**Supplementary information**

**Modeling the effect of blood vessel bifurcation ratio on occlusive thrombus formation**

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**Mathematical derivation for shear rate on thrombus surface**

Equation for shear rate on thrombus surface for a fully developed steady Newtonian flow reads as

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| --- | --- |
| $$\dot{γ\_{th}}=\frac{8v\_{th}}{D\_{12}}$$ | $$(A1)$$ |

$$\dot{γ\_{th}}=Shear rate on thrombus surface$$

$$v\_{th}=velocity near thrombus$$

$$D\_{12}=Diameter of thrombotic region$$

$$D\_{12}=\left(1-x\right)\*D\_{1};x-occlusion ratio, D\_{1}-Diameter of vessel before occlusion$$

$$D\_{1}=r\_{1}\*D;r\_{1}-Bifurcation ratio$$

Substituting the above in equation $(A1)$

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| --- | --- |
| $$\dot{γ\_{th}}= \frac{8 v\_{th}}{\left(1-x\right)r\_{1}D}$$$$where v\_{th}= \frac{Q\_{1}}{A\_{th}}$$ | $$(A2)$$ |

$$Q\_{1}=Blood flow rate through thrombotic vessel$$

$$A\_{th}=Open area for flow in occluded vessel $$

$$Q\_{1}=Q. \frac{R\_{2}}{R\_{2}+R\_{1}};where Q= \frac{∆P}{R\_{t}}$$

$$A\_{th}= \frac{πD\_{12}^{4}}{4}= \frac{π\left(1-x\right)^{2}D\_{1}^{2}}{4}= \frac{π\left(1-x\right)^{2}r\_{1}^{2}D^{2}}{4}$$

$$∴v\_{th}= \frac{∆P}{R\_{t}}\*\frac{R\_{2}}{R\_{2}+R\_{1}}\*\frac{4}{π\left(1-x\right)^{2}r\_{1}^{2}D^{2}}$$

Substituting these in$ (A2)$, we get

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| $$\dot{γ\_{th}}=\frac{8}{\left(1-x\right)r\_{1}D}\frac{∆P}{R\_{t}}\frac{R\_{2}}{R\_{2}+R\_{1}}\frac{4}{π\left(1-x\right)^{2}r\_{1}^{2}D^{2}}$$ |  |
| $$\dot{γ\_{th}}=\frac{32 ∆P}{πD^{3} R\_{t}}\frac{R\_{2}}{(R\_{2}+R\_{1})\* \left(1-x\right)^{3}r\_{1}^{3}}$$ | $$(A3)$$ |

This could be further simplified as the first few terms in $(A3)$ could be grouped as the inlet shear rate in the parent vessel.

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| --- | --- |
| $$Inlet shear rate, \dot{γ}= \frac{8\*v}{D}= \frac{8\* ∆P}{R\_{t}\*D\*\frac{πD^{2}}{4}}= \frac{32 ∆P }{πD^{3}R\_{t}}$$So equation $(A3)$ is simplified to,  |  |
| $$\dot{γ\_{th}}= \dot{γ}\frac{R\_{2}}{(R\_{2}+R\_{1})\* \left(1-x\right)^{3}r\_{1}^{3}}$$$$R\_{1}=resistance of thrombotic vessel$$ | $$(A4)$$ |
| $$R\_{1}=R\_{11}+R\_{12}+R\_{13}= \frac{128 μL\_{11}}{πD\_{1}^{4}}+\frac{128 μL\_{12}}{πD\_{12}^{4}}+\frac{128 μL\_{13}}{πD\_{1}^{4}};$$$$L\_{11}, L\_{13}- Lengths of non thrombotic regions of vessel; $$$$L\_{12}-Length of thrombus $$$$μ-Blood viscosity$$$$D\_{1}-Diameter of vessel, $$$$D\_{12}-Open diameter of thrombotic region of vessel$$$$D\_{12}=\left(1-x\right)\*D\_{1}$$ |  |

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| --- | --- |
| $$R\_{1}=R\_{11}+R\_{12}+R\_{13}= \frac{128 μL\_{11}}{πD\_{1}^{4}}+\frac{128 μL\_{12}}{πD\_{12}^{4}}+\frac{128 μL\_{13}}{πD\_{1}^{4}}$$ | $$(A5)$$ |

The variables used for resistances, diameters and lengths in these equations are depicted in Figures 3(a), (b) and (c).

Simplifying $\left(A5\right) $we obtain $R\_{1}$ as

|  |  |
| --- | --- |
| $$R\_{1}= \frac{128 μ}{π} λ\_{1}; where λ\_{1}= \frac{L\_{11}}{D\_{1}^{4}}+\frac{L\_{12}}{D.r\_{1}.\left(1-x\right)^{4}}+\frac{L\_{13}}{D\_{1}^{4}}$$ | $$(A6)$$ |

Similarly $R\_{2}$can be written as

|  |  |
| --- | --- |
| $$R\_{2}= \frac{128 μ}{π} λ\_{2}; where λ\_{2}= \frac{L\_{2}}{D\_{2}^{4}}; $$ |  |

For equation $(A4)$ we need $\frac{R\_{2}}{R\_{2}+R\_{1}}$

|  |  |
| --- | --- |
| $$\frac{R\_{2}}{R\_{2}+R\_{1}}= \frac{λ\_{1}}{λ\_{1}+λ\_{2}} $$ | $$(A7)$$ |

Substituting $(A7)$ in$ (A4)$, we get equation (9) (in the manuscript) to describe the shear rate on the thrombus surface at different stages of occlusion.

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| $$\dot{γ\_{th} }=\dot{γ}\frac{λ\_{1}}{(λ\_{1}+λ\_{2})\*\left(1-x\right)^{3}r\_{1}^{3}}$$ | $$(9)$$ |

This model describes the instantaneous shear rate on thrombus surface at any stage of occlusion defined by occlusion ratio$ x$.