Supplemental material

Sustainable Supplier Selection and Order Allocation: A Fuzzy Approach

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1.0 Supplementary literature review

Table 1 summarizes the literature on sustainable supplier selection. Table 2 summarizes the recent studies on supplier selection that have considered risk in the problem. Although some studies consider risk in supplier selection, the concurrent consideration of sustainable supplier selection, risk, and inflation has not been investigated.

Table 1 Summary of sustainable supplier selection literature

Author	year	Economic	Environmental	social	Order allocation
P.K. Humphreys et al.	2003	√	√	×	×
Bai and Sarkis	2009	√	√	√	×
kozkan and Cifci	2010	√	√	√	×
Foerstl et al.	2010	√	√	×	×
Leppelt et al.	2011	√	√	√	×
Goebel et al.	2012	×	√	√	×
Azadnia et al.	2012	√	√	√	×
Amindoust et al.	2012	√	√	√	×
Govindan et al.	2012	√	√	√	×
Igarashi et al.	2013	√	√	×	×

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Azadnia et al.	2013	√	√	√	×
Öztürk and Özçelik	2014	√	√	√	×
Azadnia et al.	2014	√	√	√	√
Ifeyinwa et al.	2014	√	√	√	×
Sarkis et al.	2014	√	√	√	×
Ghadimi and Heavey	2014	√	√	√	×
Tavana et al.	2015	√	√	√	×
Kaur et al.	2016	√	√	√	×
Zimmer et al.	2016	√	√	√	×
Orji and Wei	2015	√	√	×	×
Awasthi	2015	√	√	√	×
Luthra et al.	2017	√	√	√	×
Zhou et al.	2016	√	√	×	×
Shalke et al.	2018	√	√	√	×
Fallahpour et al.	2017	√	√	√	×
Kannan	2017	√	√	√	√
Awasthi et al.	2018	√	√	√	×
Ghadimi et al.	2018	√	√	√	√
Rashidi and Cullinane	2018	√	√	√	×
Arabsheybani et al.	2018	√	√	√	√
Liu et al.	2018	√	√	√	×
Kannan	2018	√	√	√	×
Xu et al.	2019	√	√	√	×
Ghadimi et al.	2019	√	√	√	×

Moheb-alizadeh and Handfield	2019	√	√	√	×
Pishchulov et al.	2019	√	√	√	×
Mohammed et al.	2019	√	√	√	√

Table 2 Recent studies on supplier selection and order allocation with risk

Author	Single-objective	Bi- objective	Multi-objective	Risk	Inflation	Fuzzy Approach
Kannan et al .(2013)	×	×	√	×	×	√
Sheikhalishahi and Torabi (2014)	×	×	√	√	×	√
Bakeshlou et al.(2014)	×	×	√		×	√
Azadnia et al.(2014)	×	×	√	×	×	√
Sarkis et al.(2014)	×	×	√	×	×	√
Gold et al.(2015)	×	×	×	√	×	√
Nekooie et al. (2015)	×	×	√	√	×	×
Zimmer et al (2015)	×	×	×	×	×	
Kaur et al.(2015)	√	×	×	×	×	×
Orji et al (2015)	×	×	×	×	×	√
Pramanik et al (2016)	×	×	×	×	×	√
Yazdani et al (2016)	×	×	√	×	×	×
Song et al.(2016)	×	×	×	√	×	×
Mavi et al.(2016)	×	×	√	√	×	√
Azadnia.(2016)	×	×	√	×	√	×
Hamdan et al.(2016)	×	×	√	×	×	√
Wan et al.(2016)	×	×	×	√	×	√
Shalke et al.(2016)	×	×	√	×	×	
Zhou et al.(2016)	×	×	√	×	×	√
Tamošaitienė et al.(2017)	×	×	√	×	×	×
Vahidi et al (2017)	×	√	×	√	×	
Yoon et al.(2017)	×	×	√	√	×	×
Suprasongsin et al.(2017)	×	×	√	√	×	√
Zimmer et al.(2017)	×	×	×	√	×	√
Khojasteh-Ghamari et al. (2017)	√	×	×	√	×	×
Turk et al.(2017)	×	×	√	√	×	√
Awasthi et al (2017)	×	×	×	√	×	√
Dupont et al.(2017)	√	×	×	√	×	×
Liming Yao(2017)	×	×	×	×	×	√
Gupta et al .(2017)	×	×	×	×	×	√
Moheb-Alizadeh et al.(2017)	×	×	√	×	×	√
Fallahpour et al.(2017)	×	×	×	×	×	√
Kannan (2017)	×	×	×	×	×	√
Wang et al.(2018)	×	×	×	√	×	√
Taleizadeh et al .(2018)	×	×	√	×	×	√
Cheraghalipour et al.(2018)	×	√	×	√	×	×
Arabsheybani et al.(2018)	×	×	√	√	×	√
Paksoy et al.(2019)	×	×	×	√	×	√
Habibi et al.(2019)	×	×	√	×	√	√
Moheb-Alizadeh et al.(2019)	×	×	√	×	×	√
Alikhani et al. (2019)	√	×	×	√	×	√

