**Supplemental Online Material**

**Table S1.** Improved EON quadrature of 384 (points/4π). The 48 discretized directions over an octant of the sphere are presented in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | weight † |  |  |  |
| 1 | 2.7133901557034558e-4 | 2.2250255888577393e-2 | 2.2250255888577393e-2 | 9.9950480350310756e-1 |
| 2 | 2.7133901557034558e-4 | 2.2250255888577393e-2 | 9.9950480350310756e-1 | 2.2250255888577393e-2 |
| 3 | 2.7133901557034558e-4 | 9.9950480350310756e-1 | 2.2250255888577393e-2 | 2.2250255888577393e-2 |
| 4 | 2.4747658377021711e-3 | 1.5063842157143472e-1 | 1.5063842157143472e-1 | 9.7704459053460473e-1 |
| 5 | 2.4747658377021711e-3 | 1.5063842157143472e-1 | 9.7704459053460473e-1 | 1.5063842157143472e-1 |
| 6 | 2.4747658377021711e-3 | 9.7704459053460473e-1 | 1.5063842157143472e-1 | 1.5063842157143472e-1 |
| 7 | 3.1302276351586317e-3 | 5.2345326834952976e-1 | 5.2345326834952976e-1 | 6.7230450817199669e-1 |
| 8 | 3.1302276351586317e-3 | 5.2345326834952976e-1 | 6.7230450817199669e-1 | 5.2345326834952976e-1 |
| 9 | 3.1302276351586317e-3 | 6.7230450817199669e-1 | 5.2345326834952976e-1 | 5.2345326834952976e-1 |
| 10 | 4.3735792205142644e-3 | 3.6464927043714109e-1 | 3.6464927043714109e-1 | 8.5677407707009990e-1 |
| 11 | 4.3735792205142644e-3 | 3.6464927043714109e-1 | 8.5677407707009990e-1 | 3.6464927043714109e-1 |
| 12 | 4.3735792205142644e-3 | 8.5677407707009990e-1 | 3.6464927043714109e-1 | 3.6464927043714109e-1 |
| 13 | 8.6581574778795299e-4 | 9.9164907955830643e-1 | 1.2591820170388069e-1 | 2.7869508263054451e-2 |
| 14 | 8.6581574778795299e-4 | 9.9164907955830643e-1 | 2.7869508263054451e-2 | 1.2591820170388069e-1 |
| 15 | 8.6581574778795299e-4 | 1.2591820170388069e-1 | 2.7869508263054451e-2 | 9.9164907955830643e-1 |
| 16 | 8.6581574778795299e-4 | 1.2591820170388069e-1 | 9.9164907955830643e-1 | 2.7869508263054451e-2 |
| 17 | 8.6581574778795299e-4 | 2.7869508263054451e-2 | 9.9164907955830643e-1 | 1.2591820170388069e-1 |
| 18 | 8.6581574778795299e-4 | 2.7869508263054451e-2 | 1.2591820170388069e-1 | 9.9164907955830643e-1 |
| 19 | 1.5472421060451036e-3 | 2.7641148244519852e-2 | 3.2807233804759719e-1 | 9.4424811778033616e-1 |
| 20 | 1.5472421060451036e-3 | 2.7641148244519852e-2 | 9.4424811778033616e-1 | 3.2807233804759719e-1 |
| 21 | 1.5472421060451036e-3 | 3.2807233804759719e-1 | 9.4424811778033616e-1 | 2.7641148244519852e-2 |
| 22 | 1.5472421060451036e-3 | 3.2807233804759719e-1 | 2.7641148244519852e-2 | 9.4424811778033616e-1 |
| 23 | 1.5472421060451036e-3 | 9.4424811778033616e-1 | 2.7641148244519852e-2 | 3.2807233804759719e-1 |
