S. MCDM techniques

Since 1970, various MCDM methods have been gradually developed, which have become a basis for selection, ranking, screening, prioritization, and classification of finite and available alternatives. To apply these techniques, input data is converted to a matrix in which rows and columns are defined as alternatives and effective criteria for alternative selection, respectively. In these techniques, the performance of each alternative is not only dependent on the criterion level considered for describing alternatives but also experts' priorities on criteria determination would be applied for decision making (Koksalan, 2011; Tzeng and Huang, 2011; Xu, 2015).

S.1. ORESTE technique

This technique was first presented by Marc Roubens under the general title of "Organization, Rangement Et Synthese De Donnees Relationnelles". If A is considered as a finite set of m alternatives, the alternatives could be analyzed by a set C containing k criteria. In this technique, the relative importance of each criterion is not determined by its weight, but, with a preference structure on criteria set C defined as a weak order. This preference structure is defined as a complete and transitive relation including I and P relations, where I (indifference) and P (preference) are symmetric and antisymmetric, respectively. Also, for each criterion of j=1, ..., k, a preference structure is defined on set A, which is transitive like set C, and is constructed from preference and indifference relations. Consequently, the first and second preference structures are constructed based on relative importance of criteria and criteria set according to each criterion, respectively. In the following, Besson's average ranking method is applied to initial ranking based on these structures. So, regarding the ranking, all criteria and alternatives are initially referred to as 1to k (k criterion) and 1 to m (m alternative), respectively. Then, the most and least allocated

number is averaged with a similar preference (P) or indifference (I) based on preference structure. Therefore, preferences are converted to ranks applying Besson's average ranking method. The rank of each criterion and the rank for each alternative over a criterion are introduced by r_k and $r_k(m)$, respectively (Isabelle and Pastijn, 2002). The aggregation ranking method includes three following steps (Roubens, 1982):

Step A

First, an arbitrary matrix called position matrix (P matrix) is constructed in which, the decision alternatives with respect to the criteria are ordered in each column from the best to the worst. The rows are also ordered by criteria ranks. By projection of position matrix on its main diagonal, better positions are located to the left-hand side of the main diagonal, while worse ones are located to the right-hand side. In the following, a zero value origin is introduced at the end of the left-hand side of the main diagonal and all the projections then, the distances of the projections from the origin ($d(0, m_k)$) are calculated by Eqs. (S-1) and (S-2):

$$if\ a\ P_k\ b\ then\ d(0,a_k) < d(0,b_k) \quad \text{(S-1)}$$

$$if\ r_1(a) = r_2(b)\ \ and\ 1\ P\ 2\ then\ d(0,a_1) < d(0,b_2) \quad \text{(S-2)}$$

The projection action of distances may be performed in different ways. At direct linear projection, rk and $r_k(m)$ for alternative m at k criterion are used according to Eq. (S-3):

$$d(0,m_k) = \frac{1}{2}[r_k + r_k(m)] \quad \text{(S-3)}$$

At indirect linear projection, projection distances from the origin are defined as Eq. (S-4):

$$d'(0,m_k) = \alpha r_k + (1-\alpha) r_k(m) \quad \text{(S-4)}$$

Eq. (S-5) is used at nonlinear projection with normalized weights of α and 1-α:

$$d''(0,m_k) = \sqrt[R]{(\alpha r_k^R + (1-\alpha) r_k(m)^R)} \quad \text{(S-5)}$$

The distance of d corresponding to the R values could be defined as Eq. (S-6) if Eq. (S-5) is considered to be used:

$$R = \begin{cases} -1 \rightarrow d'' = Geometric\ mean \\ 1 \rightarrow d'' = Weighted\ arithmetic\ mean \\ -\infty \rightarrow d'' = \min(r_k, r_k(m)) \\ 2 \rightarrow d'' = Average\ of\ squares \\ +\infty \rightarrow d'' = \max(r_k, r_k(m)) \end{cases} \quad \text{(S-6)}$$

Step B

In this step, using Besson's average ranking method R(m_k) rank is given to d(0,m_k) (D matrix) as shown in Eq. (S-7):

$$R(a_1) < R(a_2) \quad if \quad d(0,a_1) < d(0,b_2) \quad \text{(S-7)}$$

The obtained global ranks are ranged in the closed interval (Eq. (S-8)):

$$1 < R(m_k) < m_k \quad \text{(S-8)}$$

Step C

The global rank of each criterion is summed separately for all the alternatives (R matrix). The final aggregation for every alternative such as m is calculated by Eq. (S-9):

$$R(m) = \sum_{K=1}^{K} R(m_k) \quad \text{(S-9)}$$

Therefore, an incremental sequential structure based on R(m) is achieved, as presented by Eqs. (S-10) and (S-11):

$$R(a) < R(b) \ \ then\ a\ P\ b \quad \text{(S-10)}$$

$$R(a) = R(b) \ \ then\ a\ I\ b \quad \text{(S-11)}$$

In this structure, a better rank is given to the alternative with a smaller R(m).

