Supplementary Materials for "A kernel nonparametric quantile estimator for right-censored competing risks data"

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Appendix B Detailed Results of Simulation Studies

Table B. 1: Performance comparison between the kernel smoothed quantile estimator $\widetilde{Q}_1(p)$ with $h = 0.05$ and the Peng and Fine's quantile estimator $\widehat{Q}_1(p)$, based on 1000 simulated samples.

			$C\% = 15\%$				$C\% = 30\%$			
n	Method	p	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
		True	0.427	0.637	0.833	1.048	0.427	0.637	0.833	1.048
100	$\widehat{Q}_1(p)$	Bias	0.005	0.006	0.009	0.015	0.007	0.006	0.009	0.019
		SD	0.075	0.086	0.100	0.133	0.077	0.089	0.107	0.151
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	0.003	0.005	0.009	0.016	0.006	0.007	0.010	0.017
		SD	0.069	0.083	0.096	0.127	0.073	0.086	0.103	0.141
		ESE	0.068	0.081	0.100	0.140	0.070	0.086	0.109	0.162
		RMSE	**0.859**	**0.922**	**0.934**	**0.913**	**0.894**	**0.924**	**0.939**	**0.874**
200	$\widehat{Q}_1(p)$	Bias	0.005	0.003	0.005	0.007	0.006	0.004	0.006	0.010
		SD	0.051	0.058	0.069	0.095	0.053	0.061	0.073	0.100
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	0.002	0.003	0.005	0.009	0.003	0.004	0.006	0.012
		SD	0.047	0.056	0.067	0.092	0.049	0.058	0.071	0.097
		ESE	0.048	0.056	0.068	0.092	0.049	0.059	0.074	0.103
		RMSE	**0.869**	**0.917**	**0.950**	**0.943**	**0.847**	**0.914**	**0.945**	**0.955**
400	$\widehat{Q}_1(p)$	bias	0.002	-0.000	0.001	0.001	0.003	0.001	0.003	0.003
		SD	0.036	0.041	0.049	0.061	0.038	0.043	0.053	0.066
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	-0.000	-0.000	0.001	0.003	0.000	0.001	0.003	0.005
		SD	0.034	0.039	0.047	0.061	0.035	0.041	0.051	0.065
		ESE	0.034	0.039	0.048	0.063	0.035	0.041	0.051	0.069
		RMSE	**1.181**	**1.102**	**1.059**	**1.023**	**1.189**	**1.092**	**1.076**	**1.027**

Bias: the empirical bias; SD: the empirical standard error; ESE: the average bootstrap estimated standard error; RMSE: MSE($\widetilde{Q}_1(p)$)/MSE($\widehat{Q}_1(p)$).

Table B. 2: Performance comparison between the kernel smoothed quantile estimator $\widetilde{Q}_2(p)$ with $h = 0.05$ and the Peng and Fine's quantile estimator $\widehat{Q}_2(p)$, based on 1000 simulated samples.

			$C\% = 15\%$			$C\% = 30\%$		
n	Method	p	0.1	0.2	0.3	0.1	0.2	0.3
		True	0.349	0.627	0.995	0.349	0.627	0.995
100	$\widehat{Q}_2(p)$	Bias	0.002	0.015	0.045	0.010	0.018	0.040
		SD	0.084	0.128	0.272	0.085	0.137	0.269
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	0.007	0.017	0.003	0.008	0.022	-0.020
		SD	0.077	0.124	0.225	0.079	0.133	0.253
		ESE	0.082	0.138	0.262	0.085	0.146	0.285
		RMSE	**0.844**	**0.947**	**0.666**	**0.863**	**0.964**	**0.874**
200	$\widehat{Q}_2(p)$	Bias	0.007	0.012	0.039	0.006	0.012	0.039
		SD	0.059	0.095	0.212	0.060	0.098	0.211
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	0.006	0.014	0.034	0.006	0.015	0.015
		SD	0.055	0.092	0.165	0.056	0.095	0.174
		ESE	0.056	0.093	0.186	0.058	0.098	0.206
		RMSE	**0.873**	**0.941**	**0.613**	**0.883**	**0.937**	**0.661**
400	$\widehat{Q}_2(p)$	Bias	0.001	0.002	0.008	0.002	0.002	0.011
		SD	0.041	0.062	0.124	0.042	0.065	0.133
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	0.001	0.004	0.017	0.001	0.004	0.017
		SD	0.038	0.060	0.120	0.039	0.063	0.126
		ESE	0.039	0.062	0.123	0.040	0.065	0.134
		RMSE	**0.865**	**0.916**	**0.956**	**0.864**	**0.922**	**0.910**

Bias: the empirical bias; SD: the empirical standard error; ESE: the average bootstrap estimated standard error; RMSE: MSE($\widetilde{Q}_2(p)$)/MSE($\widehat{Q}_2(p)$). Note: since $P(\epsilon = 2) = 0.4$, we only report $p = 0.1, 0.2, 0.3$.

