

Supplementary Material

Using Response Times for Joint Modeling of Response and Omission Behavior

Esther Ulitzsch, Matthias von Davier, & Steffi Pohl

1.1 Parameter Recovery

We calculated the mean bias to describe the discrepancies between the sample estimates and the corresponding population values. It is defined as the average deviation of the sample estimate from the corresponding population value:

$$\frac{\sum_{r=1}^{n_{rep}} \hat{\phi}_{pr} - \phi_p}{n_{rep}} \quad (1.1)$$

where $\hat{\phi}_{pr}$ is the sample estimate of population parameter p for the r th replication, ϕ_p is the corresponding population value of parameter p , and n_{rep} is the number of replications

Root mean squared error (RMSE) was calculated to aggregate information on bias and variability into a single measure and is defined as

$$\sqrt{\frac{\sum_{r=1}^{n_{rep}} (\hat{\phi}_r - \phi)^2}{n_{rep}}} \quad (1.2)$$

The RMSE is equal to the square root of the squared bias plus the variance of the estimates across replications. Small RMSE values indicate that there is only a small or no bias and the estimates do not vary strongly across replications.

Mean bias and RMSE were calculated for all parameter types separately. Note that mean bias and RMSE are unstandardized measures and can as such not be compared across parameter types.

Table 1.1. Mean bias of person parameter variances

$\rho_{\cdot\eta}$	N	K	Omitted	$var(\tau)$	$var(\xi)$	$var(\eta)$
$\rho_{\cdot\eta} = 0$	375	10	5%	0.00	0.77	0.13
			17%	0.00	0.57	0.08
		30	5%	0.01	0.93	0.05
			17%	0.00	0.28	0.06
	750	10	5%	0.00	0.52	0.05
			17%	0.00	0.21	0.03
		30	5%	0.00	0.47	0.04
			17%	0.00	0.25	0.04
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.00	0.83	-0.04
			17%	0.00	0.47	0.01
		30	5%	0.00	0.69	0.04
			17%	0.00	0.37	0.02
	750	10	5%	-0.00	0.23	-0.01
			17%	0.00	0.21	0.00
		30	5%	0.00	0.19	0.01
			17%	0.00	0.16	0.01

Note: τ : speed; ξ : omission propensity; η : omission speed; $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items. Ability variance $var(\theta)$ was set to unity for model identification.

Table 1.2. RMSE of person parameter variances

$\rho_{\cdot\eta}$	N	K	Omitted	$var(\tau)$	$var(\xi)$	$var(\eta)$
$\rho_{\cdot\eta} = 0$	375	10	5%	0.01	1.77	0.32
			17%	0.01	1.20	0.18
		30	5%	0.01	1.34	0.22
			17%	0.02	0.75	0.12
	750	10	5%	0.01	1.15	0.18
			17%	0.01	0.70	0.10
		30	5%	0.01	0.87	0.13
			17%	0.01	0.61	0.09
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.01	1.80	0.26
			17%	0.02	1.11	0.18
		30	5%	0.02	1.21	0.19
			17%	0.02	0.76	0.14
	750	10	5%	0.01	0.83	0.19
			17%	0.01	0.66	0.11
		30	5%	0.01	0.85	0.15
			17%	0.01	0.59	0.10

Note: τ : speed; ξ : omission propensity; η : omission speed; $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items. Ability variance $var(\theta)$ was set to unity for model identification.