S.2. MAPPAC technique

This multi objective ranking technique was first introduced by Matarazzo (1986) on the basis of a pairwise comparison of alternatives relative to each pair of criteria, defining preference and indifference relations. This technique is implemented in three following steps (Matarazzo, 1986; Martel and Matarazzo, 2005; Erdal Dincer, 2011):

Step A

In this technique, v_{ij} value describing the performance of αj relative to K_i is given to each K_i (C matrix). The quantitative weight of w_i is assigned to K_i describing its importance as presented in Eq. (S-12):

$$\sum_{i=1}^{n} w_i = 1 \quad \text{(S-12)}$$

Then, for each K_i a value function is created and to determine the value of $u(v_{ij})$ for each v_{ij} expression $0<u(v_{ij})<1$ is used.

Step B

Then, the basic preference criteria $\pi_{gh}(w_e, w_f)$ (P matrix) between each pair of alternatives (w_e and w_f) according to each pair of criteria (K_g and K_h) are calculated by Eq. (S-13):

$\pi_{gh}(w_e, w_f) =$

$$\begin{cases} 1 \; if \; v(v_{ge}) > v(v_{gf}) \; \cap \; v(v_{he}) > v(v_{hf}) \\ 0 \; if \; v(v_{ge}) < v(v_{gf}) \; \cap \; v(v_{he}) < v(v_{hf}) \\ \frac{1}{2} if \; v(v_{ge}) = v(v_{gf}) \; \cap \; v(v_{he}) = v(v_{hf}) \\ \frac{w_g(v(v_{ge})-(v_{gf}))}{w_g(v(v_{ge})-(v_{gf}))+w_h((v(v_{hf})-(v_{he}))} \; if \; \left(v(v_{ge}) > v(v_{gf}) \cap v(v_{he}) \leq v(v_{hf})\right) \cup \\ (v(v_{ge}) = v(v_{gf}) \cap v(v_{he}) < v(v_{hf})) \\ \frac{w_g(v(v_{he})-(v_{hf}))}{w_g(v(v_{gf})-(v_{ge}))+w_h((v(v_{he})-(v_{hf}))} \; if \; \left(v(v_{ge}) \leq v(v_{gf}) \cap v(v_{he}) > v(v_{hf})\right) \cup \\ (v(v_{ge}) < v(v_{gf}) \cap v(v_{he}) \geq v(v_{hf})) \end{cases} \quad \text{(S-13)}$$

Eqs. (S-14) and (S-15) are used to calculate π_{ef} and π_e for α_e:

$$\pi_{ef} = \sum_{i<j} \pi_{ij}(\alpha_e, \alpha_f) \frac{w_i + w_j}{m-1} \quad \text{(S-14)}$$

$$\pi_e = \sum_{\alpha_f \in A \setminus \alpha_e} \pi_{ef} \quad \text{(S-15)}$$

Step C

α_e associated with the biggest πe is chosen as the optimum alternative, and again πe is calculated as the best alternative. This is a repetitive process for ranking all the alternatives. Similarly, first the minimum of optimum alternative is chosen from A. Then, this alternative is removed from A and πe is recalculated. The remaining of α_e along with the smallest π_e is chosen as the best second alternative. Likewise, this process is repeated until total ranking of all alternatives. These ascending and descending rankings are combined to achieve a very weak ranking of A.

