**Appendix A**

**Proof of Proposition 1**: When the predicted savings per unit is given, taking the derivative of expression (1) on , we know that the profit function of the manufacturer is concave in . For

,

let , we derive that the optimal retail price of the manufacturer is

 .

From expression (A.1), we easily know that the optimal retail price is positively related to CEA, and have Proposition 1(1). Observing expression (A.1), we have Proposition 1(2). The proof is completed. □

**Proof of Proposition 2**:Let , substituting  into expression (1), the profit function of the manufacturer is

 .

From expression (A.2), we have  and. For , we easily know that the profit function of the manufacturer is concave. Let , we have $\hat{r\_{b}}=\frac{A(bp\_{e}+τ)r\_{0}^{2}}{2bk-r\_{0}^{2}(bp\_{e}+τ)^{2}}$. Assuming that $\hat{r\_{b}}$ is an interior solution, i.e., $\hat{r\_{b}}<r\_{0}$, which is equivalent to . From the assumption in Section 3.1, we have $r\_{b}^{\*}=\hat{r\_{b}}$, i.e.,

 .

Substituting  into expression (A.2), we have the optimal profit of the manufacturer

 .

From the expressions (A.3) and (A.4), we easily know that the optimal predicted savings per unit and profit of the manufacturer are both positively related to CEA, and we derive Proposition 2(1). Observing the expressions (A.3) and (A.4), we find that the initial energy level per unit can enlarge the impacts of CEA on the optimal predicted savings per unit and profit of the manufacturer, and we derive Proposition 2(2). Proposition 2(3) is obvious. The proof is completed. □

**Proof of Proposition 3**: When shared savings is preferred by the manufacturer, taking the predicted savings per unit  and the fraction of actual savings per unit  as given, we take the derivative of expression (2), and easily prove that the profit function of the manufacturer is concave. Further we have the optimal retail price of the manufacturer

 .

Similar to the proof of Proposition 1, from expression (A.5), we derive Proposition 3. The proof is completed. □

**Proofs of Propositions 4 and 5:** Substituting  into expression (2), we have the profit function of the ESCO

 .

From (A.5), we have . To ensure that the profit function is strictly concave, it needs

,

i.e., , where. Let , we have

 $\hat{r\_{s}}(φ)=\frac{p\_{e}r\_{0}^{2}A(1-φ)}{2(αk-B\_{1})}$,

To ensure that the optimal predicted savings per unit is an interior solution, i.e., $\hat{r\_{s}}(φ)<r\_{0}$, it needs . Assuming , we have $r\_{s}^{\*}(φ)=\hat{r\_{s}}(φ)$.

Substituting expression (A.7) into (2), we have the profit function of the manufacturer

 .

From (A.8), we have . For  and , the profit function of the manufacturer is strictly concave. Let , we have the optimal fraction of predicted savings per unit

 .

Substituting expression (A.9) into expression (A.8), we have the optimal profit of the manufacturer

 .

Taking the derivative of expression (A.10), we derive Proposition 5(3). For , substituting  into the previous expression, we have .

For , we have . Substituting the previous two expressions into expression (A.7), we have the optimal predicted savings per unit of the ESCO

.

Substituting  and  into expression (A.6), we have the optimal profit of the ESCO

.

Taking the derivative of the optimal predicted savings per unit and profit of the ESCO, we have , , , and . Similar to the proof of Proposition 2, we can derive Proposition 4.

From expressions (A.9) and (A.10), we know that the optimal fraction of actual savings per unit is negatively related to CEA, and the optimal profit of the manufacturer is positively related to CEA, and we derive Proposition 5(2). Proposition 5(1) is obvious. The proof is completed. □

**Proofs of Propositions 6 and 7:** Let , with choices of self-saving and shared savings, comparing the optimal profits of the manufacturer under the two energy-saving modes, we have . When , we have . For  and ,  is equivalent to , where . We can easily know . Then, if , we have , and the manufacturer prefers self-saving; if , we have , and the manufacturer prefers shared savings.

We have  from . For  and (), we have . We can easily prov that when , i.e., , we have , i.e.,  is the increasing function in CEA. Concluding the above, we have Proposition 7.

From the proof of Proposition 7, we know that  is the increasing function in CEA defined in . Then, the minimum value of the threshold , and the maximum value of the threshold . For , when the investment cost factor ratio is sufficiently large, i.e.,, for all the CEA, we always have . The manufacturer prefers shared savings, i.e., CEA has no impacts on the optimal choice between energy-saving modes for the manufacturer. We have proof Proposition 6(1). Otherwise, when the investment cost factor ratio is sufficiently small i.e., , similar to the big investment cost factor ratio case, we can easily proof Proposition 6(2). Proposition 6(3) is also easily proved. Concluding the above, we have Proposition 6. The proof is completed. □

**Proof of Proposition 8:** From expression (7), we have the profit function of the manufacturer

.

Taking the derivative of expression (A.11), we have

.

Taking the derivative of the above expression, we have , and know that the profit function of the manufacturer is strictly concave in retail price. Let , taking the predicted savings per unit  and the fraction of excessive savings  as given, the optimal retail price of the manufacturer

 .

Observing the above expression, we easily know the first part of Proposition 8. From expression (A.12), we have . For , we have , i.e., the second part of Proposition 8 is also proved. Concluding the above, we have Proposition 8. □

**Proof of Proposition 9:** From expression (8), we have the profit function of the ESCO

 .