| 24 | 1.5472421060451036e-3 | 9.4424811778033616e-1 | 3.2807233804759719e-1 | 2.7641148244519852e-2 |
| 25 | 2.3832288775456662e-3 | 8.0884418745966909e-1 | 3.8677548893897996e-2 | 5.8674962941979590e-1 |
| 26 | 2.3832288775456662e-3 | 8.0884418745966909e-1 | 5.8674962941979590e-1 | 3.8677548893897996e-2 |
| 27 | 2.3832288775456662e-3 | 3.8677548893897996e-2 | 5.8674962941979590e-1 | 8.0884418745966909e-1 |
| 28 | 2.3832288775456662e-3 | 3.8677548893897996e-2 | 8.0884418745966909e-1 | 5.8674962941979590e-1 |
| 29 | 2.3832288775456662e-3 | 5.8674962941979590e-1 | 8.0884418745966909e-1 | 3.8677548893897996e-2 |
| 30 | 2.3832288775456662e-3 | 5.8674962941979590e-1 | 3.8677548893897996e-2 | 8.0884418745966909e-1 |
| 31 | 3.5205752634728772e-3 | 7.3867548144924025e-1 | 3.5409633503112433e-1 | 5.7356282883678827e-1 |
| 32 | 3.5205752634728772e-3 | 7.3867548144924025e-1 | 5.7356282883678827e-1 | 3.5409633503112433e-1 |
| 33 | 3.5205752634728772e-3 | 3.5409633503112433e-1 | 5.7356282883678827e-1 | 7.3867548144924025e-1 |
| 34 | 3.5205752634728772e-3 | 3.5409633503112433e-1 | 7.3867548144924025e-1 | 5.7356282883678827e-1 |
| 35 | 3.5205752634728772e-3 | 5.7356282883678827e-1 | 7.3867548144924025e-1 | 3.5409633503112433e-1 |
| 36 | 3.5205752634728772e-3 | 5.7356282883678827e-1 | 3.5409633503112433e-1 | 7.3867548144924025e-1 |
| 37 | 3.6575247841537903e-3 | 1.5970861147690090e-1 | 3.6166764655266084e-1 | 9.1852581501946907e-1 |
| 38 | 3.6575247841537903e-3 | 1.5970861147690090e-1 | 9.1852581501946907e-1 | 3.6166764655266084e-1 |
| 39 | 3.6575247841537903e-3 | 3.6166764655266084e-1 | 9.1852581501946907e-1 | 1.5970861147690090e-1 |
| 40 | 3.6575247841537903e-3 | 3.6166764655266084e-1 | 1.5970861147690090e-1 | 9.1852581501946907e-1 |
| 41 | 3.6575247841537903e-3 | 9.1852581501946907e-1 | 1.5970861147690090e-1 | 3.6166764655266084e-1 |
| 42 | 3.6575247841537903e-3 | 9.1852581501946907e-1 | 3.6166764655266084e-1 | 1.5970861147690090e-1 |
| 43 | 3.7339906998552367e-3 | 5.9296347373652011e-1 | 7.8514467677154377e-1 | 1.7872368436115927e-1 |
| 44 | 3.7339906998552367e-3 | 5.9296347373652011e-1 | 1.7872368436115927e-1 | 7.8514467677154377e-1 |
| 45 | 3.7339906998552367e-3 | 7.8514467677154377e-1 | 1.7872368436115927e-1 | 5.9296347373652011e-1 |
| 46 | 3.7339906998552367e-3 | 7.8514467677154377e-1 | 5.9296347373652011e-1 | 1.7872368436115927e-1 |
| 47 | 3.7339906998552367e-3 | 1.7872368436115927e-1 | 5.9296347373652011e-1 | 7.8514467677154377e-1 |
| 48 | 3.7339906998552367e-3 | 1.7872368436115927e-1 | 7.8514467677154377e-1 | 5.9296347373652011e-1 |