S.3. ELECTRE technique

ELECTRE technique was developed by Buchanan et al. (1999) in response to the deficiencies of decision making methods. ELECTRE technique is categorized as compensatory models, in which the exchange is carried out between the criteria, and an exchange in a criterion is compensated by another criterion or criteria in an opposite direction. This technique is also called as concordance

sets; so that all alternatives may be evaluated by outranking comparisons. The interaction with a decision maker is the advantage of this technique over some others. In other methods, the decision maker has no interference over the response after gathering the data and giving weights and the results are considered the criterion for decision making while in this method, the decision maker has the capability to deal with the analysis method and orienting it. The procedure steps of this technique are as follows (Tzeng and Huang, 2011):

Step A

In this step, the decision matrix is changed to a normalized decision matrix (N_D) using Euclidean norm according to Eq. (S-16):

$$N_D = [n_{ij}] \quad and \quad n_{ij} = \frac{r_{ij}}{\sqrt{\sum_{j=1}^{m} r_{ij}^2}} \tag{S-16}$$

Where r_{ij} is decision matrix elements, n_{ij} is the elements of the normalized decision matrix, and m is the number of alternatives. If the decision matrix includes criteria with negative or positive desirable aspects, then the measured values of negative criteria should get reversed for assimilating the calculation interpretation. Consequently, the bigger and smaller values of the decision matrix imply the more and less desirability of the criterion, respectively.

Step B

According to Eq. (S-17), the V matrix is the product of N_D matrix multiplied by V_{mn} diagonal matrix (the criteria weight) in which only the elements of main diagonal are non-zero.

$$V = N_D * W_{mn} = \begin{bmatrix} V_{11} & \cdots & V_{1n} \\ \vdots & \ddots & \vdots \\ V_{m1} & \cdots & V_{mn} \end{bmatrix} \tag{S-17}$$

where, m and n are criteria numbers and alternative numbers, respectively. The non-zero elements of the main diagonal of V_{mn} matrix are directly determined by the decision maker or the methods such as Shannon Entropy, Eigenvector, Weighted Least Square, and LINMAP.

Step C

In this step, all the alternatives are evaluated in pairs over all the criteria and concordance and discordance sets are formed according to Eqs. (S-18) and (S-19):

$$S_{KL} = \{j | r_{kj} \geq r_{1j}\} \quad j = 1, 2, 3, \ldots, m \tag{S-18}$$

$$D_{KL} = \{j | r_{kj} < r_{1j}\} \quad j = 1, 2, 3, \ldots, m \tag{S-19}$$

The concordance set S_{KL} is formed from all the criteria in which alternative K has desirability and preference over alternative L. Contrariwise, the discordance set D_{KL} includes the criteria in which alternative L is more desirable over alternative K.

Concordance matrix is a m×m matrix with non-element diagonal. Other elements are calculated based on weight summation for criteria of concordance sets of K and L (Eq. (S-20)):

$$I_{KL} = \sum_{j \in S_{K,L}} W_j \tag{S-20}$$

Concordance criteria I_{KL} reflects the relative importance of alternative K over L, such that $0<I_{KL}<1$.

Discordance matrix is also a m×m matrix with non-element diagonal. Non-diagonal elements are calculated using weighted normalized matrix V according to Eq. (S-21):

$$NI_{KL} = \frac{Max_{j \in D_{KL}} |V_{Kj} \quad V_{Lj}|}{Max_{j \in j} |V_{Kj} \quad V_{Lj}|} \tag{S-21}$$

Matrix NI_{KL} implies the ratio on lack of desirability for discordance sets of K and L to the whole discordance in criteria. It should be noted that, although existing data in I and NI obviously are different but complementing each other.

Step D

First, decision making threshold ($\bar{I}$) is defined. The threshold for concordance matrix indicates that decision maker prefers alternative K over L after a certain value. A conventional method for calculation of decision making threshold is averaging the concordance matrix using Eq. (S-22):

$$\bar{I} = \sum_{K=1}^{m} \sum_{L=1}^{m} \frac{I_{KL}}{m(m\ \ 1)} \quad \text{(S-22)}$$

If the elements of matrix $\bar{I}$ are bigger than the threshold, that unit element in matrix F (effective concordance matrix) could take 1 (Eq. (S-23)), otherwise, it could take 0 (Eq. (S-24)):

$$If\ \ I_{KL}\ \geq\ \bar{I}\ \ \rightarrow\ \ F_{KL} = 1 \quad \text{(S-23)}$$

$$If\ \ I_{KL}\ <\ \bar{I}\ \ \rightarrow\ \ F_{KL} = 0 \quad \text{(S-24)}$$

F is a non-element diagonal matrix so, each unit element implies an efficient and dominant alternative over one another.