Table 1.3. Mean bias of person parameter correlations

$\rho_{\cdot\eta} = 0$	375	10	5%	0.00	0.02	0.02	-0.01	-0.01	-0.04
			17%	0.00	0.01	0.01	-0.01	-0.01	0.01
		30	5%	-0.01	0.00	0.00	0.01	0.00	0.01
			17%	0.00	0.00	-0.01	-0.00	-0.00	0.00
	750	10	5%	0.00	0.00	0.01	-0.01	0.00	0.00
			17%	0.00	0.01	-0.01	0.00	0.00	0.00
		30	5%	0.00	-0.01	0.00	0.00	0.01	0.00
			17%	0.00	-0.01	0.01	0.00	0.00	-0.02
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.00	0.02	0.04	-0.01	-0.01	-0.06
			17%	0.01	0.01	0.01	-0.02	-0.02	-0.01
		30	5%	-0.01	-0.01	0.00	0.00	0.00	-0.01
			17%	-0.01	-0.01	0.00	0.00	0.00	-0.01
	750	10	5%	0.00	0.00	0.00	0.00	-0.01	-0.03
			17%	0.00	0.01	0.01	0.00	-0.01	0.00
		30	5%	0.00	0.00	0.00	0.00	0.00	-0.01
			17%	0.00	-0.01	0.00	0.00	0.00	0.00

Note: θ : ability; τ : speed; ξ : omission propensity; η : omission speed; $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.4. RMSE of person parameter correlations

$\rho_{\cdot\eta} = 0$	375	10	5%	0.05	0.06	0.15	0.07	0.13	0.21
			17%	0.06	0.06	0.11	0.06	0.08	0.13
		30	5%	0.05	0.05	0.11	0.06	0.11	0.14
			17%	0.05	0.04	0.08	0.05	0.07	0.09
	750	10	5%	0.04	0.05	0.12	0.05	0.07	0.14
			17%	0.04	0.05	0.07	0.04	0.06	0.09
		30	5%	0.03	0.04	0.09	0.04	0.08	0.11
			17%	0.04	0.03	0.06	0.04	0.05	0.07
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.06	0.07	0.11	0.07	0.07	0.10
			17%	0.05	0.06	0.07	0.05	0.05	0.04
		30	5%	0.05	0.05	0.06	0.05	0.05	0.04
			17%	0.05	0.04	0.06	0.05	0.03	0.03
	750	10	5%	0.04	0.05	0.08	0.05	0.04	0.05
			17%	0.04	0.04	0.05	0.04	0.04	0.03
		30	5%	0.04	0.04	0.05	0.04	0.03	0.03
			17%	0.04	0.03	0.04	0.04	0.02	0.02

Note: θ : ability; τ : speed; ξ : omission propensity; η : omission speed; $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.5. Mean bias for different true values of item discrimination parameters ν

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				0.750	1.125	1.500	1.875	2.205
$\rho_{\cdot\eta} = 0$	375	10	5%	0.05	0.02	0.07	0.10	0.21
			17%	0.03	0.07	0.13	0.10	0.24
		30	5%	0.06	0.09	0.13	0.17	0.22
			17%	0.07	0.11	0.14	0.18	0.24
	750	10	5%	0.01	0.04	0.07	0.07	0.08
			17%	0.02	0.02	0.04	0.05	0.08
		30	5%	0.04	0.05	0.06	0.08	0.11
			17%	0.04	0.04	0.07	0.09	0.12
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.06	0.07	0.08	0.18	0.21
			17%	0.02	0.05	0.10	0.16	0.50
		30	5%	0.06	0.08	0.13	0.16	0.23
			17%	0.06	0.11	0.14	0.20	0.27
	750	10	5%	0.01	0.03	0.07	0.08	0.05
			17%	0.03	0.05	0.05	0.08	0.09
		30	5%	0.03	0.04	0.06	0.07	0.09
			17%	0.03	0.04	0.07	0.08	0.12

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.6. RMSE for different true values of item discrimination parameters ν