† Total sum of weights is normalized as .



**Figure S1.** Improved EON quadrature of 384 (points/4π).

**Table S2.** Analytical values of .

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | 　 |  |  | 　 |  |  | 　 |  |  |
| 0 | 0 | 0 | [π/2](https://ja.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E0+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 0 | 0 | [π/14](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 8 | 0 | 0 | [π/18](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E8+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 10 | 0 | 0 | [π/22](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E10+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 1 | 0 | 0 | [π/4](https://ja.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E1+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 1 | 0 | [8/105](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 7 | 1 | 0 | [16/315](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E7+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 9 | 1 | 0 | [128/3465](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E9+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 2 | 0 | 0 | [π/6](https://ja.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E2+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 2 | 0 | [π/70](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 2 | 0 | [π/126](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 8 | 2 | 0 | [π/198](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E8+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 1 | 1 | 0 | [1/3](https://ja.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E1+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 3 | 3 | 0 | [4/105](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 3 | 0 | [16/945](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 8 | 1 | 1 | [1/99](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E8+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 3 | 0 | 0 | [π/8](https://ja.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 1 | 1 | [1/35](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 1 | 1 | [1/63](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 7 | 3 | 0 | [32/3465](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E7+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 2 | 1 | 0 | [π/16](https://ja.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E2+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 3 | 2 | 1 | [2/105](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 4 | 0 | [π/210](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E4+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 4 | 0 | [π/462](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E4+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 1 | 1 | 1 | [1/8](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E1+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 2 | 2 | 2 | [π/210](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E2+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 2 | 1 | [8/945](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 5 | 0 | [64/10395](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E5+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 4 | 0 | 0 | [π/10](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 7 | 0 | 0 | [π/16](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E7+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 3 | 1 | [2/315](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 7 | 2 | 1 | [16/3465](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E7+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 3 | 1 | 0 | [2/15](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 1 | 0 | [5π/256](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 2 | 2 | [π/630](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 3 | 1 | [2/693](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 2 | 2 | 0 | [π/30](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E2+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 2 | 0 | [π/96](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 3 | 3 | 2 | [4/945](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 4 | 1 | [8/3465](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E4+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 2 | 1 | 1 | [1/15](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E2+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 3 | 0 | [π/128](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 9 | 0 | 0 | [π/20](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E9+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 2 | 2 | [π/1386](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 5 | 0 | 0 | [π/12](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E0+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 1 | 1 | [1/48](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 8 | 1 | 0 | [7π/512](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E8+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 5 | 3 | 2 | [16/10395](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 4 | 1 | 0 | [π/32](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 2 | 1 | [π/256](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 7 | 2 | 0 | [π/160](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E7+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 4 | 2 | [π/2310](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E4+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 3 | 2 | 0 | [π/48](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 3 | 3 | 1 | [1/96](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 7 | 1 | 1 | [1/80](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E7+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 4 | 3 | 3 | [4/3465](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E3+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) |
| 3 | 1 | 1 | [1/24](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E1+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 3 | 2 | 2 | [π/384](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 6 | 3 | 0 | [π/256](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |
| 2 | 2 | 1 | [π/96](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E2+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 | 　 | 5 | 4 | 0 | [π/320](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E4+*+(cos%5Btheta%5D)%5E0+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |
| 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 6 | 2 | 1 | [π/512](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E6+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |
| 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 5 | 3 | 1 | [1/240](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |
| 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 4 | 4 | 1 | [3π/2560](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E4+*+(cos%5Btheta%5D)%5E1+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |
| 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 5 | 2 | 2 | [π/960](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E5+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E2+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |
| 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 4 | 3 | 2 | [π/1280](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E4+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E2+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |
| 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 　 | 3 | 3 | 3 | [1/480](https://www.wolframalpha.com/input/?i=Integral+(sin%5Btheta%5D*cos%5Bphi%5D)%5E3+*+(sin%5Btheta%5D*sin%5Bphi%5D)%5E3+*+(cos%5Btheta%5D)%5E3+*sin%5B%CE%B8%5D,+%7Bphi,0,pi%2F2%7D,+%7Btheta,0,pi%2F2%7D) | 　 | 　 | 　 | 　 | 　 |



**Figure S2.** Comparison of relative sensitivity coefficients of among PARTISN, MCNP6.2, and TSUNAMI-3D (4440 HEU plates).

**Figure S2.** Comparison of relative sensitivity coefficients of among PARTISN, MCNP6.2, and TSUNAMI-3D (4440 HEU plates) (continued).





**Figure S2.** Comparison of relative sensitivity coefficients of among PARTISN, MCNP6.2, and TSUNAMI-3D (4440 HEU plates) (continued).

**Calculation conditions:**

MCNP6.2: as-built geometry, ENDF-B/VII.1, IFP method, nsrck=50000, ikz=400, kct=10400,
 blocksize=5;

TSUNAMI-3D: as-built geometry, ENDF-B/VII.1, CE CLUTH method, npg=50000,
gen=10400, nsk=400, cfp=5.

**Table S3.** Numerical results of MCNP6.2.

|  |  |  |  |
| --- | --- | --- | --- |
| EU plates |  (-) |  (pcm) |  (μs) |
| 4440 | 0.99087 | ± | 0.00004† | 795 | ± | 3 | 35.310 | ± | 0.020 |
| 4320 | 0.97861 | ± | 0.00004 | 802 | ± | 3 | 36.097 | ± | 0.021 |
| 4200 | 0.96438 | ± | 0.00004 | 803 | ± | 3 | 37.149 | ± | 0.021 |
| 4080 | 0.95624 | ± | 0.00004 | 802 | ± | 3 | 39.253 | ± | 0.022 |
| 3960 | 0.95182 | ± | 0.00004 | 807 | ± | 3 | 40.388 | ± | 0.022 |
| 3840 | 0.92801 | ± | 0.00004 | 806 | ± | 3 | 42.486 | ± | 0.024 |

†1σ statistical error.

**Calculation conditions:**

MCNP6.2: as-built geometry, ENDF-B/VII.1, IFP method, nsrck=50000, ikz=400, kct=10400, blocksize=5.