Table B. 3: Performance comparison between the kernel smoothed quantile estimator $\widetilde{Q}_1(p)$ with $h = 0.10$ and the Peng and Fine's quantile estimator $\widehat{Q}_1(p)$, based on 1000 simulated samples.

			$C\% = 15\%$				$C\% = 30\%$			
n	Method	p	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
		True	0.427	0.637	0.833	1.048	0.427	0.637	0.833	1.048
100	$\widehat{Q}_1(p)$	Bias	0.005	0.006	0.009	0.015	0.007	0.006	0.009	0.019
		SD	0.075	0.086	0.100	0.133	0.077	0.089	0.107	0.151
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	-0.004	0.004	0.010	0.019	-0.002	0.006	0.012	0.018
		SD	0.065	0.080	0.094	0.118	0.068	0.083	0.100	0.129
		ESE	0.063	0.078	0.097	0.129	0.065	0.082	0.105	0.146
		RMSE	**0.758**	**0.853**	**0.893**	**0.800**	**0.778**	**0.865**	**0.884**	**0.733**
200	$\widehat{Q}_1(p)$	Bias	0.005	0.003	0.005	0.007	0.006	0.004	0.006	0.010
		SD	0.051	0.058	0.069	0.095	0.053	0.061	0.073	0.100
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	-0.006	0.002	0.007	0.015	-0.005	0.003	0.008	0.018
		SD	0.044	0.054	0.066	0.092	0.045	0.056	0.069	0.097
		ESE	0.044	0.054	0.067	0.090	0.045	0.057	0.072	0.098
		RMSE	**0.758**	**0.851**	**0.926**	**0.969**	**0.732**	**0.849**	**0.910**	**0.958**
400	$\widehat{Q}_1(p)$	Bias	0.002	-0.000	0.001	0.001	0.003	0.001	0.003	0.003
		SD	0.036	0.041	0.049	0.061	0.038	0.043	0.053	0.066
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	-0.008	-0.001	0.003	0.009	-0.008	0.000	0.004	0.011
		SD	0.031	0.037	0.046	0.061	0.032	0.039	0.049	0.065
		ESE	0.031	0.038	0.046	0.063	0.032	0.040	0.050	0.068
		RMSE	**0.772**	**0.843**	**0.902**	**1.009**	**0.757**	**0.847**	**0.885**	**1.007**

Bias: the empirical bias; SD: the empirical standard error; ESE: the average bootstrap estimated standard error; RMSE: MSE($\widetilde{Q}_1(p)$)/MSE($\widehat{Q}_1(p)$).

Table B. 4: Performance comparison between the kernel smoothed quantile estimator $\widetilde{Q}_2(p)$ with $h = 0.10$ and the Peng and Fine's quantile estimator $\widehat{Q}_2(p)$, based on 1000 simulated samples.

			$C\% = 15\%$			$C\% = 30\%$		
n	Method	p	0.1	0.2	0.3	0.1	0.2	0.3
		True	0.349	0.627	0.995	0.349	0.627	0.995
100	$\widehat{Q}_2(p)$	Bias	0.002	0.015	0.045	0.010	0.018	0.040
		SD	0.084	0.128	0.272	0.085	0.137	0.269
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	0.007	0.025	-0.040	0.008	0.028	-0.074
		SD	0.072	0.124	0.177	0.074	0.129	0.203
		ESE	0.077	0.128	0.201	0.080	0.132	0.224
		RMSE	**0.739**	**0.960**	**0.431**	**0.764**	**0.925**	**0.635**
200	$\widehat{Q}_2(p)$	Bias	0.007	0.012	0.039	0.006	0.012	0.039
		SD	0.059	0.095	0.212	0.060	0.098	0.211
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	0.005	0.021	0.004	0.005	0.023	-0.024
		SD	0.053	0.092	0.122	0.053	0.095	0.145
		ESE	0.053	0.091	0.143	0.054	0.094	0.161
		RMSE	**0.784**	**0.987**	**0.323**	**0.789**	**0.971**	**0.469**
400	$\widehat{Q}_2(p)$	Bias	0.001	0.002	0.008	0.002	0.002	0.011
		SD	0.041	0.062	0.124	0.042	0.065	0.133
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	-0.001	0.010	0.021	-0.001	0.011	0.010
		SD	0.036	0.059	0.095	0.037	0.063	0.107
		ESE	0.036	0.062	0.097	0.037	0.065	0.109
		RMSE	**0.762**	**0.930**	**0.614**	**0.761**	**0.947**	**0.648**

Bias: the empirical bias; SD: the empirical standard error; ESE: the average bootstrap estimated standard error; RMSE: MSE($\widetilde{Q}_2(p)$)/MSE($\widehat{Q}_2(p)$). Note: since $P(\epsilon = 2) = 0.4$, we only report $p = 0.1, 0.2, 0.3$.

