**Appendix 1.**

install.packages("Metrics")

install.packages("GA")

library("Metrics")

library("GA")

# Data Set

p <- ncol(longley) - 1

n <- nrow(longley)

ux <- longley[,1:p]

uy <- longley[,p+1]

x <- scale(ux)

y <- scale(uy)

I <- diag(p)

S = (t(x) %\*% x)

Sinverse = solve(S)

BetaL <- Sinverse %\*% t(x) %\*% y

lambda <- diag(eigen( S )$values)

Q <- eigen( S )$vectors

Z <- x %\*% Q

Alfa <- t(Q) %\*% BetaL

e= y - Z %\*% Alfa

SigmaSquareHat <- t(e) %\*% e /(n-p)

prm <- 0

uft <- 1

TPE <- "TPE"

Ridge <- "Ridge"

Liu <- "Liu"

NONE <- "NONE"

fMSE <- 1

fMAE <- 2

fMAPE <- 3

fSMSE <- 4

SEED <- 1234

# returns condition number for k and d parameters

condition\_number <- function(k,d)

{

Sk <- S + k\*I

Skd <- S + k\*d\*I

Kor <- S %\*% solve(Skd) %\*% Sk

ev <- eigen( Kor )$values

cond <- sqrt(ev[1] / ev[p])

return(cond)

}

# returns VIF vaules for k and d parameters

vif <- function(k,d)

{

Sk <- S + k\*I

Skd <- S + k\*d\*I

Skinverse <- solve(Sk)

m <- (n-1) \* Skinverse %\*% Skd %\*% Sinverse %\*% Skd %\*% Skinverse

VIF <- diag(m)

return(VIF)

}

# Fitness Function for Genetic Algorithm GA1

f <- function(k,d)

{

cn <- condition\_number(k,d)

if (cn<=10) { cn <- 0 }

vf <- max(vif(k,d))

if (vf<=10) { vf <- 0 }

Penalty <- cn + vf

BetaKD <- solve( S + k\*I ) %\*% ( t(x) %\*% y + k\*d\*BetaL )

yhat <- x %\*% BetaKD

Performance <- switch(uft, mse(y, yhat), mae(y, yhat), mapee(y, yhat), SMSE(k,d) )

return( Performance + Penalty ) # Return Fitness

}

# Fitness Function for Genetic Algorithm GA2

fd <- function(d)

{

k <- prm

f(k,d)

}

# Fitness Function for Genetic Algorithm GA3

fk <- function(k)

{

d <- prm

f(k,d)

}

GA1 <- function(kmax) # k and d are variables

{

GA <- ga(type = "real-valued", fitness = function(x) -f(x[1], x[2]),

min = c(0, 0), max = c(kmax, 1), pcrossover=0.6, monitor = NULL,

popSize = 100, maxiter = 200, elitism=20, pmutation=0.1, seed=SEED)

summary(GA)

# assign k and d values from GA

k<-summary(GA)$sol[1,1]

d<-summary(GA)$sol[1,2]

uy<- -1\*summary(GA)$fitness

BetaKD<-solve((t(x) %\*% x)+ k\*I) %\*% ( t(x) %\*% y + k\*d\*BetaL )

yhat <- x %\*% BetaKD

sn <- array( c(uy, mse( y , yhat ),condition\_number(k,d),k,d, BetaKD) , dim=c(p+5,1) )

sn

}

GA2 <- function(k) # k is constant and d is variable

{

prm <<- k

GA <- ga(type = "real-valued", fitness = function(x) -fd(x[1]),

min = c(0), max = c(1), monitor = NULL, pcrossover=0.6,

popSize = 100, maxiter = 200, elitism=20, pmutation=0.1, seed=SEED)

summary(GA)

