \\
&= \left(\frac{1}{(\lambda_n - \lambda_{n-1})} - 1\right)\sigma^4_t(\kappa_t - 1).
\end{aligned}$$

The second equality follows from imposing the null hypothesis of $\sigma^2_{1,t} = ... = \sigma^2_{M+1,t} = \sigma^2_t$. As for the third equality, note that $\sum_{m=1}^{M+1}(\lambda_m - \lambda_{m-1}) = 1$, suggesting that, under the null,

[1] As we show in Remark 1 to the proof of Proposition 1, the results reported in Proposition 1 apply also to F_n and M_n.

$\sum_{m=1}^{M+1}(\lambda_m - \lambda_{m-1})\sigma_{m,t}^2 - \sigma_{n,t}^2 = \sum_{m=1}^{M+1}(\lambda_m - \lambda_{m-1})\sigma_t^2 - \sigma_t^2 = 0$. Hence, the results reported in Proposition 1 for $\tau_{mn,t}$ apply also to $\tau_{n,t}$.

Table 2: Rejection frequencies and averages for M_{mn} and F_{mn}, respectively.

		$\delta = 0$		$\delta = 0.5$		$\delta = 1$	
N	Case	M_{mn}	F_{mn}	M_{mn}	F_{mn}	M_{mn}	F_{mn}
50	1	0.7	4.4	10.9	14.9	56.2	36.8
100	1	2.6	4.7	58.1	28.3	100	67
200	1	4	4.8	98.3	51.5	100	93
50	2	0.7	4.4	10.9	14.9	56.2	36.8
100	2	2.6	4.7	58.1	28.3	100	67
200	2	4	4.8	98.3	51.5	100	93
50	3	0.7	4.4	10.9	14.9	56.2	36.8
100	3	2.6	4.7	58.1	28.3	100	67
200	3	4	4.8	98.3	51.5	100	93

Notes: The results reported for M_{mn} are the 5% rejection frequencies. The results reported for F_{mn} are average test values across replications. $\sigma_{m,t}^2 = \delta_i \sigma_t^2$, where $\delta_i = 1 + \delta 1(i \geq 0.5N)$ and case 1–3 refer to the model assumed for σ_t^2.

1.2 Monte Carlo simulations

In this section, we investigate briefly the small-sample accuracy of the asymptotic results reported in Proposition 1. The data generating process (DGP) used for this purpose is very simple and is given by

$$y_{i,t} = \sigma_{m,t}\varepsilon_{i,t}, \tag{7}$$

where $\varepsilon_{i,t} \sim N(0,1)$. Hence, since in this section we are only interested in the testing of the group-wise variances, we chose a DGP in which the heteroskedasticity is the key feature; the results for the unit root testing problem are reported in the main paper (see Section 4). As in that paper, three cases regarding the time-variation of $\sigma_{m,t}^2$ are considered: (1) $\sigma_t^2 = 1$; (2) $\sigma_t^2 = 1 - 1(t \geq \lfloor T/2 \rfloor)3/4$; (3) $\sigma_t^2 = 1 - 3/4(1 + \exp(-(t - \lfloor T/2 \rfloor)))$. Cross-section variation is induced by setting $\sigma_{m,t}^2 = \delta_i \sigma_t^2$, where $\delta_i = 1 + \delta 1(i \geq 0.5N)$. Hence, in this DGP there are two groups, and the difference between them is governed by δ. If $\delta = 0$, then the variances are the same, whereas if $\delta \neq 0$, then they are not. The significance level is set to 5% and the number of replications is 3,000.

Table 3: Correct selection frequencies for the QML approach.