2.0 Complexity and generalization of the developed model

The model presented in this research is developed based on the previous literature. The model includes six different objective functions and some constraints. Table 3 shows the different components of the model. Mathematical models are typically investigated based on a literature review and developed based on specific needs of companies, researchers, practitioners, managers, and so on. A model can be applied to a specific case study and provide feasible and sensible results. However, generalization should be addressed in the development of the model. A model can be applied as a benchmark when it is able to work with different sets of data and different sizes of problem. To address the issue of generalization, the model is applied to a large scale case study to investigate its applicability and find the solution time. The results of the large scale case study are shown in the Section 6 of this supplementary material.

Table 3 Quantitative and computational specifications of the model

Number of decision variables	7
Number of constraints	12 constraints including n 14 constraints including j 16 constraints including t 4 constraints including q 2 constraints including r 1 constraint including b 1 constraint including α
Number of binary variables	3 binary variables
Operating system features	Processor: Intel ® Core ™ i3 CPU Installed memory (RAM): 4.00 GB (3.87 GB usable) System Type: 64-bit Operating System
CPU time	The discussed model: less than 2 minutes The large-scale model: 23-27 minutes
Software	LINGO 11 software

The model provides a feasible and optimal solution in a reasonable time (less than 2 minutes as shown in Table 3). To consider a model as a benchmark, it is necessary to check its generalization condition. Based on the outputs from the large-scale case study and the CPU time, the model also provides a feasible and optimal solution in a reasonable time (23-27 minutes). Figure 1 shows the CPU usage of the operating system in the process of solving both models (small- and large-scale). The results of the large scale application can be seen in the Section 6 of this supplementary material.