# assign k and d values from GA

d<-summary(GA)$sol[1,1]

uy<- -1\*summary(GA)$fitness

BetaKD<-solve((t(x) %\*% x)+ k\*I) %\*% ( t(x) %\*% y + k\*d\*BetaL )

yhat <- x %\*% BetaKD

sn <- array( c(uy, mse( y , yhat ),condition\_number(k,d),k,d, BetaKD) , dim=c(p+5,1) )

sn

}

GA3 <- function(d, kmax) # k is variable and d is constant

{

prm <<- d

GA <- ga(type = "real-valued", fitness = function(x) -fk(x[1]),

min = c(0), max = c(kmax), monitor = NULL,pcrossover=0.6,

popSize = 100, maxiter = 200, elitism=20, pmutation=0.1, seed=SEED)

summary(GA)

# assign k and d values from GA

k<-summary(GA)$sol[1,1]

uy<- -1\*summary(GA)$fitness

BetaKD<-solve((t(x) %\*% x)+ k\*I) %\*% ( t(x) %\*% y + k\*d\*BetaL )

yhat <- x %\*% BetaKD

sn <- array( c(uy, mse( y , yhat ),condition\_number(k,d),k,d, BetaKD) , dim=c(p+5,1) )

sn

}

##const <- "NONE" ## TPE

##const <- d ; param <-1 ## d=1, OLS

##const <- k ; param <-0 ## k=0, OLS

##const <- d ; param <-0 ## d=0, Ridge

##const <- k ; param <-1 ## k=1, Liu

##const <- k ; k<-100000 ## k=infinite, Shrinkage

##const <- k ; k<-0.008565 ## k fixed

##const <- Ridge ## Ridge

##const <- Liu ## Liu

##const <- TPE ## TPE

gaTPE <- function( const="NONE", param=0, fittype=1)

{

uft <<- fittype

kmax <- 10

a <- switch(const,

NONE = GA1(kmax),

TPE = GA1(kmax),

Ridge = GA3(0,kmax),

Liu = GA2(1),

k = GA2(param),

d = GA3(param,kmax)

)

