**Supplementary material**

Model 1 (): Scheduling and packaging unit size to minimise cost of lost units

The first objective function involves three expressions. The first one is related to the cost of lost units by dispatching an amount of item () greater than the demand (). The number of lost units per item is .

To meet , it is necessary to dispatch packages of size and packages remain. If this quantity is greater or equal to one, a package of size has to be delivered; otherwise a package of size . Therefore, the size of the last delivered package is . However, this package is released if there is missing demand once packages were dispatched, which is . Therefore, the total delivered amount of item is

The number of lost units of item is equal to

The total cost of lost units for all items is

The second term is related with the final inventory per item. The final inventory of item is equal to , that is

Thus, the total inventory cost for all items is

The last term is the earliness/tardiness penalties due to the packages delivery, which can be expressed as follows

Therefore, the total cost of the production-packaging system is

The second objective function refers to the kilograms of carbon dioxide (CO2) emitted due to energy utilisation of the production-packaging system. CO2 is given by the power consumed by resources operation of the production-packaging system. Energy consumption consists of three parts: 1) idle energy consumption during the resource setup ; 2) processing/packaging energy consumption from or , until the power reaches the steady state from a time ; and 3) energy consumption during the steady state from until the next setup. Figure 3 presents the characteristic behaviour of electric power.

The behaviour of the electric power used versus elapsed time of resource at station between setups is given by a general characteristic curve . This function depends on the type of resource and its specifications. The accumulated electric power from the start time to a time for processing one lot throughout the proposed production-packaging system is given by the area under , which can be expressed as the integral of , as follows

Therefore, the kilograms of CO2 emitted are given by the following expression

Model 2 (): Scheduling and packaging unit size to minimise unpacking cost

The total cost function for the second model also involves three terms. The first expression is related with the unpacking cost. A package has to be open if there is missing demand once packages of size are delivered. As mentioned before, the missing demand is equal to . Therefore, the number of packages that have to be opened () and their size () are

The total unpacking cost for all items is represented with the following function:

The earliness/tardiness penalties can be expressed as the previous model. For this model, the total inventory is given by

Since the total inventory for all items is determined, the costs of inventory () and material for packing this amount of finished product are fixed. Therefore, they are not included in the proposed model but considered in the analysis of results.

Therefore, the total cost of the production-packaging system would be:

The second objective function (kilograms of CO2) is represented with the same function shown above. The model that determines the packaging unit size to minimize the unpacking total cost is as follows:

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Figure 3