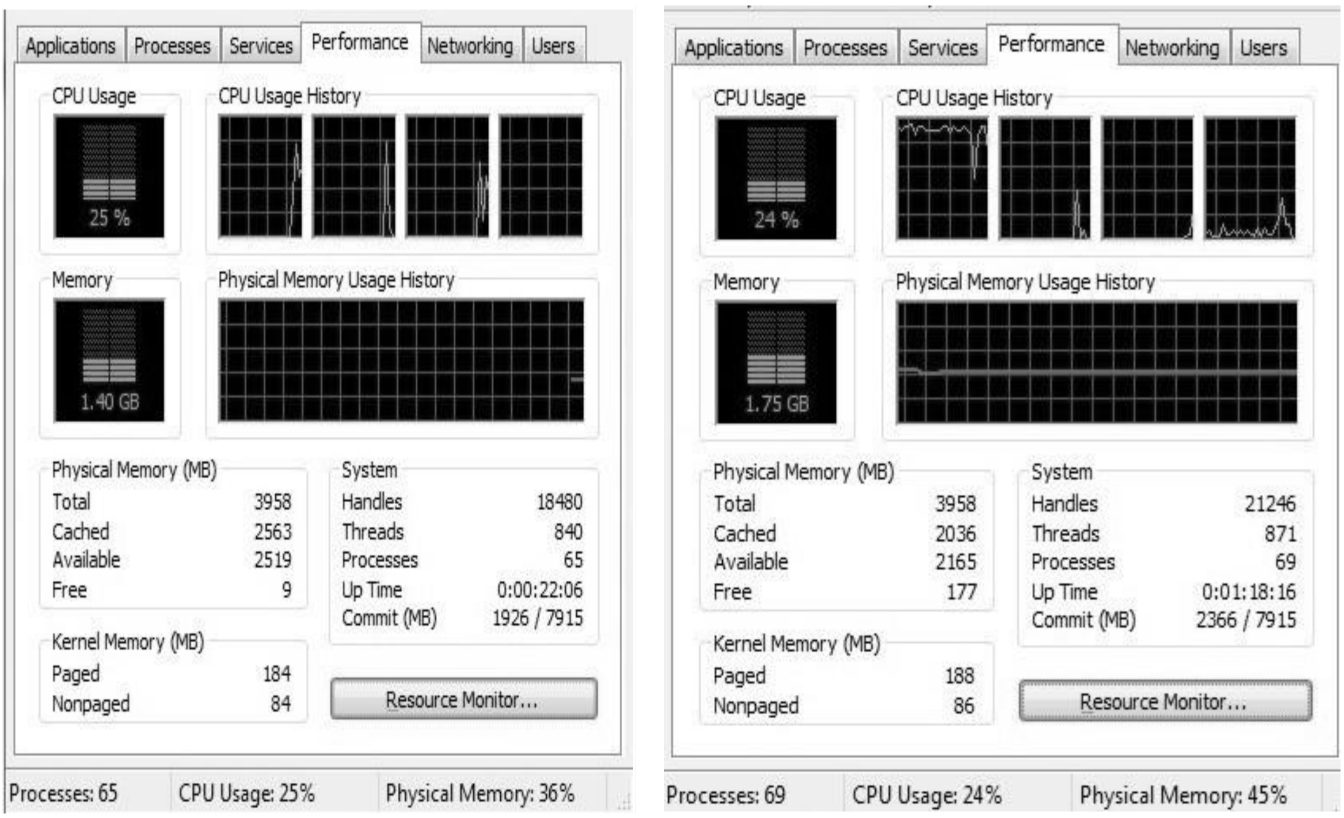

Figure 1 CPU performances (small-scale (left) and large-scale (right))

3.0 Solution Approach

This section discusses the solution approach of the proposed model and includes an introduction to fuzzy set theory, fundamental definitions and the solution approach of the proposed model.

3.1 Fuzzy Set Theory

Fuzzy set theory (Zadeh, 1965, 1973) is applied to model the decision making processes based on imprecise and vague information such as the judgment of decision makers. Qualitative features are represented by the means of linguistic variables. These features are expressed qualitatively by linguistic terms and quantitatively by fuzzy sets in the discourse universe and the membership function. Operations between linguistic variables involve the following concept.

3.2 Fundamental definitions

3.2.1 Fuzzy Set

A fuzzy set $\tilde{A}$ in X is defined by

$$\tilde{A} = \{ x \, . \, \mu_A(x) \}. \quad x \in X$$

Where $\mu_A(x) : X \rightarrow [\,0\,.\,1\,]$ is the membership function of $\tilde{A}$ and $\mu_A(x)$ is the degree of pertinence of x in $\tilde{A}$. If $\mu_A(x)$ equals 0, x does not belong to the fuzzy set $\tilde{A}$. If $\mu_A(x)$ equals 1, x completely belongs to the fuzzy set $\tilde{A}$. However, unlike classical set theory, if $\mu_A(x)$ has a value between 0 and 1, it partially belongs to the fuzzy set $\tilde{A}$. That is, the pertinence of x is true with a degree of membership given by $\mu_A(x)$(Zadeh, 1965; Zimmermann, 1991).