a

}

**Appendix 2.** Simulation results for n = 50

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **n** | **σ** | **ρ** | **OLS** | **HK** | **HKB** | **IHK** | **LW** | **DMM** | **DCL** | **ITR** | **AM** | **GM** | **HM** | **GA-MSE** | **GA-MAE** | **GA-MAPE** |
| 50 | 0.01 | 0.9 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0221 | 0.0220 | **0.0211** |
| 50 | 0.01 | 0.99 | 0.0321 | 0.0321 | 0.0320 | 0.0320 | 0.0321 | 0.0320 | 0.0320 | 0.0320 | **0.0318** | 0.0320 | 0.0320 | 0.0728 | 0.0728 | 0.0720 |
| 50 | 0.01 | 0.999 | 0.0349 | 0.0348 | 0.0344 | 0.0345 | 0.0349 | 0.0343 | 0.0345 | 0.0343 | 0.0357 | **0.0340** | 0.0345 | 0.2900 | 0.2899 | 0.2897 |
| 50 | 0.01 | 0.9999 | 0.0554 | 0.0542 | 0.0504 | 0.0505 | 0.0552 | 0.0498 | 0.0541 | 0.0508 | 0.0530 | **0.0493** | 0.0511 | 0.3249 | 0.3249 | 0.3249 |
| 50 | 0.1 | 0.9 | 0.0244 | 0.0243 | 0.0238 | 0.0239 | 0.0244 | 0.0237 | 0.0238 | 0.0237 | **0.0221** | 0.0234 | 0.0239 | 0.0244 | 0.0229 | 0.0243 |
| 50 | 0.1 | 0.99 | 0.0538 | 0.0526 | 0.0486 | 0.0488 | 0.0536 | 0.0480 | 0.0485 | 0.0479 | 0.0530 | **0.0466** | 0.0493 | 0.0794 | 0.0794 | 0.1037 |
| 50 | 0.1 | 0.999 | 0.2577 | 0.2293 | 0.1739 | **0.1679** | 0.2513 | 0.2910 | 0.1994 | 0.2435 | 0.2081 | 0.1900 | 0.1945 | 0.2911 | 0.2913 | 0.2952 |
| 50 | 0.1 | 0.9999 | 2.3213 | 1.5913 | 0.7488 | 0.3384 | 1.8471 | 24.7923 | 1.1209 | 1.0618 | 1.1253 | 1.1299 | 1.1986 | 0.3252 | 0.3255 | **0.3241** |
| 50 | 0.5 | 0.9 | 0.0906 | 0.0860 | 0.0733 | **0.0730** | 0.0898 | 0.0771 | 0.0753 | 0.0762 | 0.1055 | 0.0731 | 0.0754 | 0.0906 | 0.0826 | 0.0911 |
| 50 | 0.5 | 0.99 | 0.6125 | 0.4969 | 0.3137 | 0.2771 | 0.5764 | 1.4524 | 0.6572 | 0.5183 | 0.3984 | 0.3731 | 0.3956 | 0.2196 | **0.2172** | 0.2559 |
| 50 | 0.5 | 0.999 | 5.7769 | 3.6529 | 1.5141 | 0.7405 | 3.4584 | 142.1349 | 8.0085 | 2.1304 | 2.5879 | 2.5912 | 2.7537 | 0.3171 | 0.3159 | **0.3142** |
| 50 | 0.5 | 0.9999 | 57.5127 | 33.8984 | 12.6022 | 0.3416 | 4.4409 | 533.0888 | 96.5549 | 15.9244 | 24.2803 | 24.3529 | 26.0521 | 0.3355 | 0.3348 | **0.3289** |
| 50 | 1 | 0.9 | 0.3033 | 0.2686 | 0.2032 | 0.1985 | 0.2960 | 0.3124 | 0.2484 | 0.3131 | 0.3175 | 0.2306 | 0.2269 | 0.3033 | 0.2694 | **0.1734** |
| 50 | 1 | 0.99 | 2.3723 | 1.6349 | 0.7798 | 0.5251 | 1.9130 | 31.9785 | 5.0514 | 1.3729 | 1.2190 | 1.1587 | 1.2403 | 0.6493 | 0.6123 | **0.3986** |
| 50 | 1 | 0.999 | 23.0558 | 13.7844 | 5.2472 | 1.8705 | 5.7074 | 193.0659 | 89.7865 | 7.2036 | 9.8772 | 9.8856 | 10.5656 | 0.3980 | 0.3910 | **0.3537** |
| 50 | 1 | 0.9999 | 236.526 | 138.4276 | 50.6315 | 0.3633 | 2.5345 | 792.7644 | 606.0598 | 59.6831 | 98.1111 | 98.2425 | 106.0607 | 0.3670 | 0.3655 | **0.3520** |