This matrix is measured by the threshold value of $N\bar{I}$. To calculate $N\bar{I}$, discordance matrix is averaged according to Eq. (S-25):

$$N\bar{I} = \sum_{K=1}^{m} \sum_{L=1}^{m} \frac{NI_{KL}}{m(m\ \ 1)} \quad \text{(S-25)}$$

If each element in $N\bar{I}$ is bigger than the threshold, that unit element in matrix G (effective discordance matrix) could take 0 (Eq. (S-26)), otherwise, it could take 1 (Eq. (S-27)):

$$If\ \ NI_{KL} >\ \ N\bar{I}\ \ \rightarrow\ \ G_{KL} = 0 \quad \text{(S-26)}$$

$$If\ \ NI_{KL}\ \leq\ N\bar{I}\ \ \rightarrow\ \ G_{KL} = 1 \quad \text{(S-27)}$$

Step E

Matrix H is composed of common elements of F and G matrices, according to Eq. (S-28):

$$H_{KL} = F_{KL} * G_{KL} \quad \text{(S-28)}$$

This matrix implies the order of relative preferences of alternatives. It means that, if H_{KL}=1, alternative K is a preference over L, both in terms of concordance and discordance criteria.

Step F

K is an effective alternative, if at least for one L, $H_{KL}=1$ and for all Ls, $H_{KL}=0$. The simultaneous observation of these two clauses is likely to be low probable, so that the effective alternative could be determined directly. Each column of matrix H with at least a one-unit element is eliminated because that column is under the preference of row(s). Therefore, the preference and the desirable alternative are accompanied with a column at most having 0 or with a row at least having 1.

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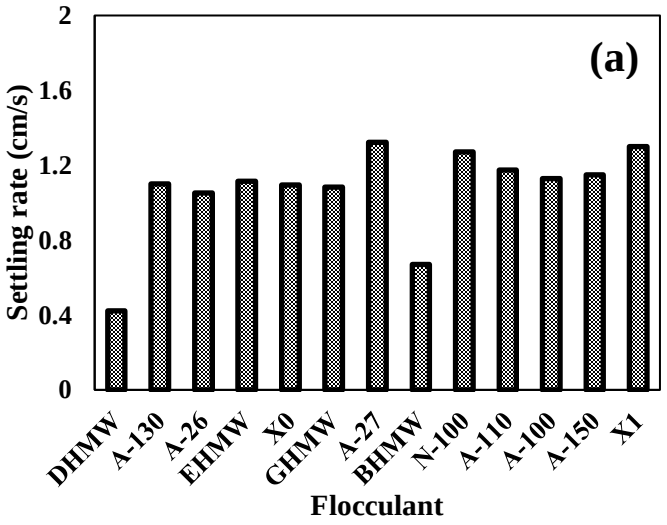
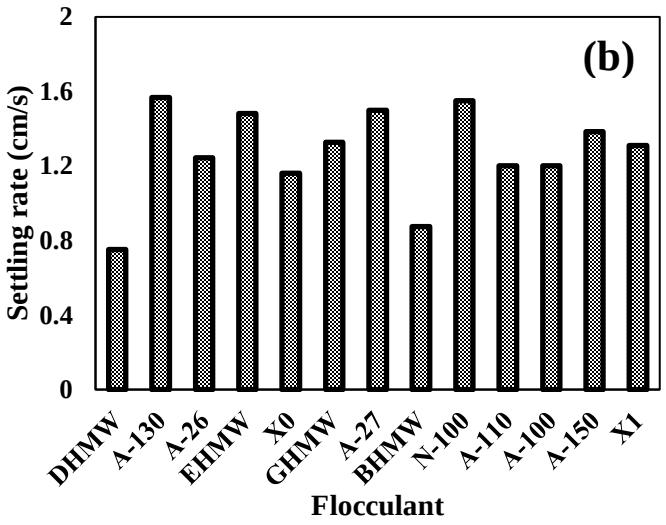
R=

