Supplement to "Managing Component Degradation in Series Systems for Balancing Degradation Through Reallocation and Maintenance"

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S.1 Stochastic Response Surface Method

We rewrite the stochastic response surface method proposed in Regis and Shoemaker (2007) in the context of our problem in Algorithm S.1. To emphasis that the long-run average operational cost C is a function of the reallocation and replacement thresholds (L, M) and it is evaluated from simulations, we write it in Algorithm S.1 as $\hat{C}(L, M)$. Recall that for a fixed combination of (L, M), the simulation is replicated for m times. The estimated cost is then computed by Equation (12) (known degradation rate) or (13) (unknown degradation rate) in the main text. If the stochastic kriging model is used as the response surface, we use the two-stage estimation method in Ankenman et al. (2010) for fitting the nm simulation outputs, where we assume a Gaussian correlation structure as with Zhang et al. (2016). If the radial basis function (RBF) is used as the response surface, we use the method in Regis and Shoemaker (2007) for the data fitting.

At step 4 of Algorithm S.1, we need to give a score to each candidate point in Ω_n to select which points being further evaluated from simulations. In this study, we follow the procedure in Regis and Shoemaker (2007) to evaluate a candidate point from two criteria. First, we compute $s_n^{\max} = \max\{s_n(L, M) : (L, M) \in \Omega_n\}$ and $s_n^{\min} = \min\{s_n(L, M) : (L, M) \in \Omega_n\}$. For each $(L, M) \in \Omega_n$, let

$$V_n^{(1)} = \begin{cases} \frac{s_n(L,M) - s_n^{\min}}{s_n^{\max} - s_n^{\min}}, & s_n^{\max} \neq s_n^{\min} \\ 1, & \text{otherwise.} \end{cases}$$

It estimates the function value of candidate points from the fitted response surface. A smaller value of $V_n^{(1)}$ implies that the candidate point may yield a smaller value of $C(L, M)$. Meanwhile, we compute $\Delta_n(L, M) = \min_{i=1,\cdots,n} ||(L, M) - (L_i, M_i)||$ for each candidate in Ω_n, where $||\cdot||$ denotes the Euclidean norm. The function $\Delta_n(L, M)$ determines the minimum distance from previously evaluated points, which we want to maximize in order to promote the global search (Regis and Shoemaker, 2007). Let $\Delta_n^{\max} = \max\{\Delta_n(L, M) : (L, M) \in \Omega_n\}$ and $\Delta_n^{\min} = \min\{\Delta_n(L, M) : (L, M) \in \Omega_n\}$. For each $(L, M) \in \Omega_n$, also let

$$V_n^{(2)} = \begin{cases} \frac{\Delta_n^{\max} - \Delta_n(L,M)}{\Delta_n^{\max} - \Delta_n^{\min}}, & \Delta_n^{\max} \neq \Delta_n^{\min} \\ 1, & \text{otherwise.} \end{cases}$$

Then the score $\mathcal{W}^{(n)}$ for each $(L, M) \in \Omega_n$ is expressed as a weighted sum, which is given by

$$\mathcal{W}^{(n)} = w_n V_n^{(1)} + (1 - w_n) V_n^{(2)}, \tag{S.1}$$

where w_n is a weight whose value may be change at each iteration.

In Sections 6.1 and 6.2 of the main text, we set $n_0 = 10$, $k = 2,000$, $n_{\max} = 1,010$, and $\alpha = 0.01$ for both numerical examples. As with Regis and Shoemaker (2007), we use a weight pattern of $\langle 0.2, 0.4, 0.6, 0.9, 0.95, 1\rangle$ so that the value of w_n is cycled within the six numbers. In Section 6.1 of the main text, the initial evaluation points for both the stochastic kriging and the RBF model are kept the same in each replication to stipulate the fair comparison. In the Monte Carlo simulations, we replicate for $m = 10$ times for each fixed combination of (L, M) and compute their mean to approximate the long-run average operational cost. For all replications, the time horizon of the simulation is set as $H = 1,000$. In Section 6.2 of the main text, we also conduct a numerical study assuming the degradation rate $\boldsymbol{\mu}$ is unknown. In this case, we realize $B = 50$ samples of $\boldsymbol{\mu}$ based on its posterior distribution in each replication of the simulations.