N	T	Case	$\delta = 0$	$\delta = 0.5$	$\delta = 1$
10	100	1	90	39.9	90.2
20	100	1	83	48.3	96.5
10	200	1	89.7	64.4	99.1
20	200	1	82.5	83.1	100
10	100	2	90.4	44.1	91.4
20	100	2	80.7	51.2	97.2
10	200	2	91.9	67	99.1
20	200	2	84	85.9	100
10	100	3	90	44.2	91.3
20	100	3	80.9	51.3	97.2
10	200	3	91.8	67.1	99.1
20	200	3	84.2	86.1	100

Notes: The results reported in the table refers to the percentage of times the QML approach was able to pick both the number of groups and their members. See Table 1 for an explanation of the rest.

The results are summarized in Table 2. While the numbers reported for M_{mn} are the 5% rejection frequencies, those reported for F_{mn} are the average test values from across the replications. The critical value used for the latter test is 1.96, suggesting that the average should be close to 5% under the null. Also, since for these tests T is relatively unimportant, in the simulations we fix $T = 50$, and focus on the effect of N. It is seen that while F_{mn} is correctly centered at 5%, M_{mn} has a tendency to under-reject the null of equal variances. We also see that this under-rejection tendency disappears, albeit quite slowly, as N increases. The best power is generally obtained by using M_{mn}, which is not totally unexpected considering that it is an order statistic. As expected, there is no difference in the results depending on the specification of σ_t^2.

2 Monte Carlo results for the QML approach

In this section we consider some results obtained by applying the QML approach discussed in Section 3.3 of the main paper to the DGP considered in Section 1.2. Thus, while M_{mn} and F_{mn} assume knowledge of the groups, the QML approach does not. In Table 3 we report the frequency by which the QML approach is able to pick both the number of groups and their members correctly. The first thing to note is that the correct selection frequencies are very high and that they tend to increase with N and T. The correct selection frequencies are lower for $\delta = 0.5$ than for $\delta = 1$, which is to be expected, because a smaller break is more difficult to discern.

3 Additional Monte Carlo results for the unit root tests

In this section we report some Monte Carlo results for the unit root tests that are discussed in Section 4 of the main paper, but not reported. In particular, two sets of results are reported. The results reported in Section 4 of the main paper are based on using the QML approach to select the groups. The first set of results reported in this section, contained in Tables 4–6, are for the case when $\sigma_{m,t}^2 = \sigma_t^2$ is known to the researcher, which seems like a natural benchmark for determining the "cost" of having to estimate the groups. Except for the fact that $\sigma_{m,t}^2 = \sigma_t^2$, the DGP used here is the same as in the main paper. The second set of results, reported in Tables 7–9, are for the case when $\sigma_{m,t}^2 = \sigma_{i,t}^2 = \delta_i \sigma_t^2$, where σ_t^2 is as in cases 1–3 above (Section 1.2) and $\delta_i \sim U(1,2)$. Thus, in this case, $N_m - N_{m-1} = 1$, and therefore the number of groups is equal to N. As mentioned in the paper, when $N_m - N_{m-1} = 1$, we have $\Delta \hat{R}_{wi,t} = \hat{w}_{i,t} \hat{\epsilon}_{i,t} = \text{sign}(\hat{\epsilon}_{i,t})$, and sign-based tests have been shown to be robust to certain types of heteroskedasticity (see Demetrescu and Hanck, 2011). Hence, for the group selection to be worthwhile, there should be some benefits to accounting for the grouping. The second set of results is therefore interesting for determining the value of accounting for the group structure. Again, except for the change in $\sigma_{m,t}^2$, the DGP is the same as in Section 4 of the main paper.

As in Section 1.2, the significance level is set to 5% and the number of replications is 3,000. The results contained in Tables 4–6 can be summarized as follows:

- The size results (Table 4) are almost identical to the ones reported in Table 1 of the main paper. Hence, in terms of size accuracy, there seem to be no cost to estimating the groups.

- As expected, the test based on assuming a single homoskedastic group (see Tables 5 and 6) has higher power than the corresponding test based on two estimated groups (see Tables 2 and 3 of the main paper). However, gain in power is only marginal. For example, the largest gain in power for the τ test in the case when $a = b = -1$ and $\kappa = 0$

is no more than 3.4%. Hence, again, there seem to be little or no cost to estimating the groups.