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				0.750	1.125	1.500	1.875	2.250
$\rho_{\cdot\eta} = 0$	375	10	5%	0.16	0.19	0.29	0.38	0.45
			17%	0.19	0.23	0.32	0.35	0.47
		30	5%	0.15	0.25	0.23	0.30	0.35
			17%	0.19	0.22	0.30	0.31	0.43
	750	10	5%	0.11	0.14	0.18	0.20	0.31
			17%	0.16	0.17	0.18	0.24	0.32
		30	5%	0.11	0.15	0.18	0.17	0.27
			17%	0.12	0.15	0.17	0.24	0.25
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.23	0.21	0.30	0.35	0.36
			17%	0.21	0.20	0.29	0.38	0.53
		30	5%	0.18	0.20	0.25	0.32	0.37
			17%	0.19	0.23	0.27	0.38	0.46
	750	10	5%	0.13	0.16	0.18	0.22	0.29
			17%	0.11	0.17	0.17	0.28	0.27
		30	5%	0.11	0.14	0.16	0.17	0.27
			17%	0.13	0.13	0.17	0.20	0.24

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.7. Mean bias for different true values of item difficulty parameters b

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				-1.00	-0.50	0.00	0.50	1.00
$\rho_{\cdot\eta} = 0$	375	10	5%	-0.01	0.01	0.02	0.05	0.09
			17%	-0.03	0.02	0.03	0.05	0.04
		30	5%	-0.02	-0.01	0.00	0.01	0.03
			17%	-0.03	-0.01	0.00	0.01	0.03
	750	10	5%	0.00	0.00	0.00	0.00	0.02
			17%	-0.01	0.02	0.01	0.02	0.03
		30	5%	-0.02	-0.01	-0.01	0.00	0.01
			17%	-0.02	-0.01	0.00	0.01	0.01
$\rho_{\cdot\eta} \neq 0$	375	10	5%	-0.02	-0.02	0.02	0.03	-0.01
			17%	-0.05	-0.03	0.00	0.00	0.02
		30	5%	-0.01	0.00	0.01	0.03	0.03
			17%	-0.02	0.00	0.00	0.03	0.03
	750	10	5%	-0.02	-0.01	0.00	0.00	0.02
			17%	-0.01	-0.02	0.00	0.00	0.00
		30	5%	-0.02	-0.01	-0.02	0.00	0.00
			17%	-0.02	-0.01	0.00	0.00	0.01

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.8. RMSE for different true values of item difficulty parameters b

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				-1.00	-0.50	0.00	0.50	1.00
$\rho_{\cdot\eta} = 0$	375	10	5%	0.16	0.18	0.16	0.15	0.26
			17%	0.24	0.18	0.17	0.24	0.21
		30	5%	0.18	0.16	0.16	0.16	0.18
			17%	0.17	0.17	0.18	0.16	0.20
	750	10	5%	0.10	0.13	0.08	0.12	0.14
			17%	0.14	0.12	0.12	0.12	0.15
		30	5%	0.12	0.11	0.10	0.13	0.12
			17%	0.13	0.12	0.13	0.12	0.13
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.19	0.18	0.16	0.16	0.18
			17%	0.19	0.18	0.17	0.17	0.17
		30	5%	0.17	0.14	0.16	0.16	0.19
			17%	0.16	0.17	0.18	0.18	0.20
	750	10	5%	0.11	0.10	0.12	0.12	0.12
			17%	0.16	0.13	0.12	0.15	0.16
		30	5%	0.12	0.12	0.12	0.11	0.12
			17%	0.12	0.11	0.10	0.12	0.12

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.9. Mean bias for different true values of time intensity parameters β

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				3.50	3.75	4.00	4.25	4.50
$\rho_{\cdot\eta} = 0$	375	10	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
		30	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
	750	10	5%	0.00	0.00	0.00	0.00	0.00
			17%	-0.01	0.00	0.00	-0.01	-0.01
		30	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
		10	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
		30	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
		30	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
	750	10	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
		30	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
		10	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00
		30	5%	0.00	0.00	0.00	0.00	0.00
			17%	0.00	0.00	0.00	0.00	0.00

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.10. RMSE for different true values of time intensity parameters β