2.2.2 Fuzzy Numbers

A fuzzy number is a fuzzy set where the membership function of the conditions of normality

$$\sup \widetilde{A} \; [x]_{x \in X} \; = 1 \tag{25}$$

and of convexity

$$\tilde{A} \; [\, \lambda x_1 + (1 - \lambda) x_2 \geq \min[A(x_1) \, . \, A(x_2) \,]] \tag{26}$$

For all $x_1, x_2 \in X$ and all $\lambda \in [0,1]$. The triangular fuzzy number is commonly used in decision making due to its intuitive membership function, $\mu_A(x)$, given by

$$\mu_A(x) = \begin{cases} 0 & x < l, \\ \dfrac{x - l}{m - l} & a \leq x \leq m, \\ \dfrac{u - x}{u - m} & m \leq x \leq u, \\ 0 & x > u, \end{cases} \tag{27}$$

Where *l*, *m*, and *u* are real numbers with $l < m < u$ (Figure 2a). Outside the interval $[l, u]$, the pertinence degree is null, and m represents the point where the pertinence degree is maximum. Trapezoidal fuzzy numbers are also frequently applied in decision making processes, as illustrated in Figure 2b (Zimmermann, 1991; Kahraman, 2008).

A fuzzy number can be defined in different forms considering the nature of the problem in hand. According to Markowski and Mannan (2008), any shape of membership function could be applied in reliability analysis of engineering systems. Among different shapes of membership functions, the triangular and trapezoidal shapes are widely used. Arithmetic operations on fuzzy numbers are performed following the fuzzy set theory rules and the extension principle (Zadeh,

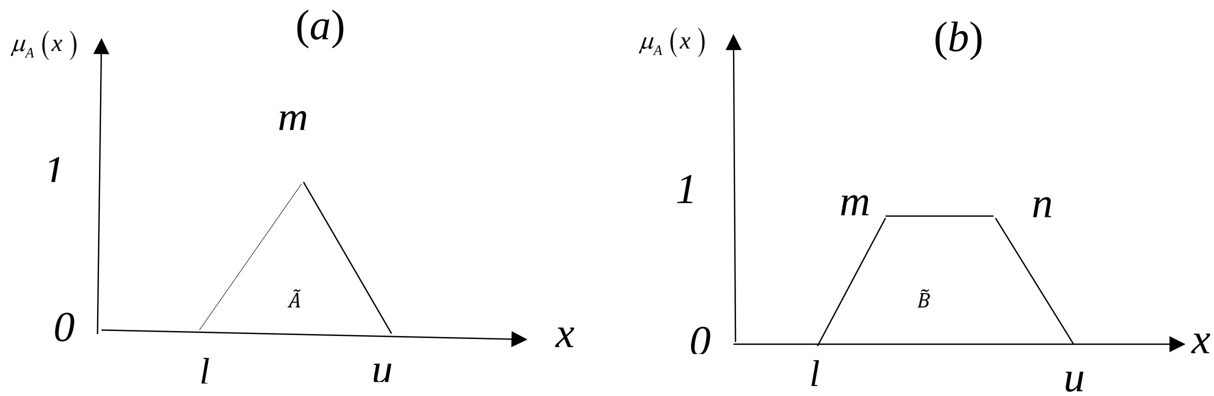

Figure 2 Triangular and trapezoidal fuzzy

1975; Zimmermann, 2011). Shu, Cheng, and Chang (2006), Bowles and Pelaez (1995), Liang and Wang (1993), and Misra and Weber (1990) have described different arithmetic operations on fuzzy numbers based on the extension principle.

