**Supplemental material of “Transport consistent diffusion coefficient for CMFD acceleration and comparison of convergence properties” by A. Yamamoto et al.**

Detail derivation for the linearized Fourier analysis is described in this supplemental material. Though part of the derivation is described in the paper, complete derivation is provided for completeness of this document. The following derivation is based on Refs.[3], [4], [8], and [10].

## 1. Fundamental formulations

The neutron transport equation in one-dimensional slab geometry using the step characteristics approximation is written as:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

where

: direction cosine for direction *n*,

: angular flux,

: scalar flux,

: mesh width,

: macroscopic total cross section,

: macroscopic scattering cross section,

: isotropic neutron source,

: weighting factor to calculate average angular flux for direction *n*,

: quadrature weight,

: index of iteration,

: index of mesh, indicates mesh interface between and ,

: index of direction.

In the CMFD acceleration, the following difference equation is used for the neutron net current.

|  |  |
| --- | --- |
|  |  |

where,

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

and

: diffusion coefficient, : index of coarse mesh, indicates coarse mesh interface between and , and a coarse mesh include *p* fine meshes.

Using Eq.(4), the CMFD equation is written as:

|  |  |
| --- | --- |
|  |  |

where .

Once the CMFD solution is obtained, the scalar flux is normalized as:

|  |  |
| --- | --- |
|  |  |

Equations (1), (6)-(9) consist a set for the CMFD acceleration.

## 2. Fourier analysis

## 2.1 Relation of angular flux errors in transport equation

The following expansions are used:

|  |  |
| --- | --- |
|  |  |

Substitute Eq.(10) into Eq.(1) and divide both sides by , we have:

|  |  |
| --- | --- |
|  |  |

Equation (11) can be written as:

|  |  |
| --- | --- |
|  |  |

where the following relation is considered:

|  |  |
| --- | --- |
|  |  |

Equation (12) can be further simplified as:

|  |  |
| --- | --- |
|  |  |

where .

Substituting Eq. (10) into Eq. (3), we have:

|  |  |
| --- | --- |
|  |  |

Equations (14) and (15) represent errors during the transport iterations.

## 2.2 Relation of scalar flux errors in CMFD equation

Next, we will derive similar formulation for the CMFD equation. Multiplying both sides of Eq.(6) by , we have the following:

|  |  |
| --- | --- |
|  |  |

Inserting Eqs.(7) and (10) into Eq.(16), we have:

|  |  |
| --- | --- |
|  |  |

where is considered in the last formulation.

Similarly, we have:

|  |  |
| --- | --- |
|  |  |

Substituting Eqs. (10), (17), and (18) into Eq.(8) and neglecting , we have:

|  |  |
| --- | --- |
|  |  |

By rearranging Eq.(19), we have:

|  |  |
| --- | --- |
|  |  |

By integrating Eq.(1) with angle, we have:

|  |  |
| --- | --- |
|  |  |

where Eq.(3) is used. Summing Eq.(21) for *p* fine cells consisting a coarse mesh of *i*, we have:

|  |  |
| --- | --- |
|  |  |

where the following relations are used:

|  |  |
| --- | --- |
|  |  |

Applying Eq.(10) to Eq.(23), we have:

|  |  |
| --- | --- |
|  |  |

Substituting Eq.(24) to Eq.(20) , we have:

|  |  |
| --- | --- |
|  |  |

Substituting Eq.(10) into Eq.(9), we have:

|  |  |
| --- | --- |
|  |  |

Equations (25) and (26) represent the error by the CMFD equation.

## 2.3 Summary of angular flux and scalar flux errors in transport and CMFD equations

To sum up the derivation so far, Eqs.(27)-(30) are used to represent errors during transport iterations with the CMFD acceleration:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |

## 2.4 Applying Fourier expansion to errors

Now we apply the Fourier expansion as follows:

|  |  |
| --- | --- |
|  |  |

where

: growth factor of error mode,

, , , , : expansion coefficients,

: error mode frequency in order to make dimensionless,

: imaginary number,

: center position of coarse mesh *i*,

: center position of fine mesh *k*,

: position of boundary between fine meshes *k* and *k*+1.

