**Derivation of pooled T**

*Notation*

Let be the mean response of group .

Let be the observed response for subject of group .

Denote as the common number of observations per group and as the total number of groups (meaning there are a total of observations).

Let be the variance of group .

Let be the sample mean for group .

Let be the unbiased sample variance.

Let denote the pooled variance estimate (since each arm has observations).

Let .

Let, and similarly and .

*Likelihood*

Since the joint likelihood is is

Then, for all groups and observations, since the are independent, the joint likelihood is

Then, using the prior , we see that the joint posterior distribution is

Following Gelman (pg. 66) we desire to integrate out

Denote where . Then note the following:

1. and thus.
2. and therefore for any .

Then substituting with , the integral becomes

Note that the integral is an unnormalized gamma integral, thus:

Thus we see the joint posterior of is

Note that this is a multivariate t-distribution with parameter where is the identity matrix (note that is not the covariance matrix). This is because, for and the density of such a multivariate T is proportional to

Note that and also

Thus , and we see that as previously stated.

Finally, since the are assumed independent, the posterior marginal distribution for any is given as which means that