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				3.50	3.75	4.00	4.25	4.50
$\rho_{\cdot\eta} = 0$	375	10	5%	0.04	0.03	0.03	0.03	0.03
			17%	0.02	0.03	0.03	0.03	0.03
		30	5%	0.03	0.03	0.03	0.03	0.03
			17%	0.03	0.03	0.03	0.03	0.03
	750	10	5%	0.02	0.02	0.02	0.02	0.02
			17%	0.02	0.02	0.02	0.02	0.02
		30	5%	0.02	0.02	0.03	0.02	0.03
			17%	0.02	0.02	0.02	0.02	0.02
		10	5%	0.03	0.02	0.03	0.03	0.03
			17%	0.03	0.03	0.03	0.04	0.03
		30	5%	0.03	0.03	0.03	0.03	0.03
			17%	0.04	0.03	0.03	0.03	0.03
$\rho_{\cdot\eta} \neq 0$	750	10	5%	0.02	0.01	0.02	0.02	0.02
			17%	0.02	0.02	0.02	0.02	0.02
		30	5%	0.02	0.02	0.02	0.02	0.02
			17%	0.02	0.02	0.02	0.02	0.02

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.11. Mean bias for different true values of omission difficulty parameters a

$\rho_{\cdot\eta}$	N	K	Omitted	2.00	2.50	3.00	3.50	True Parameter							
$\rho_{\cdot\eta} = 0$	375	10	5%					0.09	0.15	0.13	0.19	0.23			
			17%	0.04	0.05	0.07	0.08	0.11							
		30	5%					0.12	0.15	0.20	0.21	0.30			
			17%	-0.01	0.00	0.02	0.03	0.04							
		10	5%					0.09	0.14	0.13	0.15	0.18			
			17%	-0.01	0.01	-0.01	0.00	0.04							
	750	30	5%					0.07	0.07	0.11	0.09	0.11			
			17%	0.04	0.04	0.04	0.04	0.07							
		10	5%					0.15	0.17	0.22	0.30	0.40			
			17%	0.07	0.06	0.05	0.07	0.12							
		30	5%					0.09	0.11	0.12	0.16	0.21			
			17%	0.01	0.01	0.03	0.02	0.05							
$\rho_{\cdot\eta} \neq 0$	750	10	5%					0.03	0.05	0.09	0.03	0.08			
			17%	0.04	0.02	0.05	0.05	0.08							
		30	5%					0.05	0.05	0.07	0.07	0.12			
			17%	0.01	0.00	0.01	0.01	0.02							

Note: Since omission difficulties were varied in order to yield different omission rates, different values were considered for conditions omission rates of 5% and 17%. $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.12. RMSE for different true values of omission difficulty parameters a

ρ_{η}	N	K	Omitted	True Parameter									
				2.00	2.50	3.00	3.50	4.25	4.50	4.75	5.25	5.75	6.25
$\rho_{\eta} = 0$	375	10	5%	0.21	0.24	0.27	0.28	0.40	0.33	0.48	0.49	0.55	0.61
			17%	0.22	0.22	0.23	0.26	0.35	0.29	0.38	0.46	0.49	0.61
		30	5%	0.16	0.17	0.17	0.19	0.30	0.22	0.34	0.36	0.39	0.46
			17%	0.15	0.16	0.17	0.18	0.23	0.22	0.25	0.29	0.30	0.37
	750	10	5%	0.26	0.27	0.30	0.29	0.46	0.38	0.54	0.53	0.64	0.75
		30	5%	0.20	0.22	0.24	0.25	0.34	0.30	0.36	0.41	0.50	0.58
	750	10	5%	0.18	0.17	0.20	0.18	0.23	0.24	0.25	0.31	0.29	0.39
			17%	0.15	0.16	0.16	0.18	0.23	0.21	0.26	0.29	0.33	0.39
		30	5%	0.15	0.16	0.16	0.18	0.23	0.21	0.26	0.29	0.33	0.39
			17%	0.15	0.16	0.16	0.18	0.23	0.21	0.26	0.29	0.33	0.39
	750	10	5%	0.18	0.17	0.20	0.18	0.23	0.24	0.25	0.31	0.29	0.39
		30	5%	0.15	0.16	0.16	0.18	0.23	0.21	0.26	0.29	0.33	0.39

