Supplementary Material for
"A computational bootstrap procedure to compare two dependent time series"

Lei Jin, Li Cai, and Suojin Wang

Since the asymptotic null distribution is not tractable, we simulated 1000000 realizations of the limiting random variable of the test statistic given in Theorem 1. The $(1-\alpha) \times 100$-th percentile of the 1000000 realizations was then obtained as the approximation to the critical value based on the asymptotic null distribution. At the significance level of $\alpha = 0.05$, the critical values are 3.589 for $R = 5$ and 4.009 for $R = 10$, respectively.

With 1000 replications, the empirical type I error rates in percentage at significance level $\alpha = 0.05$ for the test using the critical values are given in Table S.1. The results indicate that the proposed test has good type I error rates for both Gaussian and non-Gaussian time series when T is not too small.

TABLE S.1: Empirical type I error rates in percentage at significance level $\alpha = 0.05$ with the critical values of the asymptotic null distribution are used.

T	64	128	256	1024	64	128	256	1024
	$R = 5$							
Innovations	Gaussian				t_{10}			
Model (a)	7.0	5.0	6.1	5.7	7.0	6.0	5.4	5.3
Model (b)	6.4	6.6	5.8	5.0	5.2	4.5	5.0	5.4
Model (c)	5.3	7.0	6.9	5.8	7.2	4.7	6.0	6.3
Model (d)	6.1	6.8	5.5	5.9	6.3	5.3	4.6	5.0
Model (e)	6.8	5.0	5.9	6.3	6.6	6.8	5.7	5.3
Model (f)	6.9	5.8	5.8	4.8	6.4	6.6	6.0	4.5
Innovations	t_5				χ^2_2			
Model (a)	6.2	4.3	5.6	5.1	7.0	3.6	5.2	4.0
Model (b)	4.9	3.8	4.0	3.4	5.7	4.4	5.2	4.7
Model (c)	6.6	6.2	4.0	4.6	4.8	4.9	4.9	4.6
Model (d)	6.0	3.8	5.6	5.1	8.0	4.3	3.6	5.5
Model (e)	6.2	4.4	4.9	6.2	5.0	3.6	5.9	5.0
Model (f)	5.6	6.1	4.9	6.9	6.7	4.3	4.3	4.6
	$R = 10$							
Innovations	Gaussian				t_{10}			
Model (a)	9.0	5.0	6.6	5.6	9.2	5.6	5.0	5.4
Model (b)	8.5	6.3	5.6	5.2	9.0	5.3	5.5	6.1
Model (c)	8.4	7.6	7.2	5.7	7.5	4.8	4.8	5.9
Model (d)	8.3	4.9	5.7	6.6	8.1	6.2	5.8	4.7
Model (e)	7.0	7.1	6.2	5.7	8.8	5.7	6.9	6.4
Model (f)	7.6	5.2	5.2	5.1	9.0	5.7	5.9	5.6
Innovations	t_5				χ^2_2			
Model (a)	10.3	5.7	4.3	4.5	9.8	5.6	4.7	4.9
Model (b)	8.6	5.0	4.1	5.2	10.1	6.4	5.2	5.3
Model (c)	8.1	5.1	4.3	5.9	10.6	6.0	5.4	5.1
Model (d)	7.7	7.4	5.9	4.9	8.9	5.6	5.5	4.6
Model (e)	7.8	5.8	5.1	5.3	9.1	5.1	4.7	4.6
Model (f)	10.7	5.9	5.1	5.3	9.8	5.0	5.0	5.8