Estimating the Maximum Likelihood Root Mean Square Error of Approximation (RMSEA) with Non-normal Data: A Monte-Carlo Study

Supplementary materials

In the body of the paper, continuous non-normal data were generated following Vale and Maurelli (1983). In these supplementary materials, we provide results using an alternative approach to generate non-normal data (Muthén & Kaplan, 1985). In this case, multivariate normal data are generated first. These data are then discretized into five categories according to a set of thresholds to achieve the desired marginal skewness and kurtosis. The resulting data is treated as continuous. The conditions and replications (5000) are the same in both cases, except that for the Muthén and Kaplan approach we used: skewness = 0, kurtosis = 0 (i.e., normal); skewness = 0, kurtosis = 2; and skewness = -2, kurtosis = 3.18. We also performed an additional set of simulations using the Vale and Maurelli (1983) approach using these skewness and kurtosis values (3 was used instead of 3.18, though). The results obtained in the latter case are available from the authors upon request.

Returning to the results obtained using the Muthén and Kaplan approach, more specifically, the five category data were generated as follows: First, we generated multivariate normal data **z**\* with mean zero and the two factor correlation structure described in the body of the manuscript. The normal variables are categorized via the threshold relationship *Yi* = *ki* if , *ki* = 0, ..., 4, where . The thresholds were selected so that the items had the desired marginal skewness and kurtosis.

The population covariance matrices were computed using:

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Finally, the population RMSEAML was obtained by minimizing *FML* in equation 4 of the body of the manuscript. The reader can verify in the tables below that concurrent results are obtained regardless of the mechanism used to generate the data, MLMV emerges as the method of choice. However, using the Muthén and Kaplan approach all methods yield somewhat more accurate results.

A drawback of the Muthén-Kaplan approach to generate non-normal data is that the population RMSEA need not be zero in the presence of non-normality even when the model is 'correctly specified'. This can be seen in the tables below. When ρ = 1 but the data are non-normal, the population RMSEA does not equal zero. Put differently, the minimum of *FML* in equation (3) of the article does not equal zero. The reason is that the model is not correctly specified even when ρ = 1 (McDonald, 1999). We are fitting a linear model (a factor analysis model) to a non-linear model (the ordinal factor analysis model) although the test statistics have very little power to distinguish them (Maydeu-Olivares, Cai, & Hernández, 2011). When the data resulting from the discretization has low skewness and kurtosis, the test statistics have essentially no power to distinguish a factor analysis model from an ordinal factor analysis model (RMSEA = 0). In contrast, when the discretized data has higher levels of skewness and kurtois, the test statistics have some power (RMSEA = .002 or .003, but not 0).

References

Maydeu-Olivares, A., Cai, L., & Hernández, A. (2011). Comparing the Fit of Item Response Theory and Factor Analysis Models. *Structural Equation Modeling*, *18*(3), 333–356. https://doi.org/10.1080/10705511.2011.581993

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Muthén, B., & Kaplan, D. (1985). A comparison of some methodologies for the factor analysis of non-normal Likert variables. *British Journal of Mathematical and Statistical Psychology*, *38*(2), 171–189. https://doi.org/10.1111/j.2044-8317.1985.tb00832.x

Vale, C. D., & Maurelli, V. A. (1983). Simulating multivariate nonnormal distributions. *Psychometrika*, *48*(3), 465–471. https://doi.org/10.1007/BF02293687