The first step to solve the model is to convert it to an equivalent crisp one. Some of the parameters of the objective functions and constraints are fuzzy numbers. Therefore, the problem includes imprecise objectives and imprecise constraints, simultaneously. So, the problem is considered to be a possibilistic programming problem. The grade of possibility indicates the subjective or objective degree of the occurrence of an event. It is important to realize this distinction while modelling fuzziness/imprecision in mathematical programming problems. Possibilistic decision making models provide an important capability in handling practical decision making problems. Madadi and Wong (2014) suggested a solution procedure to solve this type of fuzzy multi-objective, multi-product and multi-period models. According to this study, two steps should be considered to solve the problem. Firstly, treating imprecise objective functions (optimality issue) and secondly treating imprecise constraints (feasibility issue). Supplier selection problems are application specific. That is, the appropriate constraints and the relative importance of the objectives vary with the problem setting. Therefore, it is not possible to model a single specific functional form to be appropriate for all potential scenarios. Triangular fuzzy numbers are considered in this research. Therefore, as these fuzzy numbers are included in objective functions, to treat the imprecise objective function, it is possible to express them using a triangular possibilistic distribution. In general, a multi-objective problem may be formulated as follows:

$$
\begin{aligned}
Min \quad & Z=\left[Z_1(x,y),Z_2(x,y),...,Z_p(x,y)\right] \\
s.t. \quad & f_i(x,y)\geq b_i \qquad i=1,...,n \\
& g_i(x,y)\geq b_j \qquad j=1,...,m \\
& x\geq 0 \\
& y\in(0,1)
\end{aligned} \tag{28}
$$

Where $Z_1(x,y),Z_2(x,y),...,Z_p(x,y)$ are the objective functions to be optimized. $f_i(x,y)\geq b_i$ and $g_i(x,y)\geq b$ are the set of system constraints. The proposed model of this research follows this formulation. Considering three different cases in the fuzzy environment, each Z_i is divided into three forms to represent pessimistic, most likely, and optimistic values.

To obtain $Z^1{}_i$, $Z^2{}_i$ and $Z^3{}_i$, all fuzzy parameters in each objective function of Z_i are set at their pessimistic, most likely, and optimistic values, respectively. Therefore, all 6 objective functions should be expressed in the form of $Z_i=(Z^1{}_i,Z^2{}_i,Z^3{}_i)$. Equations 29 to 46 shows the pessimistic, most likely, and optimistic values of objective functions. Equation 29, 30 and 31 are pessimistic, most likely, and optimistic values of the first objective function.

$$
\begin{aligned}
Z^1{}_1 = & \sum_n\sum_j\sum_t X_{njt}P^1{}_{njt}+\sum_j\sum_t O^1{}_jY_{jt}+\sum_t\sum_n\sum_q h^1{}_{qn}LV_{qnt}+ \\
& \sum_{\mathrm{A}}\sum_t\sum_{j\in B}\sum_{b\in B}\sum_n dd_{bj}TR_{bjn}F_{bj\alpha t}X_{njt}+\sum_t\sum_q\sum_n \Pi^1{}_{qn}L_{qnt}+\sum_t\sum_n\sum_q\sum_j P^1{}_{njt}Z_{jqnt}
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
Z^2{}_1 = & \sum_n\sum_j\sum_t X_{njt}P^2{}_{njt}+\sum_j\sum_t O^2{}_jY_{jt}+\sum_t\sum_n\sum_q h^2{}_{qn}LV_{qnt}+ \\
& \sum_{\mathrm{A}}\sum_t\sum_{j\in B}\sum_{b\in B}\sum_n dd_{bj}TR_{bjn}F_{bj\alpha t}X_{njt}+\sum_t\sum_q\sum_n \Pi^2{}_{qn}L_{qnt}+\sum_t\sum_n\sum_q\sum_j P^2{}_{njt}Z_{jqnt}
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
Z^3{}_1 = & \sum_n\sum_j\sum_t X_{njt}P^3{}_{njt}+\sum_j\sum_t O^3{}_jY_{jt}+\sum_t\sum_n\sum_q h^3{}_{qn}LV_{qnt}+ \\
& \sum_{\mathrm{A}}\sum_t\sum_{j\in B}\sum_{b\in B}\sum_n dd_{bj}TR_{bjn}F_{bj\alpha t}X_{njt}+\sum_t\sum_q\sum_n \Pi^3{}_{qn}L_{qnt}+\sum_t\sum_n\sum_q\sum_j P^3{}_{njt}Z_{jqnt}
\end{aligned} \tag{31}
$$