S.2 Supplementary Numerical Results

S.2.1 Supplementary Results for the Tire System

In Section 6.1 of the main text, we investigate the reallocation policy of the tire system of a forward-wheel drive (FWD) automobile. We set the reallocation, the preventive maintenance (PM), and the corrective maintenance (CM) costs as $c_{\mathrm{a}} = 1$, $c_{\mathrm{p}} = 10$, and $c_{\mathrm{c}} = 100$, respectively. The degradation rates associated with the slots are set as $\mu_1 = 2$ and $\mu_2 = 0.5$ to represent the inherent imbalanced workload. The correlation coefficient between degradation

Algorithm S.1 The stochastic response surface method to find the optimal reallocation and replacement thresholds.

Input: Degradation rate $\boldsymbol{\mu}$, covariance matrix $\boldsymbol{\Sigma}$, inspection interval δ, failure threshold D; initial number of evaluated points n_0, number of candidates point k, maximum number of evaluations $n_{\max}$, proportion of candidate points to be evaluated from simulations α.

1. Select n_0 combinations of maintenance thresholds, denoted as (L_i, M_i), $i = 1, \cdots, n_0$. The initial points are selected from the symmetric Latin hypercube design (Ye et al., 2000).
2. Evaluate the long-run average operational cost $\hat{C}(L_i, M_i)$ from Monte Carlo simulations. Set $n := n_0$ and $\mathcal{A}_n := \{(L_i, M_i), i = 1, \cdots, n\}$. Let $(L_n^\star, M_n^\star) = \arg\min_{(L,M)\in\mathcal{A}_n} \hat{C}(L, M)$ be the current optimal solution.
3. Use the a response surface model (e.g., stochastic kriging) to fit the simulation outputs, where the fitted response surface is denoted as $s_n(L, M)$.
4. Uniformly generate k random candidate points $\Omega_n = \{(L_i^{(n)}, M_i^{(n)}), i = 1, \cdots, k\}$. Give a score $\mathcal{W}_i^{(n)}$, $i = 1, \cdots, k$, for each candidate point in Ω_n by Equation (S.1).
5. Select $\lceil \alpha k \rceil$ candidate points that have the lowest score $\mathcal{W}^{(n)}$, $0 < \alpha < 1$. Run the simulation to evaluate the long-run average operational cost C for these points. Update $n := n + \lceil \alpha k \rceil$, the set of the evaluated points $\mathcal{A}_n$, and the current optimal solution $(L_n^\star, M_n^\star)$.
6. Stop the iteration if $n > n_{\max}$. Otherwise, go back to Step 3.

Output: Optimal reallocation and replacement thresholds $(L^\star, M^\star) := (L_n^\star, M_n^\star)$.

Table S.1: The cost $C(L, M)$ as a function of L and M (corresponding to Figure 3 in the main text). Note that the reallocation is performed only when $L < M$.

L \ M	1	3	5	7	9	11	13	15
1	9.608	4.502	2.893	2.185	1.790	1.541	2.103	9.000
3	–	4.941	3.263	2.060	1.628	1.413	1.912	8.931
5	–	–	3.367	2.535	1.768	1.310	2.351	9.437
7	–	–	–	2.538	2.039	1.607	1.625	8.850
9	–	–	–	–	2.037	1.700	2.187	10.289
11	–	–	–	–	–	1.700	3.225	12.597
13	–	–	–	–	–	–	3.235	12.762
15	–	–	–	–	–	–	–	12.762

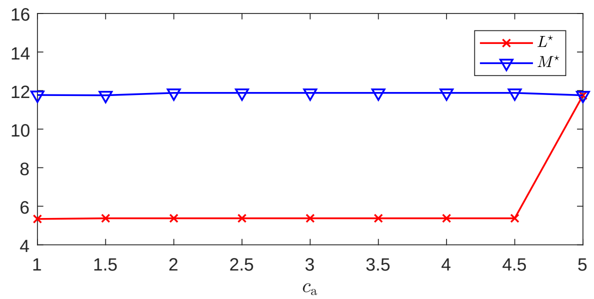


Figure S.1: The sensitivity of the optimal reallocation and replacement thresholds $(L^\star, M^\star)$ to the reallocation cost c_a, where $\mu_1/\mu_2 = 4$.

of the two tires is $\rho = 0.6$. Table S.1 summarizes the results corresponding to Figure 3 in the main text.