Note: Since omission difficulties were varied in order to yield different omission rates, different values were considered for conditions omission rates of 5% and 17%. $\rho_{\eta} = 0$ and $\rho_{\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.13. Mean bias for different true values of omission time intensity parameters δ

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				3.50	3.75	4.00	4.25	4.50
$\rho_{\cdot\eta} = 0$	375	10	5%	-0.05	-0.05	-0.06	-0.04	-0.05
			17%	0.00	0.02	0.00	0.00	0.01
		30	5%	0.01	0.02	0.01	0.01	0.00
			17%	-0.01	0.00	-0.01	0.00	0.00
	750	10	5%	0.02	-0.01	0.01	-0.01	0.00
			17%	0.00	-0.01	-0.01	0.00	-0.01
		30	5%	0.01	0.01	0.01	0.01	0.01
			17%	-0.01	-0.01	-0.01	0.00	-0.01
$\rho_{\cdot\eta} \neq 0$	375	10	5%	-0.11	-0.12	-0.09	-0.09	-0.10
			17%	0.01	0.00	-0.01	0.00	0.00
		30	5%	0.00	-0.03	-0.03	-0.01	-0.01
			17%	-0.01	-0.01	-0.01	-0.01	-0.01
	750	10	5%	-0.04	-0.03	-0.04	-0.02	-0.04
			17%	0.00	0.00	0.00	0.00	0.00
		30	5%	0.00	0.00	0.00	0.00	0.00
			17%	-0.01	0.00	0.00	-0.01	0.00

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.14. RMSE for different true values of omission time intensity parameters δ

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				3.50	3.75	4.00	4.25	4.50
$\rho_{\cdot\eta} = 0$	375	10	5%	0.37	0.33	0.37	0.32	0.31
			17%	0.17	0.17	0.15	0.20	0.18
		30	5%	0.27	0.22	0.25	0.27	0.26
			17%	0.13	0.13	0.13	0.11	0.11
	750	10	5%	0.24	0.22	0.25	0.27	0.22
			17%	0.10	0.10	0.09	0.10	0.09
		30	5%	0.17	0.17	0.17	0.17	0.18
			17%	0.09	0.09	0.08	0.08	0.07
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.31	0.33	0.32	0.34	0.37
			17%	0.16	0.11	0.14	0.13	0.11
		30	5%	0.22	0.21	0.21	0.19	0.20
			17%	0.10	0.11	0.12	0.10	0.11
	750	10	5%	0.18	0.20	0.19	0.21	0.18
			17%	0.11	0.08	0.10	0.08	0.09
		30	5%	0.13	0.13	0.14	0.16	0.14
			17%	0.08	0.08	0.08	0.07	0.07

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.15. Mean bias for different true values of time discrimination parameters α

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				2.00	2.75	3.50	4.25	5.00
$\rho_{\cdot\eta} = 0$	375	10	5%	0.01	-0.01	-0.03	0.00	0.02
			17%	-0.01	0.00	-0.01	0.00	-0.02
		30	5%	-0.01	0.00	0.00	0.00	-0.01
			17%	-0.00	0.00	0.00	-0.01	0.00
	750	10	5%	0.00	-0.01	-0.01	0.01	0.00
			17%	0.00	-0.01	0.01	0.01	-0.01
		30	5%	0.00	-0.01	-0.01	0.00	0.00
			17%	-0.01	0.00	0.00	-0.01	0.00
$\rho_{\cdot\eta} \neq 0$	375	10	5%	-0.01	-0.01	-0.02	0.01	-0.01
			17%	-0.01	-0.01	-0.01	-0.02	0.02
		30	5%	-0.01	-0.01	0.00	-0.01	-0.01
			17%	0.00	0.00	-0.01	-0.01	-0.01
	750	10	5%	0.00	-0.01	0.00	0.03	0.01
			17%	0.00	0.00	0.00	-0.02	0.00
		30	5%	0.00	0.00	0.00	-0.01	-0.01
			17%	0.00	0.00	0.00	0.00	0.00

