Supplement A1

SAS/IML program for computing the critical value and *p*-value of the extended Welch test of equivalence

```
PROC IML;
*USER SPECIFICATION PORTION;
*DESIGNATED ALPHA;                    ALPHA=0.05;
*NUMBER OF GROUPS;                    G=4;
*NULL EFFECT;                         OMEGASQ0=0.0625;
*GROUP SAMPLE SIZES;                  NVEC={10 12 13 15};
*GROUP SAMPLE MEANS;
   MEANVEC={99.8120 99.2903 100.0024 98.6407};
*GROUP SAMPLE STANDARD DEVIATIONS;
   SVEC={7.5640 5.9968 10.4808 4.5309};
*END OF SPECIFICATION PORTION;
S2VEC=SVEC##2;DF1=G-1;
WVEC=NVEC/S2VEC;NT=SUM(NVEC);
MET=SUM(WVEC#MEANVEC)/SUM(WVEC);
Q=SUM(((1-WVEC/SUM(WVEC))##2)/(NVEC-1));
DFNUE=(G#G-1)/(3#Q);
W=SUM(WVEC#(MEANVEC-MET)##2)/(DF1#(1+2#(G-2)#Q/(G#G-1)));
FCVALUE=FINV(ALPHA,DF1,DFNUE,NT#OMEGASQ0);
WPVALUE=CDF('F',W,DF1,DFNUE,NT#OMEGASQ0);
PRINT G DF1 DFNUE[FORMAT=8.4] OMEGASQ0[FORMAT=8.4];
PRINT ALPHA FCVALUE[FORMAT=8.4] W[FORMAT=8.4]
WPVALUE[FORMAT=8.4];

QUIT;
```

Supplement A2

SAS/IML program for computing the power of the extended Welch test of equivalence

```
PROC IML;
*USER SPECIFICATION PORTION;
*DESIGNATED ALPHA;                    ALPHA=0.05;
*NUMBER OF GROUPS;                    G=4;
*NULL EFFECT;                         OMEGASQ0=0.0625;
*GROUP SAMPLE SIZES;                  NVEC={10 12 13 15};
*GROUP MEANS;
   MUVEC={99.8120 99.2903 100.0024 98.6407};
*GROUP STANDARD DEVIATIONS;
   STDVEC={7.5640 5.9968 10.4808 4.5309};
*END OF SPECIFICATION PORTION;
VARVEC=STDVEC##2;DF1=G-1;NT=SUM(NVEC);
QVEC=NVEC/NT;OVEC=QVEC/VARVEC;
MUT=SUM(OVEC#MUVEC)/SUM(OVEC);
TAU=SUM(((1-OVEC/SUM(OVEC))##2)/(NVEC-1));
DFNU=(G#G-1)/(3#TAU);
OMEGASQ=SUM(OVEC#(MUVEC-MUT)##2);
FCVALUE=FINV(ALPHA,DF1,DFNU,NT#OMEGASQ0);
WPOWER=CDF('F',FCVALUE,DF1,DFNU,NT#OMEGASQ);
PRINT G DF1 DFNU[FORMAT=8.4] OMEGASQ0[FORMAT=8.4]
OMEGASQ[FORMAT=8.4];
PRINT ALPHA FCVALUE[FORMAT=8.4] WPOWER[FORMAT=8.4];

QUIT;
```

Supplement A3

SAS/IML program for computing the sample size of the extended Welch test of equivalence

```
PROC IML;
*USER SPECIFICATION PORTION;
*DESIGNATED ALPHA;                    ALPHA=0.05;
*NOMINAL POWER;                       POWER=0.80;
*NUMBER OF GROUPS;                    G=4;
*NULL EFFECT;                         OMEGASQ0=0.0625;
*GROUP RATIOS;                        RVEC={1 1 1 1};
*GROUP MEANS;
   MUVEC={99.8120 99.2903 100.0024 98.6407};
*GROUP STANDARD DEVIATIONS;
   STDVEC={7.5640 5.9968 10.4808 4.5309};
*END OF SPECIFICATION PORTION;
VARVEC=STDVEC##2;DF1=G-1;
N=5;
DO UNTIL(WPOWER>POWER | N>1000);N=N+1;
NVEC=N#RVEC;NT=SUM(NVEC);
QVEC=NVEC/NT;OVEC=QVEC/VARVEC;
MUT=SUM(OVEC#MUVEC)/SUM(OVEC);
TAU=SUM(((1-OVEC/SUM(OVEC))##2)/(NVEC-1));
DFNU=(G#G-1)/(3#TAU);
OMEGASQ=SUM(OVEC#(MUVEC-MUT)##2);
FCVALUE=FINV(ALPHA,DF1,DFNU,NT#OMEGASQ0);
WPOWER=CDF('F',FCVALUE,DF1,DFNU,NT#OMEGASQ);
END;
PRINT G DF1 DFNU[FORMAT=8.4] OMEGASQ0[FORMAT=8.4]
OMEGASQ[FORMAT=8.4];
PRINT ALPHA FCVALUE[FORMAT=8.4];
PRINT NVEC NT WPOWER[FORMAT=8.4];

QUIT;
```