	X1	**X2**	**X3**	**X4**	**X5**
A1=DHMW	2	0.95	2.99	296	31.33
A2=A130	2	0.45	7.47	78.22	28.41
A3=A26	2	0.59	7.43	154.9	32.33
A4=EHMW	2	0.95	7.87	268.5	31.84
A5=X0	2	0.70	7.72	160.5	31.48
A6=GHMW	2	0.95	7.65	208.4	30.45
A7=A27	2	0.65	9.34	36.7	32.62
A8=BHMW	2	0.95	4.74	326.9	32.11
A9=N100	2	0.49	8.98	235	29.71
A10=A110	2	0.62	8.28	251.7	31.2
A11=A100	2	0.59	7.98	276.3	30.9
A12=A150	2	0.76	8.12	83.47	31.2
A13=X1	2	0.70	9.18	120.1	31.8
A14=DHMW	4	1.89	5.3	230.9	31.16
A15=A130	4	0.90	9.64	58.49	29.24
A16=A26	4	1.19	8.77	46.52	33.02
A17=EHMW	4	1.89	10.45	90.8	31.89
A18=X0	4	1.41	8.2	74.67	31.79
A19=GHMW	4	1.89	9.37	155.8	31.39
A20=A27	4	1.30	10.59	35.04	33.51
A21=BHMW	4	1.89	6.18	257	32.05
A22=N100	4	0.98	10.95	180	30.08
A23=A110	4	1.24	8.47	128.9	32.17
A24=A100	4	1.19	8.48	184	30.97
A25=A150	4	1.51	9.76	47.74	31.83
A26=X1	4	1.41	9.25	41.03	32.51
A27=DHMW	5	2.36	6.87	108.4	30.75
A28=A130	5	1.12	11.97	27.75	29.44
A29=A26	5	1.49	9.07	30	32.57
A30=EHMW	5	2.36	11.05	31.17	31.34
A31=X0	5	1.76	9.7	60.9	32.33
A32=GHMW	5	2.36	9.94	43.67	31.49

A33=A27	5	1.62	9.44	27.79	34.17
A34=BHMW	5	2.36	6.97	230	31.44
A35=N100	5	1.23	11.92	75.12	28.49
A36=A110	5	1.55	9.3	51.89	32.39
A37=A100	5	1.49	9.8	91.47	31.61
A38=A150	5	1.89	10.39	45.29	32.61
A39=X1	5	1.76	11.72	29.53	33.08
A40=DHMW	7	3.31	8.75	54.9	32.02
A41=A130	7	1.57	12.3	26.89	28.82
A42=A26	7	2.08	11.48	17.51	32.96
A43=EHMW	7	3.31	11.3	5	32.8
A44=X0	7	2.46	11.33	31.45	31.42
A45=GHMW	7	3.31	11.51	15.13	30.86
A46=A27	7	2.27	11.19	10.7	33.89
A47=BHMW	7	3.31	6.99	184.8	31.71
A48=N100	7	1.72	13.34	55	29.51
A49=A110	7	2.18	12.37	32.38	32.26
A50=A100	7	2.08	10.92	34.42	32.46
A51=A150	7	2.65	10.59	10.72	33.1
A52=X1	7	2.46	11.29	14.9	33.54