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

 Table 1.16. RMSE for different true values of time discrimination parameters α

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				2.00	2.75	3.50	4.25	5.00
$\rho_{\cdot\eta} = 0$	375	10	5%	0.07	0.12	0.14	0.16	0.20
			17%	0.10	0.10	0.16	0.19	0.27
		30	5%	0.08	0.11	0.14	0.16	0.20
			17%	0.08	0.12	0.15	0.18	0.23
	750	10	5%	0.06	0.08	0.11	0.11	0.17
			17%	0.07	0.07	0.11	0.18	0.13
		30	5%	0.05	0.08	0.10	0.10	0.13
			17%	0.05	0.07	0.10	0.14	0.16
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.07	0.09	0.18	0.17	0.25
			17%	0.08	0.14	0.14	0.19	0.25
		30	5%	0.07	0.11	0.14	0.17	0.20
			17%	0.09	0.11	0.15	0.18	0.22
	750	10	5%	0.06	0.08	0.10	0.13	0.17
			17%	0.06	0.08	0.11	0.13	0.15
		30	5%	0.05	0.08	0.08	0.12	0.14
			17%	0.06	0.08	0.10	0.13	0.18

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.17. Mean bias for different true values of omission time discrimination parameters ω

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				1.000	1.625	2.250	2.875	3.500
$\rho_{\cdot\eta} = 0$	375	10	5%	-0.05	-0.04	-0.07	-0.02	-0.13
			17%	0.00	-0.02	-0.03	0.00	0.01
		30	5%	-0.05	-0.05	-0.08	-0.12	-0.16
			17%	0.00	-0.01	-0.02	-0.03	0.02
	750	10	5%	-0.01	0.00	0.06	0.04	-0.07
			17%	0.00	-0.01	-0.01	-0.02	0.02
		30	5%	-0.02	-0.02	-0.03	-0.04	-0.05
			17%	0.00	0.00	-0.02	-0.01	-0.01
$\rho_{\cdot\eta} \neq 0$	375	10	5%	-0.07	-0.08	-0.06	0.06	-0.31
			17%	-0.01	0.01	-0.02	-0.02	-0.02
		30	5%	-0.05	-0.04	-0.10	-0.06	-0.15
			17%	-0.01	-0.01	-0.03	-0.01	-0.01
	750	10	5%	-0.03	-0.03	-0.03	0.07	0.16
			17%	-0.01	0.00	0.01	-0.02	0.01
		30	5%	-0.02	-0.01	-0.03	-0.03	-0.04
			17%	0.00	-0.01	-0.01	-0.02	-0.02

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

Table 1.18. RMSE for different true values of omission time discrimination parameters ω

$\rho_{\cdot\eta}$	N	K	Omitted	True Parameter				
				1.000	1.625	2.250	2.875	3.500
$\rho_{\cdot\eta} = 0$	375	10	5%	0.20	0.42	1.05	0.88	1.29
			17%	0.09	0.19	0.29	0.42	0.50
		30	5%	0.23	0.47	0.58	0.85	1.14
			17%	0.10	0.19	0.27	0.35	0.45
	750	10	5%	0.12	0.43	0.60	0.63	1.05
			17%	0.07	0.16	0.21	0.28	0.46
		30	5%	0.19	0.26	0.33	0.47	0.79
			17%	0.08	0.11	0.19	0.26	0.30
$\rho_{\cdot\eta} \neq 0$	375	10	5%	0.31	0.36	0.69	0.99	1.57
			17%	0.12	0.24	0.26	0.43	0.54
		30	5%	0.24	0.40	0.79	0.92	0.95
			17%	0.09	0.25	0.24	0.34	0.45
	750	10	5%	0.15	0.29	0.55	0.88	2.30
			17%	0.07	0.13	0.18	0.36	0.29
		30	5%	0.16	0.28	0.39	0.47	0.63
			17%	0.08	0.12	0.16	0.25	0.29

Note: $\rho_{\cdot\eta} = 0$ and $\rho_{\cdot\eta} \neq 0$ denote whether omission speed is uncorrelated or correlated with ability, speed, and omission propensity; N : number of examinees; K : number of items.