The following summary applies to the results contained in Tables 7–9:

- Comparing the size results reported in Table 7 based on N groups with those reported in Table 4 based on only one group, we see that accuracy does not seem to be affected by the size of the groups, which is partly expected given the robustness of sign-based tests. Hence, in terms of size, there are no gains to be made by accounting for the size of the groups.

- While size accuracy is unaffected by the size of the groups, power is not. In fact, comparing the results based on N groups (Tables 5 and 6) with those based on one group (Tables 8 and 9), we see that the power significantly as the number of groups increases. Considering again the example when $a = b = -1$ and $\kappa = 0$, the largest difference occurs when $N = 20$ and $T = 400$, in which case the power drops from 90.5% with one group to 36.7% with N groups. Hence, while in terms of size there seem to be no point in accounting for the grouping, in terms of power accounting for the grouping can lead to substantial power gains.

All-in-all, the results reported in this section suggest that the use of the QML approach to estimate the groups has very little "cost" to it, and that the potential power gain that comes from exploring the information contained in the group structure is substantial.

References

Bai, J. (1997). Estimation of a Change Point in Multiple Regression Models. *Review of Economics and Statistics* **79**, 551–563.

Bai, J. (1999). Likelihood Ratio Tests for Multiple Structural Changes. *Journal of Econometrics* **91**, 299–323.

Demetrescu, M., and C. Hanck (2011). Unit Root Testing in Heteroskedastic Panels using the Cauchy Estimator. Forthcoming in *Journal of Business & Economic Statistics*.

Table 4: Size at the 5% level when there is only one group.

N	T	Case	τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$
			$\phi_1 = 0$			
10	100	1	7.7	0.9	5.2	1.9
20	400	1	5.8	0.3	5.2	0.6
10	100	2	7.6	1.9	5.6	3.3
20	400	2	5.7	0.6	5.3	1.4
10	100	3	7.6	1.9	5.5	3.4
20	400	3	5.7	0.7	5.3	1.4
			$\phi_1 = 0.5$			
10	100	1	8.8	0.0	5.0	0.4
20	400	1	6.3	0.1	5.1	0.1
10	100	2	9.9	0.3	5.6	1.1
20	400	2	6.7	0.1	5.3	0.3
10	100	3	10.0	0.3	5.5	1.1
20	400	3	6.7	0.1	5.3	0.3

Notes: Variance cases 1–3 refer to homogeneity, a discrete break and a smooth transition break, respectively. ϕ_1 refers to the first-order autoregressive serial correlation coefficient in the errors.

Table 5: Power at the 5% level when there is only one group.

N	T	Case	$\kappa = 0$ τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$	$\kappa = 1/2$ τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$
					$a = b = -1$					
10	100	1	33.5	3.9	6.2	2.7	9.9	1.5	5.2	2.1
20	400	1	67.7	1.3	6.5	0.7	10.7	0.4	5.1	0.6
10	100	2	48.8	6.2	6.5	4.6	14.4	3.1	5.3	3.8
20	400	2	90.4	2.7	6.6	1.7	15.6	1.1	5.1	1.4
10	100	3	48.8	6.1	6.5	4.7	14.3	3.2	5.3	3.9
20	400	3	90.5	2.8	6.6	1.7	15.7	0.9	5.1	1.4
					$a = b = -2$					
10	100	1	70.5	9.9	8.6	3.8	18.7	2.7	5.6	2.5
20	400	1	99.4	5.2	9.5	1.0	22.7	0.6	5.4	0.7
10	100	2	76.9	12.2	8.3	6.0	29.7	4.3	5.8	4.3
20	400	2	99.8	7.4	9.4	2.2	39.1	1.3	5.4	1.5
10	100	3	78.0	12.0	8.3	6.0	29.6	4.7	5.8	4.3
20	400	3	99.8	7.3	9.3	2.2	39.0	1.4	5.4	1.5
					$a = -3$, $b = -1$					
10	100	1	63.2	9.7	8.3	3.7	18.5	2.6	5.6	2.5
20	400	1	96.6	5.9	9.3	1.0	22.4	0.5	5.4	0.7
10	100	2	66.4	11.8	7.8	5.8	27.3	4.6	5.8	4.2
20	400	2	96.1	7.9	8.9	2.2	35.7	1.3	5.5	1.5
10	100	3	67.4	11.7	7.8	5.9	27.2	4.4	5.8	4.3
20	400	3	96.1	8.0	8.8	2.2	35.5	1.4	5.5	1.5