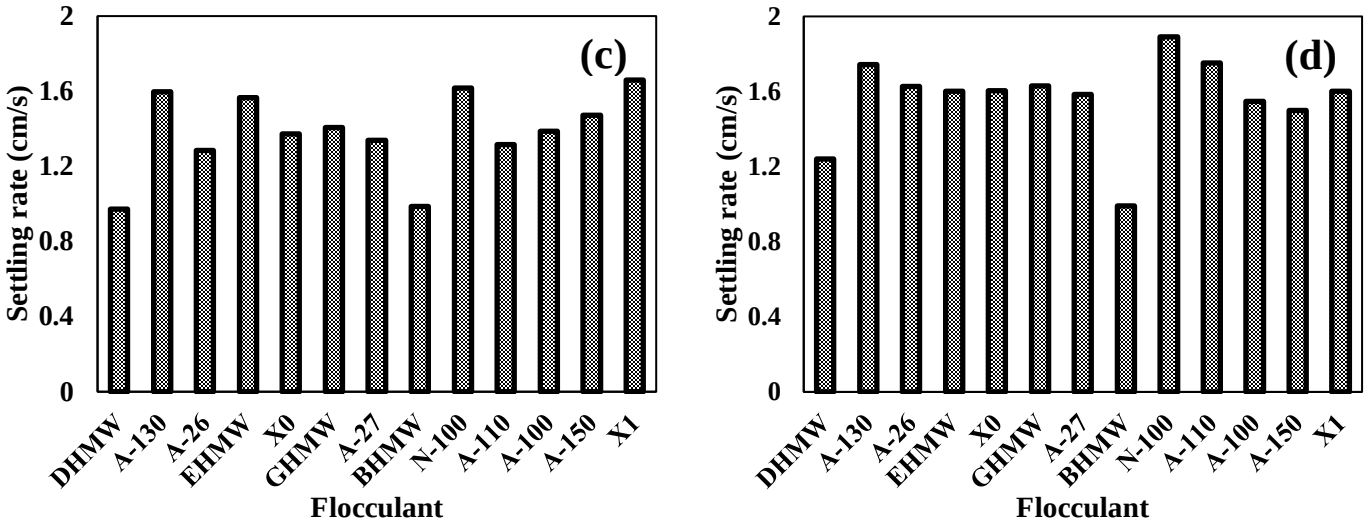


Fig. S-1. Effect of different dosages and types of flocculants on settling rate; (a) 2, (b) 4, (c) 5, and (d) 7 g/t.

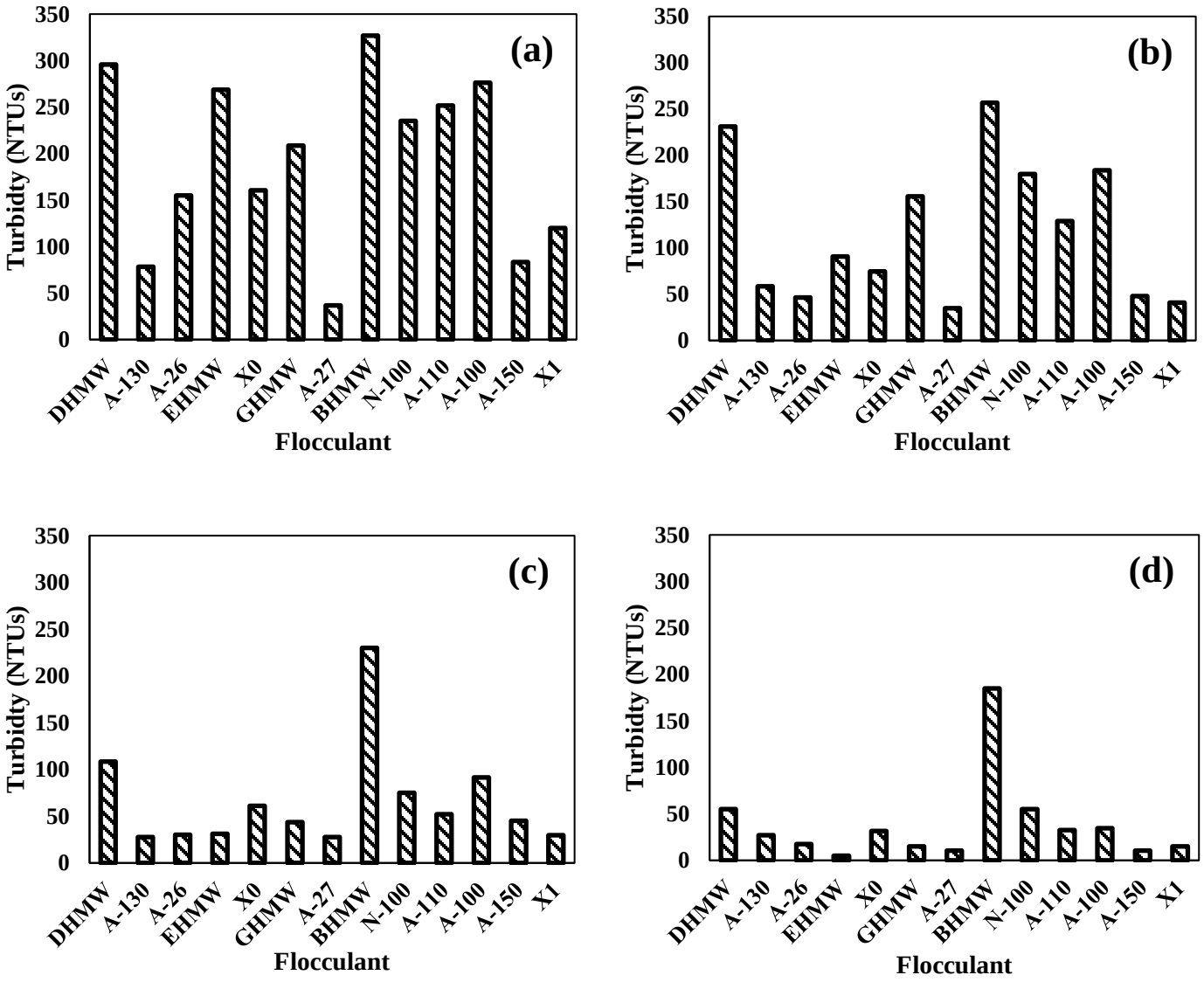


Fig. S-2. Effect of different dosages and types of flocculants on turbidity; (a) 2, (b) 4, (c) 5, and (d) 7 g/t.