1.2 Posterior Predictive Checks for the Empirical Example

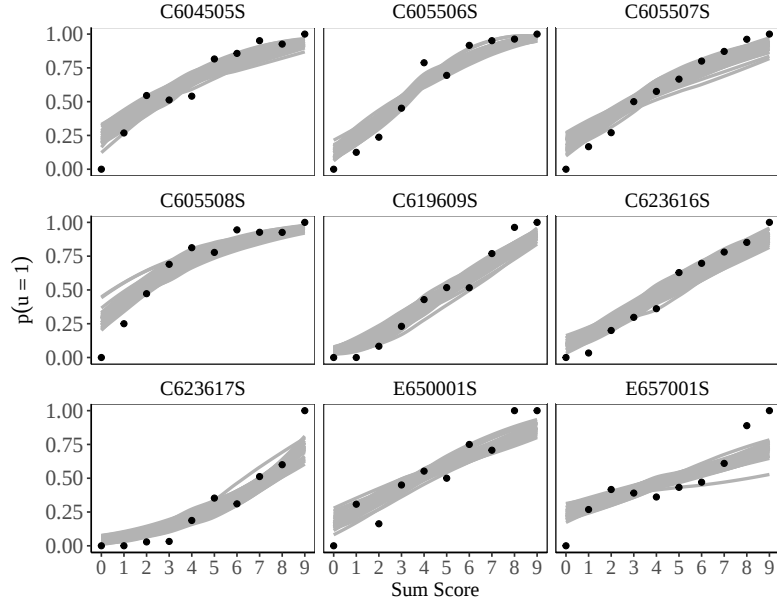


Figure 1.1. Posterior predictive probabilities correct for 30 random draws from the posterior distribution as a function of observed sum scores for nine numeracy items from PIAAC 2012. The simulated probabilities correct are superimposed on observed proportions given by black dots.

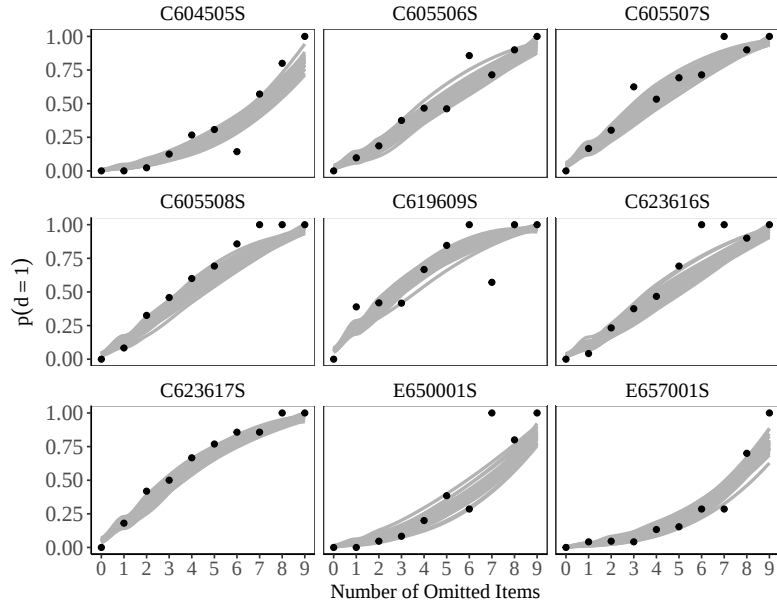


Figure 1.2. Posterior predictive probabilities of item omissions for 30 random draws from the posterior distribution as a function of observed number of omitted items for nine numeracy items from PIAAC 2012. The simulated probabilities of item omission are superimposed on observed proportions of omitted items given by black dots.