Substituting  into the expression (A.13), we have

.

Taking the derivative of the above expression, we have

.

We can easily know , i.e., the profit function of the ESCO is strictly concave in . Let , taking the predicted savings per unit  as given, we have the optimal fraction of excessive savings

$\hat{φ\_{g}^{u}}(r\_{g}^{u})=\frac{2(a-bc-bp\_{e}r\_{0}+τr\_{g}^{u})+bp\_{e}r\_{g}^{u}(2+λ^{2})}{bp\_{e}r\_{g}^{u}(2-λ)^{2}}$.

For $\hat{φ\_{g}^{u}}(r\_{g}^{u})\leq 1$, we assume . For  and , we can prove that our assumption is not correct, i.e., $\hat{φ\_{g}^{u}}(r\_{g}^{u})>1$. Then, . Concluding the above, we have Proposition 9. □

**Proof of Proposition 10:** Substituting  and  into expression (A.12) and expression (A.13), respectively, we have the profit functions of the manufacturer and the ESCO

 ,

 .

Taking the derivative of expression (A.14) in , we have

 and

. For , the profit function of the manufacturer is strictly concave. Let , we have the optimal unit predicted savings of the manufacturer

$\hat{r\_{g}^{u}}=\frac{Ar\_{0}^{2}(bp\_{e}λ+τ)}{2bαk-r\_{0}^{2}(bp\_{e}λ+τ)^{2}}$.

Assuming , we have , i.e., the optimal predicted savings per unit $r\_{g}^{u\*}=\hat{r\_{g}^{u}}$.

Substituting  into expression (A.14) and expression (A.15), respectively, the optimal profits of the manufacturer and the ESCO are, respectively,

, .

Taking the derivative of the optimal predicted savings per unit, we have . We can easily know . For , we can easily know . Observing the expressions of the optimal profits of the manufacturer and the ESCO, we have , , , , and . Concluding the above, we have Proposition 10. □

**Proof of Proposition 11:** For , if , we have , where , i.e., the manufacturer prefers guaranteed savings. If , we have , i.e., the manufacturer prefers self savings. The proof is completed. □

**Proof of Proposition 12:** Taking the derivative of , we have . We can easily prove , i.e.,  is an increasing function in CEA. In a similar way, we have . The proof is completed. □

**Proof of Proposition 13:** For , where, we assume , and it is equivalent to , where . We can easily prove . From the assumption that  is big enough and under the DU scenario, we have $A\gg (bp\_{e}+τ)r\_{0}=\sqrt{t}$. For , we have $αk\gg t$, and the size of is determined by , i.e.,. Let , assuming , we have . Then, if , and for , we have , i.e.,, and the manufacturer prefers guaranteed savings. If , and when , we have , i.e., the manufacturer prefers shared savings. Otherwise, if , we have , i.e., the manufacturer prefers guaranteed savings. Concluding the above, we have Proposition 13. □

**Appendix B**

In Section 5.2.1, we have assumed that the random variable  follows a uniform distribution on . In the following, we relax the assumption. Similar to Deng et al. (2017), we assume that the random shock to the project outcome  follows a general distribution which is supported on  with mean 0, and we denote this scenario by subscript uo. For , the optimal strategies and the optimal profits of the manufacturer and the ESCO under this scenario are same as those under the UU scenario. Then we begin to solve the game model under this scenario when guaranteed savings is preferred by the manufacturer. After that, we derive Proposition B.

**Proposition B.** The assumption that the random shock to the project outcome  follows a general distribution does not change any propositions shown under the UU scenario.

**Proof of Proposition B1:** Let , , we easily know ,. Fromexpression (12), we have the profit function of the manufacturer

.

We take the derivative of expression (12) in , and have

 .

For , we know that the profit function of the manufacturer is strictly concave in retail price. Let , taking the predicted savings per unit  and the fraction of excessive savings  as given, the optimal retail price of the manufacturer

 .

Further solving the game model, let , simplifying the expression (13), we have the profit function of the ESCO

 .

We take the derivative of expression (B.4) in , and have

.

For , we know that the profit function of the ESCO is strictly concave in . Let , we have

$\hat{φ\_{g}^{uo}}(r\_{g}^{uo})=\frac{(a-bc-bp\_{e}r\_{0}+τr\_{g}^{uo}+bλr\_{g}^{uo}p\_{e})J\_{1}+bp\_{e}(J\_{2}+J\_{3})}{2bp\_{e}J\_{2}}$.

Assuming $\hat{φ\_{g}^{uo}}(r\_{g}^{uo})>1$, we have . For  and , our aforementioned assumption is correct, i.e., $\hat{φ\_{g}^{uo}}(r\_{g}^{uo})>1$. Then,

.

Substituting and  into expression (B.1), we have the profit function of the manufacturer

 .

Taking the derivative of expression (B.5) in , we have

.

For , the profit function of the manufacturer is strictly concave. Let , we have the optimal predicted savings per unit of the manufacturer

 .

Substituting  into expressions (B.4) and (B.5), respectively, we have the optimal profits of the manufacturer and the ESCO as follows:

 , .

For , from expressions (B.3), (B.6) and (B.7), we know that the optimal strategies and profits of the manufacturer and the ESCO are same as those under the UU scenario. Further, we know the optimal choices between energy-saving modes for the manufacturer are also not changed. Concluding the proof, we have Proposition B. □