We first examine the sensitivity of the optimal control limits $(L^\star, M^\star)$ to the reallocation cost c_a and the degradation rate $\boldsymbol{\mu} = [\mu_1, \mu_2]'$. In the sensitivity analysis, the high and the low levels of c_a are set as 5 and 0.2, respectively. Meanwhile, we fix $\mu_2 = 0.5$ and set μ_1/μ_2 as 8 and 1, respectively, for the high and the low levels. Table S.2 summarizes the change of the optimal control limits $(L^\star, M^\star)$ with c_a and $\boldsymbol{\mu}$. We can see that when the degradation rate associated with the component slot is equal to each other (i.e., $\mu_1/\mu_2 = 1$), the reallocation threshold L is high. It implies that it is highly possible that we *never* reallocate the components with *balanced* degradation. On the other hand, the PM control limit M decreases in the ratio μ_1/μ_2 since the series structure of the system. Meanwhile, it is intuitive to see that the reallocation threshold L increases in the reallocation cost c_a. However, the optimal PM control limit M only slightly decreases to the reallocation cost.

Note that the reallocation threshold L is close to the replacement threshold M when $c_a = 5$ and $\mu_1/\mu_2 = 4$, meaning that it is nearly impossible to perform the reallocation before a system replacement. We further conduct a sensitivity analysis to examine the change of L to c_a with $\mu_1/\mu_2 = 4$ fixed. The result is shown in Figure S.1. Interestingly, the reallocation threshold L keeps approximately constant when c_a ranges from 1 to 4.5, implying that the reallocation is necessary in these cases. Then value of L jumps quickly when $c_a = 5$, which may mean that there is a threshold such that the reallocation is unnecessary when the reallocation cost is larger than this value.

As mentioned in Section 3.1 of the main text, the proposed maintenance policy for the two-component system does not consider the preventive replacement at the component level. It is because degradation of the two component cannot vary significantly upon replacement with reallocation. To illustrate this phenomenon, we can calculate the probability $P(\min\{z_1, z_2\} > 0.8M | \max\{z_1, z_2\} > M)$ based on the stationary distribution $\pi(z_1, z_2)$ in

Table S.2: The sensitivity of the optimal control limits $(L^\star, M^\star)$ to the reallocation cost c_a and the ratio of the degradation rate μ_1/μ_2.

c_a \ μ_1/μ_2	1	4	8
0.2	$(2.38, 13.26)$	$(5.34, 11.77)$	$(0, 10.44)$
1	$(12.75, 13.11)$	$(5.34, 11.77)$	$(4.88, 10.01)$
5	$(12.75, 13.13)$	$(11.75, 11.75)$	$(5.06, 10.13)$

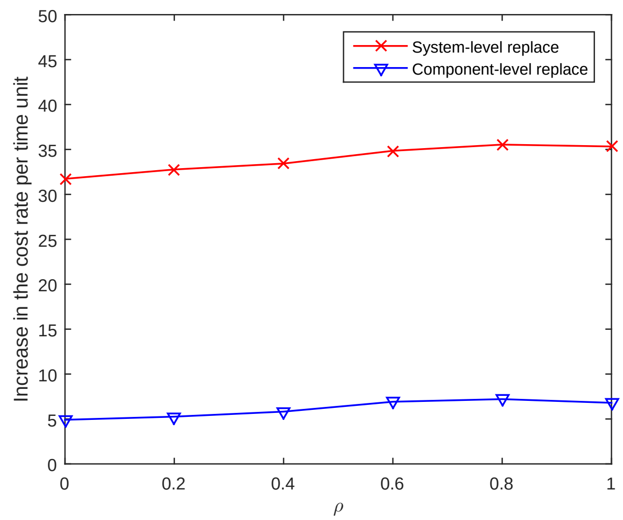


Figure S.2: The increase (in %) of the long-run average operational cost with the passive replacements only, when the correlation coefficient ρ changes. Here, $c_\mathrm{a} = 1$ and $\boldsymbol{\mu} = [2, 0.5]'$.