*Table 1. Population RMSEA and Average of RMSEA estimates across replications*

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | *p* = 16 | | | |  | *p* = 32 | | | |
| ρ | *N* | Skew.. | Kurt.. | *Pop.* | ML | MLM & MLMV | MLR |  | *Pop.* | ML | MLM & MLMV | MLR |
| .4 | 200 | -2.00 | 3.18 | **.106** | .113 | .105 | .108 |  | **.083** | .096 | .088 | .090 |
|  | 500 | -2.00 | 3.18 | **.106** | .108 | .104 | .106 |  | **.083** | .087 | .083 | .084 |
|  | 1000 | -2.00 | 3.18 | **.106** | .107 | .105 | .106 |  | **.083** | .085 | .082 | .083 |
| .8 | 200 | -2.00 | 3.18 | **.051** | .075 | .056 | .059 |  | **.043** | .073 | .056 | .058 |
|  | 500 | -2.00 | 3.18 | **.051** | .061 | .051 | .052 |  | **.043** | .055 | .045 | .045 |
|  | 1000 | -2.00 | 3.18 | **.051** | .056 | .051 | .051 |  | **.043** | .049 | .043 | .043 |
| 1 | 200 | -2.00 | 3.18 | **.003** | .059 | .027 | .030 |  | **.003** | .062 | .039 | .041 |
|  | 500 | -2.00 | 3.18 | **.003** | .036 | .012 | .013 |  | **.003** | .037 | .015 | .015 |
|  | 1000 | -2.00 | 3.18 | **.003** | .025 | .007 | .008 |  | **.003** | .026 | .007 | .008 |
| .4 | 200 | 0 | 2.00 | **.103** | .103 | .102 | .104 |  | **.081** | .084 | .083 | .084 |
|  | 500 | 0 | 2.00 | **.103** | .103 | .102 | .103 |  | **.081** | .081 | .081 | .081 |
|  | 1000 | 0 | 2.00 | **.103** | .103 | .103 | .103 |  | **.081** | .081 | .080 | .081 |
| .8 | 200 | 0 | 2.00 | **.047** | .052 | .049 | .050 |  | **.040** | .049 | .046 | .047 |
|  | 500 | 0 | 2.00 | **.047** | .049 | .047 | .047 |  | **.040** | .043 | .041 | .041 |
|  | 1000 | 0 | 2.00 | **.047** | .048 | .047 | .047 |  | **.040** | .041 | .040 | .040 |
| 1 | 200 | 0 | 2.00 | **.002** | .026 | .018 | .019 |  | **.002** | .031 | .024 | .025 |
|  | 500 | 0 | 2.00 | **.002** | .015 | .009 | .008 |  | **.002** | .017 | .009 | .009 |
|  | 1000 | 0 | 2.00 | **.002** | .010 | .006 | .006 |  | **.002** | .011 | .005 | .005 |
| .4 | 200 | 0 | 0 | **.121** | .120 | .120 | .121 |  | **.093** | .094 | .094 | .095 |
|  | 500 | 0 | 0 | **.121** | .120 | .120 | .121 |  | **.093** | .093 | .093 | .093 |
|  | 1000 | 0 | 0 | **.121** | .121 | .121 | .121 |  | **.093** | .093 | .093 | .093 |
| .8 | 200 | 0 | 0 | **.057** | .057 | .058 | .058 |  | **.047** | .050 | .051 | .051 |
|  | 500 | 0 | 0 | **.057** | .056 | .056 | .056 |  | **.047** | .047 | .047 | .047 |
|  | 1000 | 0 | 0 | **.057** | .056 | .056 | .056 |  | **.047** | .047 | .047 | .047 |
| 1 | 200 | 0 | 0 | **0** | .015 | .015 | .015 |  | **0** | .018 | .019 | .019 |
|  | 500 | 0 | 0 | **0** | .008 | .008 | .008 |  | **0** | .007 | .007 | .007 |
|  | 1000 | 0 | 0 | **0** | .005 | .005 | .005 |  | **0** | .004 | .004 | .004 |

*Notes*: pop = population value; ML = likelihood ratio (LR) test statistic; MLM = Satorra-Bentler mean adjusted LR; MLMV = Asparouhov and Muthén mean and variance adjusted LR, MLR = Asparouhov and Muthén mean adjusted LR.