Notes: κ, a and b are such that $\rho_i = \exp(c_i/N^\kappa T)$, where $c_i \sim U(a, b)$. See Table 4 for an explanation of the rest.

Table 6: Power at the 5% level when there is only one group.

N	T	Case	$\kappa = 0$: τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$	$\kappa = 1/2$: τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$
					$a = b = -5$					
10	100	1	98.3	35.8	15.9	7.4	55.9	7.6	7.7	3.4
20	400	1	100.0	35.1	21.7	2.4	75.3	1.8	6.8	0.8
10	100	2	93.7	30.2	13.6	10.1	69.2	9.5	7.6	5.3
20	400	2	100.0	27.6	18.9	4.2	94.5	3.3	6.9	1.7
10	100	3	95.3	30.0	13.8	10.2	69.4	9.6	7.5	5.3
20	400	3	100.0	27.5	19.1	4.2	94.3	3.5	6.9	1.7
					$a = b = -10$					
10	100	1	99.9	74.2	29.4	16.3	91.5	19.4	11.3	5.0
20	400	1	100.0	83.9	46.7	6.6	99.8	7.1	10.3	1.1
10	100	2	99.1	60.5	23.9	20.1	86.8	18.8	10.2	7.6
20	400	2	100.0	69.5	37.3	10.2	99.9	8.2	10.1	2.3
10	100	3	99.5	60.7	24.8	20.1	88.6	18.7	10.3	7.6
20	400	3	100.0	69.2	37.8	10.2	99.9	7.9	10.1	2.3
					$a = -15, b = -5$					
10	100	1	99.8	74.6	26.9	17.1	85.5	18.8	10.8	5.1
20	400	1	100.0	84.6	42.5	7.4	98.3	7.1	10.0	1.1
10	100	2	97.9	61.2	21.1	20.8	78.9	19.0	9.5	7.6
20	400	2	100.0	71.2	32.5	11.0	96.7	9.0	9.4	2.4
10	100	3	98.4	61.7	21.9	20.8	80.1	19.0	9.6	7.6
20	400	3	100.0	71.5	33.0	11.0	96.8	9.0	9.4	2.4

Notes: See Tables 4 and 5 for an explanation.

Table 7: Size at the 5% level when there are N groups.

N	T	Case	τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$
			$\phi_1 = 0$			
10	100	1	6.8	0.8	4.9	1.4
20	400	1	6.3	0.0	5.2	0.3
10	100	2	7.0	1.8	5.6	2.3
20	400	2	6.5	0.2	5.4	0.7
10	100	3	6.6	1.7	5.7	2.4
20	400	3	6.6	0.1	5.4	0.7
			$\phi_1 = 0.5$			
10	100	1	6.7	0.2	5.0	0.2
20	400	1	6.5	0.0	5.1	0.0
10	100	2	7.0	0.1	5.5	0.6
20	400	2	6.6	0.0	5.2	0.1
10	100	3	7.0	0.2	5.5	0.6
20	400	3	6.6	0.0	5.2	0.1

Notes: See Table 4 for an explanation.

Table 8: Power at the 5% level when there are N groups.