Equation (4) of the main text, which is 0.9166 in this numerical study. In practice, it is not recommended to replace the component frequently due to the high setup cost for replacement. Numerical comparisons provided in Section 6.1 of the main text and later in this section will further support this argument

Specifically, we have compared the proposed policy with two maintenance policies with passive replacements only in Section 6.1 of the main text. The result shows that the proposed method gives the lowest maintenance cost when $c_\mathrm{a} = 1$, $\mu_1/\mu_2 = 4$, and $\rho = 0.6$. Here, we change the parameter setting and further conduct a comprehensive comparison between these policies. First, we change the value of ρ from $0, 0.2, \cdots,$ to 1, and compare the increase of the long-run average operational cost without the reallocation. Figure S.2 shows that when the degradation is imbalanced and the reallocation cost is lower than the setup cost for replacement, the proposed method outperforms the pure replacement policies for different values of ρ. We then change the values of c_a and μ_1/μ_2. Table S.3 shows the corresponding results of comparison. We can see that when balanced degradation rates ($\mu_1 = \mu_2$) exist, the policy using component-level replacement has the highest maintenance cost. This is because under this policy, the frequency of replacement would be quite high when the degradation of the two components are *balanced*. Generally, the proposed outperforms the pure replacement policies when the reallocation cost c_{a} is relatively low. Recall that we set the setup cost for replacement as $c_{\mathrm{s}} = 3$ in the main text. When $c_{\mathrm{a}} = 5 > c_{\mathrm{s}}$, we may not need to use the reallocation to balance the component degradation. Nevertheless, c_{a} is usually smaller than c_{s} in reality as discussed in Section 6.1 of the main text.

Table S.3: The increase/decrease (in %) of the optimal long-run average operational cost with replacements only compared with the one from the reallocation strategy, when the reallocation cost c_{a} and the ratio of degradation rates μ_1/μ_2 change.

c_{a} \ μ_1/μ_2	1	4	8
0.2	$(1.81, 19.08)^*$	$(44.43, 14.51)$	$(50.72, 7.17)$
1	$(1.55, 18.78)$	$(34.85, 6.95)$	$(50.42, 6.92)$
5	$(0.25, 18.17)$	$(1.04, -20.51)$	$(3.78, -26.21)$

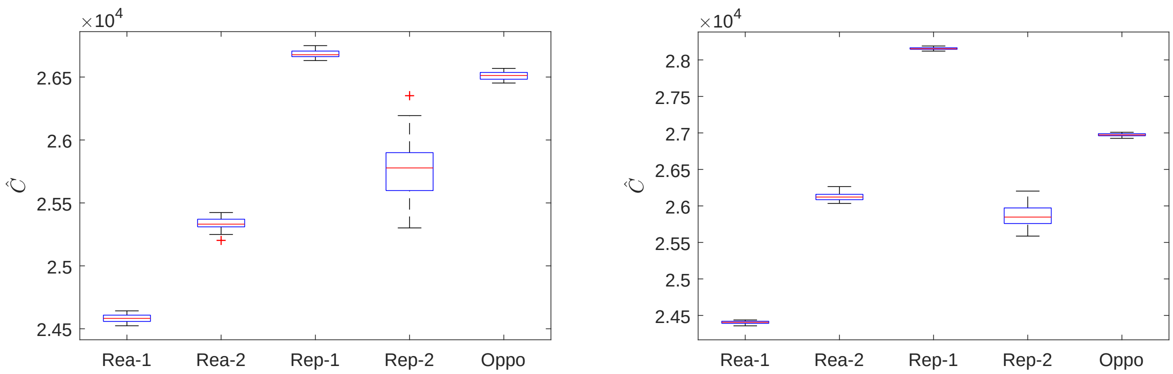
* Each entry in the parenthesis denotes the corresponding change of cost from the system-level and the component-level replacement policies introduced in Section 6.1 of the main text, respectively.

Finally, we verify the conclusion in Section 6.1 that the optimal control limit essentially manages the component degradation with the number of reallocation actions as less as possible under different parameter settings. Similarly, we examined this conclusion by changing the values of ρ, c_{a}, and μ_1/μ_2. We record the number of reallocation before the system replacement and compute the ratio of replications where only one reallocation action is performed to balance the component degradation. When ρ changes from $\{0, 0.2, \cdots, 1\}$, the corresponding ratios are 99.65%, 99.86%, 99.89%, 99.93%, 99.98%, and 99.99%, respectively. The results are insensitive to the change of ρ, again indicating the proposed model is robust to the bivariate normal assumption in the two-component case. When c_{a} and μ_1/μ_2 change, the results are shown in Table S.4. It is reasonable to see that when the degradation rates are balanced ($\mu_1 = \mu_2$), we never perform the reallocation before the system replacement. On the other hand, when $\mu_1/\mu_2 = 8$ and $c_{\mathrm{a}} = 0.2$, the ratio is also low for replications where only one reallocation action is performed. This is because when the degradation rates are extremely imbalanced and the reallocation cost is very low, it does not cost too much to perform two or more reallocation actions before the replacement. Meanwhile, when the reallocation cost is very high ($c_{\mathrm{a}} = 5$), we may also never perform the reallocation. For all the rest parameter settings, the previous conclusion holds that the reallocation strategy manages to balance the degradation with the reallocation actions as less as possible.