*Table 2. Coverage Rates for 90% Confidence Intervals around the Population RMSEA*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | | *p* = 16 | | | |  |  | | *p* = 32 | | | |
| ρ | *N* | Skew.. | Kurt.. | *Pop.* | ML | | MLM | MLMV | MLR |  | *Pop.* | ML | | MLM | MLMV | MLR |
| .4 | 200 | -2.00 | 3.18 | **.106** | .625 | | .778 | .890 | .730 |  | **.083** | .156 | | .656 | .929 | .487 |
|  | 500 | -2.00 | 3.18 | **.106** | .645 | | .803 | .854 | .700 |  | **.083** | .382 | | .644 | .812 | .605 |
|  | 1000 | -2.00 | 3.18 | **.106** | .648 | | .728 | .764 | .688 |  | **.083** | .470 | | .630 | .735 | .590 |
| .8 | 200 | -2.00 | 3.18 | **.051** | .181 | | .806 | .932 | .757 |  | **.043** | 0 | | .300 | .836 | .215 |
|  | 500 | -2.00 | 3.18 | **.051** | .388 | | .826 | .893 | .816 |  | **.043** | .013 | | .724 | .910 | .703 |
|  | 1000 | -2.00 | 3.18 | **.051** | .522 | | .820 | .855 | .814 |  | **.043** | .102 | | .742 | .862 | .736 |
| 1 | 200 | -2.00 | 3.18 | **.003** | .016 | | .778 | .944 | .708 |  | **.003** | 0 | | .111 | .808 | .056 |
|  | 500 | -2.00 | 3.18 | **.003** | .023 | | .902 | .954 | .885 |  | **.003** | 0 | | .670 | .959 | .621 |
|  | 1000 | -2.00 | 3.18 | **.003** | .033 | | .928 | .954 | .924 |  | **.003** | 0 | | .858 | .968 | .842 |
| .4 | 200 | 0 | 2.00 | **.103** | .778 | | .787 | .874 | .743 |  | **.081** | .640 | | .680 | .897 | .620 |
|  | 500 | 0 | 2.00 | **.103** | .752 | | .768 | .806 | .730 |  | **.081** | .645 | | .659 | .781 | .637 |
|  | 1000 | 0 | 2.00 | **.103** | .739 | | .759 | .780 | .718 |  | **.081** | .625 | | .636 | .707 | .620 |
| .8 | 200 | 0 | 2.00 | **.047** | .789 | | .845 | .927 | .831 |  | **.040** | .426 | | .630 | .923 | .572 |
|  | 500 | 0 | 2.00 | **.047** | .812 | | .846 | .885 | .841 |  | **.040** | .664 | | .773 | .908 | .771 |
|  | 1000 | 0 | 2.00 | **.047** | .815 | | .843 | .861 | .840 |  | **.040** | .717 | | .768 | .847 | .767 |
| 1 | 200 | 0 | 2.00 | **.002** | .688 | | .872 | .956 | .846 |  | **.002** | .138 | | .478 | .936 | .411 |
|  | 500 | 0 | 2.00 | **.002** | .747 | | .921 | .954 | .915 |  | **.002** | .313 | | .833 | .972 | .818 |
|  | 1000 | 0 | 2.00 | **.002** | .772 | | .939 | .956 | .937 |  | **.002** | .383 | | .905 | .968 | .898 |
| .4 | 200 | 0 | 0 | **.121** | .784 | | .783 | .845 | .747 |  | **.093** | .675 | | .751 | .914 | .634 |
|  | 500 | 0 | 0 | **.121** | .761 | | .806 | .837 | .725 |  | **.093** | .647 | | .677 | .782 | .625 |
|  | 1000 | 0 | 0 | **.121** | .755 | | .754 | .770 | .717 |  | **.093** | .636 | | .637 | .694 | .620 |
| .8 | 200 | 0 | 0 | **.057** | .845 | | .841 | .908 | .838 |  | **.047** | .734 | | .715 | .929 | .712 |
|  | 500 | 0 | 0 | **.057** | .840 | | .841 | .870 | .840 |  | **.047** | .781 | | .781 | .885 | .779 |
|  | 1000 | 0 | 0 | **.057** | .818 | | .819 | .832 | .818 |  | **.047** | .756 | | .756 | .814 | .756 |
| 1 | 200 | 0 | 0 | **0** | .901 | | .894 | .950 | .893 |  | **0** | .681 | | .643 | .961 | .636 |
|  | 500 | 0 | 0 | **0** | .932 | | .932 | .951 | .931 |  | **0** | .871 | | .876 | .967 | .876 |
|  | 1000 | 0 | 0 | **0** | .940 | | .942 | .951 | .941 |  | **0** | .907 | | .915 | .959 | .914 |

*Notes:* skew = skewness, kurt = excess kurtosis, pop = population value; ML = likelihood ratio (LR) test statistic; MLM = Satorra-Bentler mean adjusted LR; MLMV = Asparouhov and Muthén mean and variance adjusted LR, MLR = Asparouhov and Muthén mean adjusted LR. Shaded results indicate acceptable coverage (between .85 and .95).