**Appendix 3.** Simulation results for n = 100

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **n** | **σ** | **ρ** | **OLS** | **HK** | **HKB** | **IHK** | **LW** | **DMM** | **DCL** | **ITR** | **AM** | **GM** | **HM** | **GA-MSE** | **GA-MAE** | **GA-MAPE** |
| 100 | 0.01 | 0.9 | 0.0174 | 0.0174 | 0.0174 | 0.0174 | 0.0174 | 0.0174 | 0.0174 | 0.0174 | 0.0180 | 0.0174 | 0.0174 | 0.0174 | 0.0173 | **0.0169** |
| 100 | 0.01 | 0.99 | 0.0257 | 0.0257 | 0.0257 | 0.0257 | 0.0257 | 0.0257 | 0.0257 | 0.0257 | **0.0255** | 0.0257 | 0.0257 | 0.0653 | 0.0652 | 0.0647 |
| 100 | 0.01 | 0.999 | 0.0274 | 0.0273 | 0.0271 | 0.0271 | 0.0274 | 0.0271 | 0.0271 | 0.0271 | **0.0266** | 0.0270 | 0.0272 | 0.2874 | 0.2874 | 0.2873 |
| 100 | 0.01 | 0.9999 | 0.0353 | 0.0348 | 0.0330 | 0.0331 | 0.0352 | 0.0327 | 0.0339 | 0.0326 | 0.0351 | **0.0323** | 0.0333 | 0.3238 | 0.3238 | 0.3238 |
| 100 | 0.1 | 0.9 | 0.0184 | 0.0183 | 0.0181 | 0.0181 | 0.0184 | 0.0180 | 0.0181 | 0.0180 | 0.0219 | 0.0179 | 0.0181 | 0.0184 | 0.0171 | **0.0156** |
| 100 | 0.1 | 0.99 | 0.0345 | 0.0340 | 0.0322 | 0.0322 | 0.0345 | 0.0318 | 0.0320 | 0.0317 | 0.0487 | **0.0314** | 0.0325 | 0.0693 | 0.0693 | 0.0881 |
| 100 | 0.1 | 0.999 | 0.1190 | 0.1110 | 0.0929 | **0.0914** | 0.1182 | 0.1201 | 0.1073 | 0.1277 | 0.1229 | 0.0976 | 0.0981 | 0.2891 | 0.2892 | 0.2911 |
| 100 | 0.1 | 0.9999 | 0.9770 | 0.7075 | 0.3755 | **0.2810** | 0.9262 | 6.8974 | 0.4785 | 0.4332 | 0.4327 | 0.4380 | 0.4845 | 0.3245 | 0.3245 | 0.3229 |
| 100 | 0.5 | 0.9 | 0.0471 | 0.0455 | 0.0403 | 0.0402 | 0.0470 | 0.0408 | 0.0406 | 0.0404 | 0.0693 | **0.0392** | 0.0412 | 0.0471 | 0.0420 | 0.0434 |
| 100 | 0.5 | 0.99 | 0.2679 | 0.2349 | 0.1748 | 0.1665 | 0.2641 | 0.4478 | 0.2950 | 0.2830 | 0.2197 | 0.1876 | 0.1926 | 0.1444 | **0.1435** | 0.2057 |
| 100 | 0.5 | 0.999 | 2.4209 | 1.5269 | 0.6466 | 0.3962 | 2.1306 | 67.7477 | 4.4845 | 1.0905 | 0.8862 | 0.8813 | 1.0025 | 0.3088 | **0.3081** | 0.3086 |
| 100 | 0.5 | 0.9999 | 24.0232 | 12.9321 | 4.1886 | 0.3397 | 8.9608 | 83.1610 | 49.3876 | 7.2832 | 7.4251 | 7.4672 | 8.4781 | 0.3315 | 0.3313 | **0.3269** |
| 100 | 1 | 0.9 | 0.1380 | 0.1283 | 0.1060 | 0.1042 | 0.1372 | 0.1290 | 0.1152 | 0.1285 | 0.1708 | 0.1171 | 0.1116 | 0.1380 | 0.1220 | **0.1015** |
| 100 | 1 | 0.99 | 1.0056 | 0.7332 | 0.3934 | 0.3144 | 0.9563 | 5.7199 | 1.8887 | 0.8578 | 0.4772 | 0.4577 | 0.5040 | 0.3691 | 0.3537 | **0.2855** |
| 100 | 1 | 0.999 | 9.6410 | 5.3713 | 1.8538 | 0.6687 | 6.1652 | 56.9148 | 33.2538 | 3.1947 | 3.0812 | 3.1002 | 3.5062 | 0.3586 | 0.3565 | **0.3386** |
| 100 | 1 | 0.9999 | 96.1330 | 50.8979 | 15.9815 | 0.3496 | 8.6388 | 274.1741 | 258.8099 | 27.1874 | 29.5209 | 29.5907 | 33.6466 | 0.3512 | 0.3500 | **0.3443** |