## 2.5 Relation between and (express as a function of )

By substituting Eq.(31) into Eq.(27), we have:

|  |  |
| --- | --- |
|  |  |

where is used.

Assuming the periodic boundary condition, Eq. (32) can be written as the matrix form. In the case of *p*=4:

|  |  |
| --- | --- |
|  |  |

where . It should be noted that because of the periodic boundary condition.

Substituting Eq.(31) into Eq. (28):

|  |  |
| --- | --- |
|  |  |

Assuming the periodic boundary condition, Eq. (34) can be written as the matrix form. In the case of p=4:

|  |  |
| --- | --- |
|  |  |

Using Eqs.(33) and (35), we have:

|  |  |
| --- | --- |
| . |  |

Equation (36) can be written as:

|  |  |
| --- | --- |
|  |  |

where vectors , , and matrix are given by:

|  |  |
| --- | --- |
|  |  |

In the case of *p*=2,

|  |  |
| --- | --- |
|  |  |

In the case of *p*=1,

|  |  |
| --- | --- |
|  |  |

Equation (40) can be split into the real and imaginary parts as follows when :

|  |  |
| --- | --- |
|  |  |

The real part of Eq. (41) corresponds to in Eq.(35) of Ref.[3], i.e., the derivation so far is consistent with those in Ref.[3].

## 2.5 Relation between and (express as a function of )

Substituting Eq.(31) into Eq.(29), we have:

|  |  |
| --- | --- |
|  |  |

In order to derive the third equation in Eq.(42), the both side of the second equation is divided by , and the relation and are used.

Similarly, from Eq.(30), we have the following relation:

|  |  |
| --- | --- |
|  |  |

From Eq. (43), we have:

|  |  |
| --- | --- |
|  |  |

From Eq.(42), we have:

|  |  |
| --- | --- |
|  |  |

Substituting Eq.(44) into Eq.(45), we have:

|  |  |
| --- | --- |
|  |  |

Recall the definition of Eq.(23),

|  |  |
| --- | --- |
|  |  |

Substituting Eqs.(10) and (31) into Eq.(47), we have:

|  |  |
| --- | --- |
|  |  |

Therefore,

|  |  |
| --- | --- |
|  |  |

Substituting Eq.(49) into Eq.(46), we have:

|  |  |
| --- | --- |
|  |  |

Equation (50) can be rearranged as

|  |  |
| --- | --- |
|  |  |

In the case of *p*=4:

|  |  |
| --- | --- |
|  |  |

where

|  |  |
| --- | --- |
|  |  |

In the case of *p*=2,

|  |  |
| --- | --- |
|  |  |

where

|  |  |
| --- | --- |
|  |  |

In the case of *p*=1,

|  |  |
| --- | --- |
|  |  |

where

|  |  |
| --- | --- |
|  |  |

## 2.6 Iteration matrix for transport iteration with CMFD acceleration

Combining Eqs.(37) and (51), we finally obtain:

|  |  |
| --- | --- |
|  |  |

Thus, the iteration matrix is finally given by:

|  |  |
| --- | --- |
|  |  |

The largest real eigenvalue of the matrix dominates the convergence behavior of CMFD non-linear equation. If the absolute value of largest real eigenvalue exceeds 1.0, the CMFD acceleration diverges.

In the case of *p*=1, is simply given by:

|  |  |
| --- | --- |
|  |  |

where and are defined by Eq. (57) and the real part of Eq.(40), respectively.

In the case of *p*=4, ,, and are given by: Eqs.(53) and (38). In the case of *p*=2, ,, and are given by: Eqs.(55) and (39).