SUPPLEMENTARIES

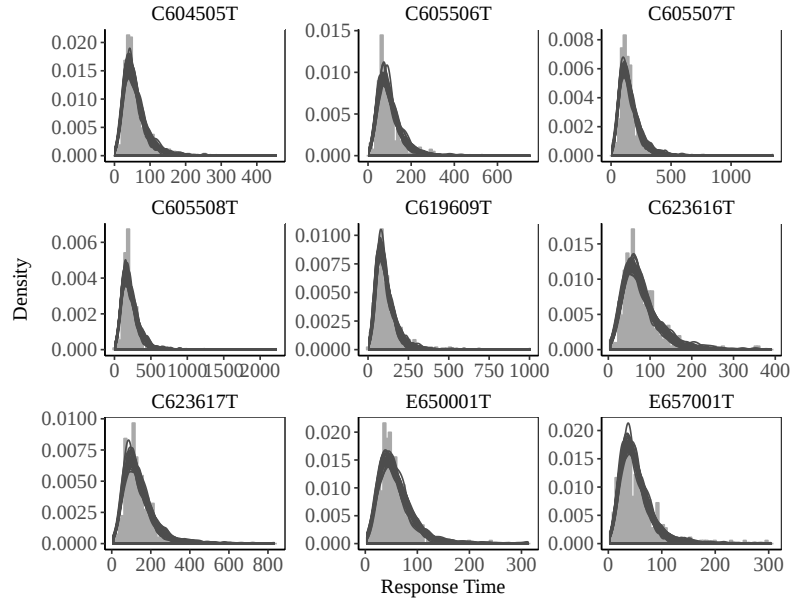


Figure 1.3. Posterior predictive distributions of observed response times for 30 random draws from the posterior distribution for nine numeracy items from PIAAC 2012. The simulated distributions are superimposed on histograms of observed response times.

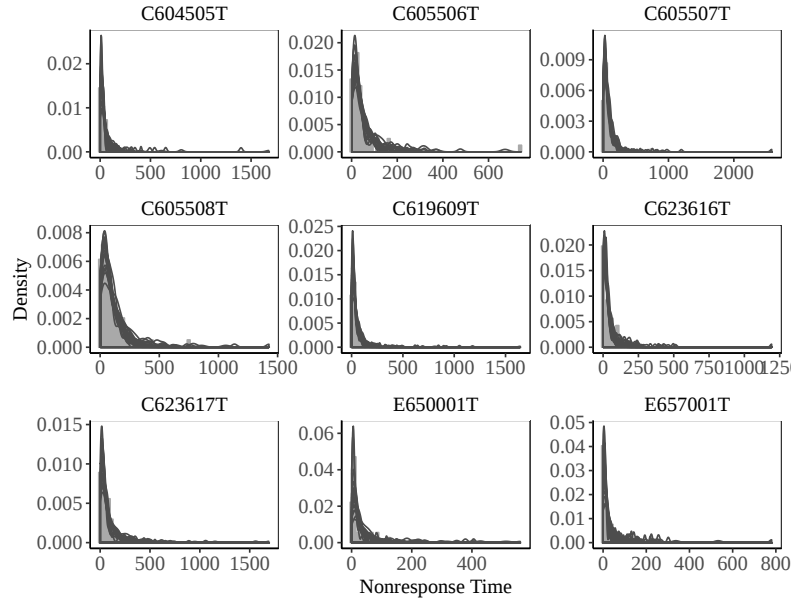


Figure 1.4. Posterior predictive distributions of observed nonresponse times for 30 random draws from the posterior distribution for nine numeracy items from PIAAC 2012. The simulated distributions are superimposed on histograms of observed nonresponse times.