			$\kappa = 0$				$\kappa = 1/2$			
N	T	Case	τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$	τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$
			$a = b = -1$							
10	100	1	16.1	3.3	5.4	2.0	8.7	1.3	4.8	1.6
20	400	1	27.2	0.5	5.5	0.4	7.6	0.0	5.2	0.3
10	100	2	19.8	5.3	5.7	3.2	9.5	2.5	5.2	2.8
20	400	2	37.6	1.4	5.5	0.9	9.6	0.3	5.1	0.7
10	100	3	19.5	5.3	5.6	3.2	9.2	2.6	5.3	2.7
20	400	3	36.7	1.3	5.5	0.9	9.3	0.3	5.1	0.7
			$a = b = -2$							
10	100	1	27.1	8.4	6.1	2.6	12.5	2.3	5.0	1.8
20	400	1	52.1	2.9	6.5	0.6	11.5	0.1	5.3	0.3
10	100	2	27.9	10.1	6.0	4.2	14.5	3.6	5.3	3.0
20	400	2	57.3	5.3	6.0	1.3	17.0	0.5	5.1	0.8
10	100	3	27.2	10.2	6.0	4.2	14.2	3.9	5.3	3.0
20	400	3	57.7	5.3	6.0	1.3	17.1	0.5	5.2	0.8
			$a = -3$, $b = -1$							
10	100	1	24.4	8.5	6.1	2.7	11.3	2.3	5.2	1.8
20	400	1	48.6	3.2	6.4	0.6	11.4	0.1	5.2	0.3
10	100	2	25.1	9.7	5.8	4.2	13.6	3.8	5.3	3.0
20	400	2	50.4	4.6	5.9	1.3	16.0	0.4	5.2	0.8
10	100	3	24.8	10.2	5.9	4.1	13.1	3.6	5.4	3.0
20	400	3	50.7	4.6	6.0	1.3	15.9	0.6	5.2	0.8

Notes: See Tables 4, 5 and 7 for an explanation.

Table 9: Power at the 5% level when there are N groups.

N	T	Case	$\kappa = 0$				$\kappa = 1/2$			
			τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$	τ	$\overline{\tau}_{IV}$	τ_i	$\tau_{IV,i}$
			$a = b = -5$							
10	100	1	42.7	33.6	7.9	5.2	22.6	5.9	5.7	2.2
20	400	1	80.3	34.6	9.1	1.4	30.6	0.6	5.7	0.4
10	100	2	37.4	29.5	7.3	7.5	25.3	8.0	5.8	3.8
20	400	2	71.1	28.1	7.9	2.6	41.4	1.7	5.6	0.9
10	100	3	38.8	29.7	7.3	7.4	25.1	8.1	5.8	3.8
20	400	3	71.5	28.4	8.0	2.6	41.6	1.7	5.5	0.9
			$a = b = -10$							
10	100	1	54.5	78.9	10.8	12.5	35.7	16.7	6.7	3.6
20	400	1	88.2	87.8	12.7	4.1	56.5	4.2	6.8	0.6
10	100	2	48.4	66.3	9.9	16.1	31.5	16.5	6.2	5.4
20	400	2	82.6	74.3	11.6	7.1	59.7	6.5	6.1	1.4
10	100	3	50.0	66.5	10.1	16.2	32.2	16.4	6.3	5.4
20	400	3	83.9	74.4	11.6	7.1	60.0	6.3	6.1	1.4
			$a = -15,\ b = -5$							
10	100	1	53.3	79.0	10.6	13.2	33.1	16.1	6.8	3.5
20	400	1	87.3	87.8	12.3	4.6	52.6	4.1	6.6	0.7
10	100	2	45.9	67.2	9.5	16.7	29.2	16.6	6.4	5.5
20	400	2	79.3	76.5	10.9	7.7	52.5	6.1	6.1	1.4
10	100	3	47.2	67.1	9.5	16.8	30.0	16.5	6.4	5.5
20	400	3	79.5	76.1	10.9	7.8	52.9	6.1	6.1	1.4

Notes: See Tables 4, 5 and 7 for an explanation.