

Supplementary Materials for: Threshold Knot Selection for Large-Scale Spatial Models With Applications to the *Deepwater Horizon* Disaster

This document contains supplementary materials for Jelsema, Kwok, and Pedada (2019). The contents of this supplement are several preliminary simulation studies that were conducted to establish the consistency of the Threshold Knot Selection (TKS) method, and to determine values for the tuning parameters. Equation and figure references throughout are consistent with the main text.

Of the three simulation designs considered, the results for designs 1 and 2 are described in the main text. This document contains further information and the full results for design 3. Figure (S1) provides examples of the three patterns of locations described in the simulation design. Tables (S1), (S2), and (S3) provide the full (aggregate) results for the simulation. Each value in the table represents the median of 30 replications.

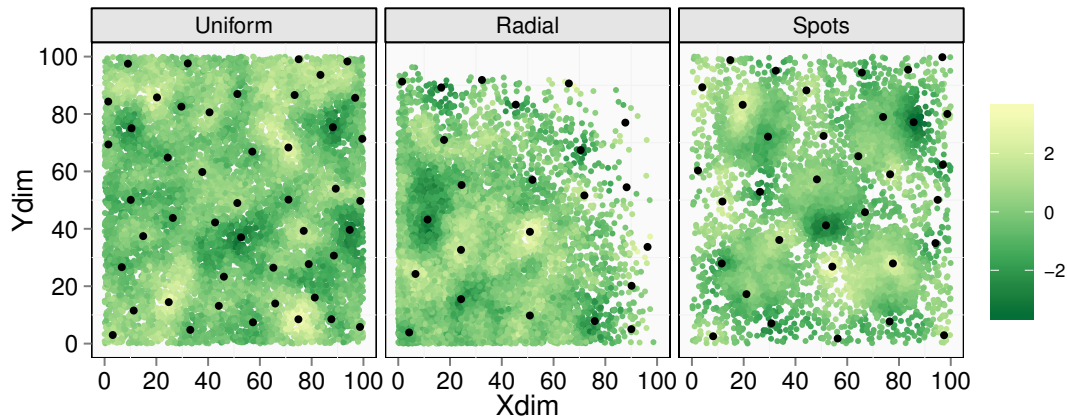


Figure S1: Examples of the Uniform, Radial, and Spots pattern of locations that were used in simulation design 3.

Pattern	Θ	Median CRPS			Median Time (s)		
		aTKS	SF	Grid	aTKS	SF	Grid
Uniform	0.001	1.14	1.48	1.39	125	189	35
Uniform	0.05	1.17	1.44	1.50	141	207	40
Uniform	5	1.22	1.42	1.41	126	168	35
Uniform	10	1.05	1.18	1.02	115	141	27
Uniform	20	0.95	1.18	1.07	110	136	26
Uniform	50	0.63	0.77	0.66	97	90	16
Uniform	100	0.58	0.63	0.55	98	83	11
Spots	0.001	1.33	1.57	1.55	174	283	91
Spots	0.05	1.26	1.69	1.52	180	293	95
Spots	5	1.49	1.60	1.80	164	242	84
Spots	10	1.24	1.20	1.16	146	204	64
Spots	20	1.09	1.25	1.04	132	181	52
Spots	50	0.65	0.67	0.57	118	134	35
Spots	100	0.58	0.59	0.57	98	95	26
Radial	0.001	1.30	1.51	1.46	157	325	81
Radial	0.05	1.46	1.64	1.65	166	294	88
Radial	5	1.40	1.56	1.48	157	267	81
Radial	10	1.13	1.15	1.09	137	219	66
Radial	20	0.93	1.02	1.01	126	208	49
Radial	50	0.60	0.68	0.60	99	126	23
Radial	100	0.62	0.63	0.54	88	101	18

Table S1: Results of simulation design 3, continuous ranked probability score and computational time between aTKS, the SF method, and the Grid method. Results shown are for $n = 10,000$.

Pattern	Θ	Median CRPS			Median Time (s)		
		aTKS	SF	Grid	aTKS	SF	Grid
Uniform	0.001	1.26	1.47	1.50	272	299	66
Uniform	0.05	1.14	1.46	1.54	288	317	70
Uniform	5	1.17	1.23	1.41	261	273	62
Uniform	10	1.12	1.22	1.30	225	225	55
Uniform	20	0.93	1.19	1.00	221	180	38
Uniform	50	0.72	0.80	0.67	196	138	26
Uniform	100	0.57	0.62	0.62	197	116	18
Spots	0.001	1.33	1.70	1.55	370	493	175
Spots	0.05	1.25	1.53	1.64	385	519	192
Spots	5	1.29	1.25	1.61	363	464	168
Spots	10	1.09	1.14	1.25	311	350	146
Spots	20	1.00	0.98	1.01	287	311	118
Spots	50	0.67	0.69	0.59	246	235	81
Spots	100	0.60	0.62	0.53	221	177	54
Radial	0.001	1.27	1.53	1.72	321	586	150
Radial	0.05	1.33	1.36	1.63	335	574	156
Radial	5	1.35	1.34	1.48	328	506	148
Radial	10	1.00	1.12	1.13	277	397	114
Radial	20	1.05	1.10	1.03	267	319	97
Radial	50	0.64	0.74	0.69	214	218	56
Radial	100	0.53	0.60	0.59	186	165	40

Table S2: Results of simulation design 3, continuous ranked probability score and computational time between aTKS, the SF method, and the Grid method. Results shown are for $n = 15,000$.

Pattern	Θ	Median CRPS			Median Time (s)		
		aTKS	SF	Grid	aTKS	SF	Grid
Uniform	0.001	1.35	1.38	1.53	453	436	105
Uniform	0.05	1.32	1.48	1.66	494	432	116
Uniform	5	1.01	1.44	1.36	442	350	96
Uniform	10	1.15	1.12	1.27	376	316	80
Uniform	20	1.07	1.21	1.05	344	244	59
Uniform	50	0.72	0.81	0.71	323	195	35
Uniform	100	0.67	0.74	0.62	314	163	26
Spots	0.001	1.18	1.80	1.55	680	826	297
Spots	0.05	1.38	1.42	1.50	669	756	304
Spots	5	1.24	1.38	1.61	626	706	296
Spots	10	1.25	1.30	1.40	533	537	231
Spots	20	1.06	1.11	1.02	502	484	203
Spots	50	0.71	0.73	0.60	409	329	120
Spots	100	0.61	0.70	0.56	347	250	87
Radial	0.001	1.41	1.44	1.63	578	885	251
Radial	0.05	1.43	1.65	1.60	570	877	250
Radial	5	1.28	1.32	1.46	533	735	237
Radial	10	1.11	1.29	1.24	464	594	166
Radial	20	0.92	1.04	1.09	416	496	144
Radial	50	0.68	0.80	0.75	370	336	96
Radial	100	0.58	0.63	0.59	310	240	64

Table S3: Results of simulation design 3, continuous ranked probability score and computational time between aTKS, the SF method, and the Grid method. Results shown are for $n = 20,000$.