*Table 3. Test of close fit results. Empirical rejection rates at a 5% significance level of a test that the RMSEA is less than or equal to its population value*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | | *p* = 16 | | | |  |  | | *p* = 32 | | | |
| ρ | *N* | Skew.. | Kurt.. | *Pop.* | ML | | MLM | MLMV | MLR |  | *Pop.* | ML | | MLM | MLMV | MLR |
| .4 | 200 | -2.00 | 3.18 | **.106** | .344 | | .108 | .059 | .180 |  | **.083** | .852 | | .336 | .076 | .496 |
|  | 500 | -2.00 | 3.18 | **.106** | .261 | | .057 | .041 | .156 |  | **.083** | .591 | | .182 | .102 | .248 |
|  | 1000 | -2.00 | 3.18 | **.106** | .224 | | .103 | .088 | .154 |  | **.083** | .444 | | .149 | .103 | .204 |
| .8 | 200 | -2.00 | 3.18 | **.051** | .824 | | .176 | .069 | .229 |  | **.043** | 1.00 | | .705 | .169 | .790 |
|  | 500 | -2.00 | 3.18 | **.051** | .613 | | .106 | .069 | .120 |  | **.043** | .987 | | .236 | .085 | .266 |
|  | 1000 | -2.00 | 3.18 | **.051** | .464 | | .091 | .074 | .097 |  | **.043** | .899 | | .153 | .087 | .164 |
| 1 | 200 | -2.00 | 3.18 | **.003** | .984 | | .224 | .057 | .294 |  | **.003** | 1.00 | | .890 | .196 | .945 |
|  | 500 | -2.00 | 3.18 | **.003** | .978 | | .099 | .047 | .116 |  | **.003** | 1.00 | | .331 | .041 | .380 |
|  | 1000 | -2.00 | 3.18 | **.003** | .968 | | .072 | .046 | .076 |  | **.003** | 1.00 | | .143 | .032 | .158 |
| .4 | 200 | 0 | 2.00 | **.103** | .128 | | .104 | .063 | .168 |  | **.081** | .303 | | .241 | .086 | .316 |
|  | 500 | 0 | 2.00 | **.103** | .128 | | .101 | .084 | .151 |  | **.081** | .205 | | .152 | .096 | .194 |
|  | 1000 | 0 | 2.00 | **.103** | .138 | | .113 | .101 | .160 |  | **.081** | .179 | | .133 | .106 | .166 |
| .8 | 200 | 0 | 2.00 | **.047** | .196 | | .121 | .063 | .139 |  | **.040** | .581 | | .370 | .081 | .428 |
|  | 500 | 0 | 2.00 | **.047** | .149 | | .090 | .070 | .097 |  | **.040** | .319 | | .162 | .074 | .170 |
|  | 1000 | 0 | 2.00 | **.047** | .134 | | .085 | .078 | .089 |  | **.040** | .236 | | .118 | .080 | .122 |
| 1 | 200 | 0 | 2.00 | **.002** | .315 | | .130 | .045 | .156 |  | **.002** | .865 | | .525 | .065 | .591 |
|  | 500 | 0 | 2.00 | **.002** | .254 | | .079 | .046 | .085 |  | **.002** | .688 | | .168 | .029 | .183 |
|  | 1000 | 0 | 2.00 | **.002** | .228 | | .061 | .044 | .064 |  | **.002** | .618 | | .095 | .032 | .102 |
| .4 | 200 | 0 | 0 | **.121** | .092 | | .095 | .069 | .145 |  | **.093** | .214 | | .177 | .068 | .273 |
|  | 500 | 0 | 0 | **.121** | .097 | | .065 | .054 | .144 |  | **.093** | .177 | | .165 | .111 | .216 |
|  | 1000 | 0 | 0 | **.121** | .108 | | .107 | .101 | .150 |  | **.093** | .173 | | .172 | .148 | .202 |
| .8 | 200 | 0 | 0 | **.057** | .102 | | .109 | .068 | .113 |  | **.047** | .247 | | .269 | .073 | .275 |
|  | 500 | 0 | 0 | **.057** | .069 | | .070 | .059 | .073 |  | **.047** | .130 | | .133 | .072 | .134 |
|  | 1000 | 0 | 0 | **.057** | .070 | | .069 | .065 | .070 |  | **.047** | .107 | | .106 | .082 | .106 |
| 1 | 200 | 0 | 0 | **0** | .100 | | .107 | .051 | .109 |  | **0** | .322 | | .361 | .040 | .367 |
|  | 500 | 0 | 0 | **0** | .069 | | .068 | .049 | .069 |  | **0** | .129 | | .124 | .033 | .124 |
|  | 1000 | 0 | 0 | **0** | .060 | | .059 | .050 | .059 |  | **0** | .094 | | .086 | .041 | .086 |

*Notes:* pop = population value; ML = likelihood ratio (LR) test statistic; MLM = Satorra-Bentler mean adjusted LR; MLMV = Asparouhov and Muthén mean and variance adjusted LR, MLR = Asparouhov and Muthén mean adjusted LR. Shaded results indicate acceptable rejection rates (between .03 and .08).