Table S.4: The ratio (in %) of simulation replications where we only perform one reallocation action before the system replacement, when c_a and μ_1/μ_2 change.

c_a \ μ_1/μ_2	1	4	8
0.2	24.53*	99.96	0.38**
1	0*	99.93	98.93
5	0*	0*	99.06

* No reallocations most time.
** More than one reallocations most time.



(a) (b)

Figure S.3: The box plot of the estimated optimal long-run average operational cost $\hat{C}$ under different maintenance policies. (a) $\sigma_i/\mu_i = 1/6$ and (b) $\sigma_i/\mu_i = 1/9$.

S.2.2 Supplementary Results for the Battery System

In Section 6.2 of the main text, we investigate the reallocation policy of the battery system of a fleet of $N = 5$ hybrid-electric vehicles. First, we assume the degradation rate $\boldsymbol{\mu} = [\mu_1, \cdots, \mu_5]'$ associated with each vehicle is known. The ratio of the degradation volatility to the degradation rate σ_i/μ_i is fixed for $i = 1, \cdots, 5$ and the results for $\sigma_i/\mu_i = 1/6$ is provided in the main text. Here, we provide the results when σ_i/μ_i equals 1/3 and 1/9, respectively. Figures S.3 shows the corresponding box plot of the estimated optimal long-run average operational cost $\hat{C}$ under the four different maintenance policies, where the reallocation policy uses two versions of $\phi(\cdot)$ shown in (10) and (11), respectively, in the main text. We can also see that the proposed reallocation strategy with $\phi(\cdot)$ specified in (10) of the main text outperforms the policies with passive replacements only under different parameter settings.

We then investigate the performance of the model when the true degradation rate associated with each hybrid-electric vehicle deviates from the nominal value. In the main text, we have presented the results when $\lambda_i/\omega_i = 1/3$ for all $i = 1, ., .., 5$. Here, the results when

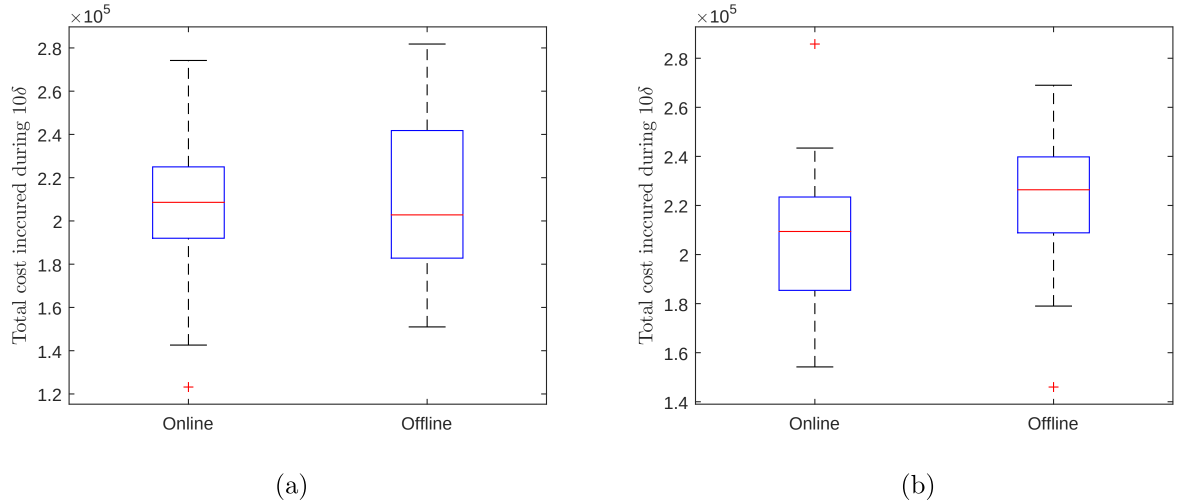


Figure S.4: The box plot of the total cost incurred during the simulation time horizon with and without considering the uncertainty in the degradation rates. (a) $\lambda_i/\omega_i = 1/6$; (b) $\lambda_i/\omega_i = 1/9$.

λ_i/ω_i equals 1/6 and 1/9 are both provided in Figure S.4.

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