Table B. 5: Performance comparison between the kernel smoothed quantile estimator $\widetilde{Q}_1(p)$ with $h = 0.15$ and the Peng and Fine's quantile estimator $\widehat{Q}_1(p)$, based on 1000 simulated samples.

			$C\% = 15\%$				$C\% = 30\%$			
n	Method	p	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4
		True	0.427	0.637	0.833	1.048	0.427	0.637	0.833	1.048
100	$\widehat{Q}_1(p)$	Bias	0.005	0.006	0.009	0.015	0.007	0.006	0.009	0.019
		SD	0.075	0.086	0.100	0.133	0.077	0.089	0.107	0.151
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	-0.021	0.003	0.013	0.022	-0.019	0.004	0.015	0.011
		SD	0.059	0.077	0.093	0.109	0.061	0.080	0.098	0.116
		ESE	0.056	0.075	0.094	0.118	0.058	0.079	0.101	0.134
		RMSE	**0.696**	**0.791**	**0.880**	**0.684**	**0.695**	**0.807**	**0.855**	**0.587**
200	$\widehat{Q}_1(p)$	Bias	0.005	0.003	0.005	0.007	0.006	0.004	0.006	0.010
		SD	0.051	0.058	0.069	0.095	0.053	0.061	0.073	0.100
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	-0.023	0.001	0.009	0.023	-0.023	0.002	0.011	0.024
		SD	0.039	0.052	0.066	0.088	0.040	0.054	0.069	0.091
		ESE	0.039	0.053	0.066	0.085	0.041	0.055	0.071	0.092
		RMSE	**0.812**	**0.792**	**0.925**	**0.915**	**0.767**	**0.793**	**0.907**	**0.881**
400	$\widehat{Q}_1(p)$	Bias	0.002	-0.000	0.001	0.001	0.003	0.001	0.003	0.003
		SD	0.036	0.041	0.049	0.061	0.038	0.043	0.053	0.066
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_1(p)$	Bias	-0.026	-0.002	0.005	0.020	-0.025	-0.001	0.007	0.022
		SD	0.028	0.036	0.045	0.063	0.029	0.038	0.048	0.066
		ESE	0.028	0.037	0.046	0.062	0.029	0.039	0.049	0.066
		RMSE	**1.074**	**0.797**	**0.883**	**1.148**	**1.017**	**0.796**	**0.861**	**1.124**

Bias: the empirical bias; SD: the empirical standard error; ESE: the average bootstrap estimated standard error; RMSE: MSE($\widetilde{Q}_1(p)$)/MSE($\widehat{Q}_1(p)$).

Table B. 6: Performance comparison between the kernel smoothed quantile estimator $\widetilde{Q}_2(p)$ with $h = 0.15$ and the Peng and Fine's quantile estimator $\widehat{Q}_2(p)$, based on 1000 simulated samples.

			$C\% = 15\%$			$C\% = 30\%$		
n	Method	p	0.1	0.2	0.3	0.1	0.2	0.3
		True	0.349	0.627	0.995	0.349	0.627	0.995
100	$\widehat{Q}_2(p)$	Bias	0.002	0.015	0.045	0.010	0.018	0.040
		SD	0.084	0.128	0.272	0.085	0.137	0.269
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	0.005	0.033	-0.120	0.007	0.032	-0.156
		SD	0.068	0.118	0.150	0.071	0.122	0.179
		ESE	0.073	0.116	0.162	0.075	0.119	0.185
		RMSE	**0.658**	**0.908**	**0.485**	**0.693**	**0.842**	**0.766**
200	$\widehat{Q}_2(p)$	Bias	0.007	0.012	0.039	0.006	0.012	0.039
		SD	0.059	0.095	0.212	0.060	0.098	0.211
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	0.002	0.034	-0.080	0.003	0.033	-0.109
		SD	0.050	0.092	0.101	0.051	0.091	0.133
		ESE	0.050	0.086	0.115	0.052	0.088	0.135
		RMSE	**0.705**	**1.062**	**0.358**	**0.712**	**0.950**	**0.645**
400	$\widehat{Q}_2(p)$	Bias	0.001	0.002	0.008	0.002	0.002	0.011
		SD	0.041	0.062	0.124	0.042	0.065	0.133
		RMSE	1.000	1.000	1.000	1.000	1.000	1.000
	$\widetilde{Q}_2(p)$	Bias	-0.003	0.023	-0.060	-0.003	0.024	-0.078
		SD	0.034	0.062	0.071	0.035	0.064	0.092
		ESE	0.034	0.063	0.076	0.035	0.064	0.094
		RMSE	**0.683**	**1.114**	**0.567**	**0.686**	**1.092**	**0.813**

Bias: the empirical bias; SD: the empirical standard error; ESE: the average bootstrap estimated standard error; RMSE: MSE($\widetilde{Q}_2(p)$)/MSE($\widehat{Q}_2(p)$). Note: since $P(\epsilon = 2) = 0.4$, we only report $p = 0.1, 0.2, 0.3$.