Supplement B1

R program for computing the critical value and *p*-value of the extended Welch test of equivalence

```
function () {
#USER SPECIFICATIONS PORTION
alpha<-0.05 #DESIGNATED ALPHA
g<-4 #NUMBER OF GROUPS
omegasq0<-0.0625 #NULL EFFECT
nvec<-c(10,12,13,15) #GROUP SAMPLE SIZES
meanvec<-c(99.8120, 99.2903, 100.0024, 98.6407) #GROUP SAMPLE MEANS
svec<-c(7.5640, 5.9968, 10.4808, 4.5309) #GROUP SAMPLE STANDARD
 DEVIATIONS
#END OF SPECIFICATION
s2vec<-svec^2
df1<-g-1
wvec<-nvec/s2vec
nt<-sum(nvec)
met<-sum(wvec*meanvec)/sum(wvec)
q<-sum(((1-wvec/sum(wvec))^2)/(nvec-1))
dfnue<-(g*g-1)/(3*q)
w<-sum(wvec*(meanvec-met)^2)/(df1*(1+2*(g-2)*q/(g*g-1)))
fcvalue<-qf(alpha,df1,dfnue,nt*omegasq0)
wpvalue<-pf(w,df1,dfnue,nt*omegasq0)
print("g, df1, dfnue, omegasq0")
print(c(g,df1,dfnue,omegasq0))
print("alpha, fcvalue, w, wpvalue")
print(c(alpha,fcvalue,w,wpvalue))
}
```

Supplement B2

R program for computing the power of the extended Welch test of equivalence

```
function () {
#USER SPECIFICATIONS PORTION
alpha<-0.05 #DESIGNATED ALPHA
g<-4 #NUMBER OF GROUPS
omegasq0<-0.0625 #NULL EFFECT
nvec<-c(10,12,13,15) #GROUP SAMPLE SIZES
muvec<-c(99.8120, 99.2903, 100.0024, 98.6407) #GROUP MEANS
stdvec<-c(7.5640, 5.9968, 10.4808, 4.5309) #GROUP STANDARD DEVIATIONS
#END OF SPECIFICATION
varvec<-stdvec^2
df1<-g-1
nt<-sum(nvec)
qvec<-nvec/nt
ovec<-qvec/varvec
mut<-sum(ovec*muvec)/sum(ovec)
tau<-sum(((1-ovec/sum(ovec))^2)/(nvec-1))
dfnu<-(g*g-1)/(3*tau)
omegasq<-sum(ovec*(muvec-mut)^2)
fcvalue<-qf(alpha,df1,dfnu,nt*omegasq0)
wpower<-pf(fcvalue,df1,dfnu,nt*omegasq)
print("g, df1, dfnu, omegasq0, omegasq")
print(c(g,df1,dfnu,omegasq0,omegasq))
print("alpha, fcvalue, wpower")
print(c(alpha,fcvalue,wpower))
}
```

Supplement B3

R program for computing the sample size of the extended Welch test of equivalence

```
function () {
#USER SPECIFICATIONS PORTION
alpha<-0.05 #DESIGNATED ALPHA
power<-0.80 #NOMINAL POWER
g<-4 #NUMBER OF GROUPS
omegasq0<-0.0625 #NULL EFFECT
rvec<-c(1,1,1,1) #GROUP RATIOS
muvec<-c(99.8120, 99.2903, 100.0024, 98.6407) #GROUP MEANS
stdvec<-c(7.5640, 5.9968, 10.4808, 4.5309) #GROUP STANDARD DEVIATIONS
#END OF SPECIFICATION
varvec<-stdvec^2
df1<-g-1
n<-5
wpower<-0
while(wpower<power & n<1000){
n<-n+1
nvec<-n*rvec
nt<-sum(nvec)
qvec<-nvec/nt
ovec<-qvec/varvec
mut<-sum(ovec*muvec)/sum(ovec)
tau<-sum(((1-ovec/sum(ovec))^2)/(nvec-1))
dfnu<-(g*g-1)/(3*tau)
omegasq<-sum(ovec*(muvec-mut)^2)
fcvalue<-qf(alpha,df1,dfnu,nt*omegasq0)
wpower<-pf(fcvalue,df1,dfnu,nt*omegasq)
}
print("g, df1, dfnu, omegasq0, omegasq")
print(c(g,df1,dfnu,omegasq0,omegasq))
print("alpha, fcvalue")
print(c(alpha,fcvalue))
print("nvec, nt, wpower")
print(c(nvec,nt,wpower))
}
```