**Appendix 4.** Simulation results for n = 250

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **n** | **σ** | **ρ** | **OLS** | **HK** | **HKB** | **IHK** | **LW** | **DMM** | **DCL** | **ITR** | **AM** | **GM** | **HM** | **GA-MSE** | **GA-MAE** | **GA-MAPE** |
| 250 | 0.01 | 0.9 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0167 | 0.0166 | **0.0159** |
| 250 | 0.01 | 0.99 | 0.0265 | 0.0264 | 0.0264 | 0.0264 | 0.0264 | 0.0264 | 0.0264 | 0.0264 | **0.0264** | 0.0264 | 0.0264 | 0.0586 | 0.0585 | 0.0583 |
| 250 | 0.01 | 0.999 | 0.0281 | 0.0281 | 0.0280 | 0.0280 | 0.0281 | 0.0280 | 0.0280 | 0.0280 | **0.0278** | 0.0280 | 0.0280 | 0.2848 | 0.2848 | 0.2847 |
| 250 | 0.01 | 0.9999 | 0.0326 | 0.0324 | 0.0317 | 0.0317 | 0.0326 | 0.0315 | 0.0317 | 0.0315 | 0.0384 | **0.0312** | 0.0317 | 0.3258 | 0.3258 | 0.3258 |
| 250 | 0.1 | 0.9 | 0.0172 | 0.0172 | 0.0171 | 0.0171 | 0.0172 | 0.0171 | 0.0171 | 0.0171 | 0.0247 | 0.0170 | 0.0171 | 0.0172 | 0.0164 | **0.0123** |
| 250 | 0.1 | 0.99 | 0.0313 | 0.0311 | 0.0303 | 0.0303 | 0.0313 | 0.0301 | 0.0302 | 0.0301 | 0.0347 | **0.0298** | 0.0304 | 0.0603 | 0.0603 | 0.0674 |
| 250 | 0.1 | 0.999 | 0.0761 | 0.0735 | 0.0651 | **0.0648** | 0.0760 | 0.0674 | 0.0672 | 0.0693 | 0.0819 | 0.0648 | 0.0666 | 0.2855 | 0.2854 | 0.2873 |
| 250 | 0.1 | 0.9999 | 0.4890 | 0.4118 | 0.2702 | **0.2417** | 0.4848 | 0.4967 | 0.3572 | 0.3622 | 0.3149 | 0.2988 | 0.3134 | 0.3261 | 0.3261 | 0.3259 |
| 250 | 0.5 | 0.9 | 0.0319 | 0.0313 | 0.0292 | 0.0292 | 0.0319 | 0.0291 | 0.0291 | 0.0288 | 0.0296 | 0.0281 | 0.0293 | 0.0319 | 0.0284 | **0.0204** |
| 250 | 0.5 | 0.99 | 0.1454 | 0.1361 | 0.1121 | 0.1097 | 0.1450 | 0.1407 | 0.1312 | 0.1374 | 0.1295 | 0.1147 | 0.1174 | **0.0962** | 0.0963 | 0.1296 |
| 250 | 0.5 | 0.999 | 1.1827 | 0.8886 | 0.4699 | 0.3546 | 1.1587 | 2.2614 | 1.5340 | 0.5851 | 0.5912 | 0.5768 | 0.6225 | **0.2939** | 0.2941 | 0.2974 |
| 250 | 0.5 | 0.9999 | 11.5350 | 7.0580 | 2.6331 | 0.5786 | 9.5246 | 29.2648 | 19.5054 | 3.7237 | 4.2298 | 4.2462 | 4.6170 | 0.3295 | 0.3294 | **0.3290** |
| 250 | 1 | 0.9 | 0.0765 | 0.0734 | 0.0639 | 0.0634 | 0.0764 | 0.0666 | 0.0650 | 0.0658 | 0.0916 | 0.0634 | 0.0651 | 0.0765 | 0.0687 | **0.0515** |
| 250 | 1 | 0.99 | 0.5009 | 0.4228 | 0.2785 | 0.2483 | 0.4967 | 0.5088 | 0.4393 | 0.4926 | 0.3281 | 0.3050 | 0.3236 | 0.2062 | 0.2077 | **0.1930** |
| 250 | 1 | 0.999 | 4.6441 | 2.9993 | 1.2264 | 0.5941 | 4.2943 | 803.2820 | 7.1598 | 2.0021 | 1.8233 | 1.8145 | 2.0066 | 0.3174 | 0.3172 | **0.3167** |
| 250 | 1 | 0.9999 | 46.0461 | 27.3346 | 9.6663 | 0.9972 | 23.7622 | 85.7520 | 86.3361 | 12.5997 | 16.3677 | 16.4017 | 17.8517 | 0.3390 | 0.3388 | **0.3385** |