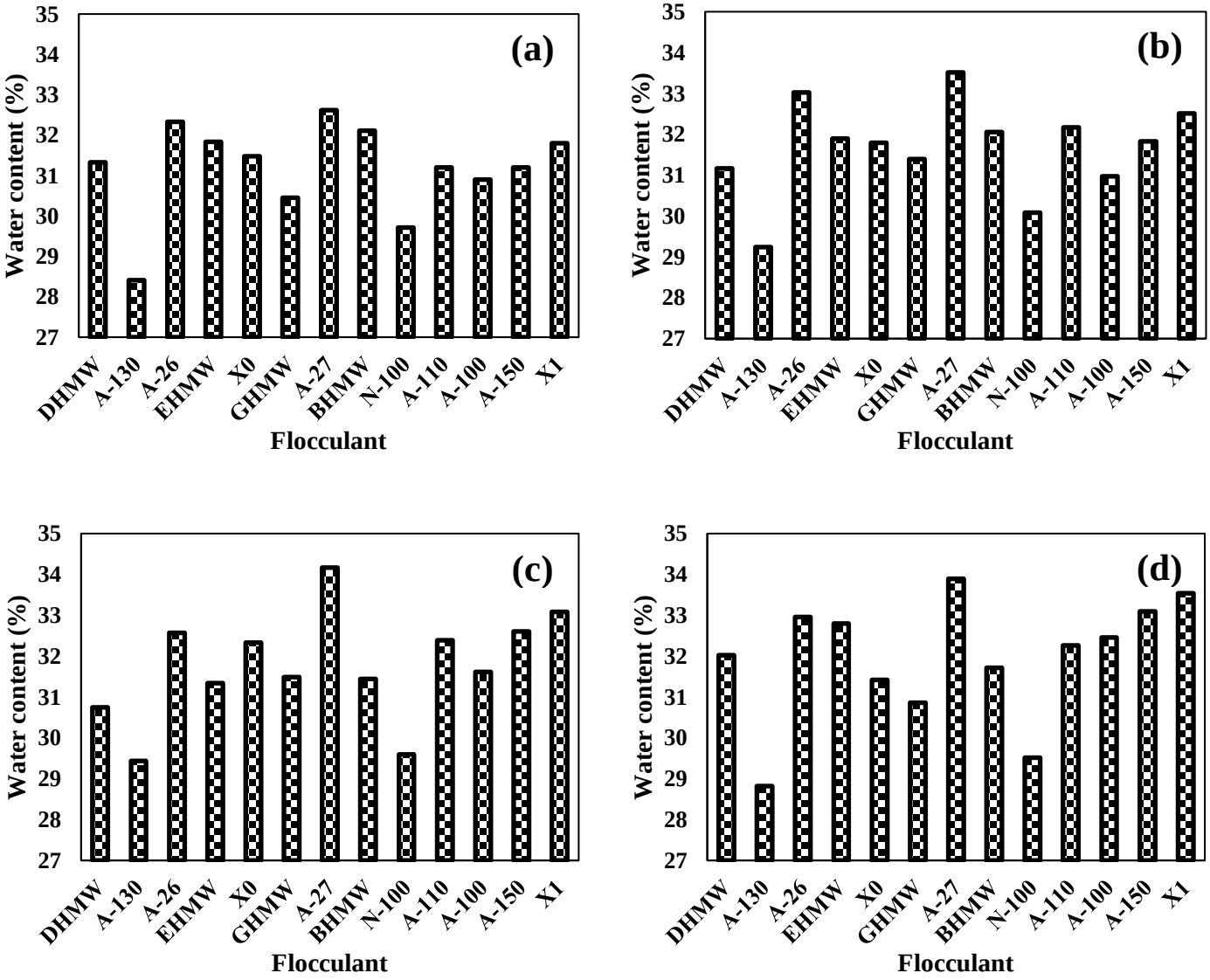


Fig. S-3. Effect of different dosages and types of flocculants on water content; (a) 2, (b) 4, (c) 5, and (d) 7 g/t.