Using the same approach which is explained above, Equations 32, 33 and 34 show the pessimistic, most likely, and optimistic values of the second objective function.

$$Z^1{}_2=\sum_n\sum_j\sum_t X_{njt}.W^1{}_{nj} \tag{32}$$

$$Z^2{}_2=\sum_n\sum_j\sum_t X_{njt}.W^2{}_{nj} \tag{33}$$

$$Z^{3}{}_{2} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.W^{3}{}_{nj} \tag{34}$$

Similarly, Equation 35, 36 and 37 are pessimistic, most likely, and optimistic values of the third objective function.

$$Z^{1}{}_{3} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.E^{1}{}_{nj} \tag{35}$$

$$Z^{2}{}_{3} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.E^{2}{}_{nj} \tag{36}$$

$$Z^{3}{}_{3} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.E^{3}{}_{nj} \tag{37}$$

Following the same steps, Equation 38, 39 and 40 are pessimistic, most likely, and optimistic values of the fourth objective function.

$$Z^{1}{}_{4} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.SO^{1}{}_{nj} \tag{38}$$

$$Z^{2}{}_{4} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.SO^{2}{}_{nj} \tag{39}$$

$$Z^{3}{}_{4} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.SO^{3}{}_{nj} \tag{40}$$

The fifth objective function can be converted to pessimistic, most likely, and optimistic values. Equation 41, 42 and 43 show the fifth objective function in these three conditions.

$$Z^{1}{}_{5} = \sum_{n}\sum_{j}\sum_{t}\sum_{r} X_{njt}.R_{rnt} \tag{41}$$

$$Z^{2}{}_{5} = \sum_{n}\sum_{j}\sum_{t}\sum_{r} X_{njt}.R_{rnt} \tag{42}$$

$$Z^{3}{}_{5} = \sum_{n}\sum_{j}\sum_{t}\sum_{r} X_{njt}.R_{rnt} \tag{43}$$

Lastly, Equation 44, 45 and 46 are pessimistic, most likely, and optimistic values of the sixth objective function.

$$Z^{1}{}_{6} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.(I^{1}P^{1}{}_{njt} + P^{1}{}_{njt}) + \sum_{j}\sum_{t}(I^{1}O^{1}{}_{j} + O^{1}{}_{j}).Y_{jt} \tag{44}$$

$$Z^{2}{}_{6} = \sum_{n}\sum_{j}\sum_{t} X_{njt}.(I^{2}P^{2}{}_{njt} + P^{2}{}_{njt}) + \sum_{j}\sum_{t}(I^{2}O^{2}{}_{j} + O^{2}{}_{j}).Y_{jt} \tag{45}$$

$$Z^3{}_6 = \sum_n \sum_j \sum_t X_{njt}.(I^3 P^3{}_{njt} + P^3{}_{njt}) + \sum_j \sum_t (I^3 O^3{}_j + O^3{}_j).Y_{jt}$$
(46)

The second step is to treat imprecise constraints. This section addresses the issue of solution feasibility. To deal with this issue, Madadi and Wong (2014) applied the approach of Jimenez et al. (2007), where all fuzzy constraints are converted to their equivalent crisp ones as follows.

$$\sum_j \sum_{k=1}^{t} X_{njk} \geq (1-\alpha)\frac{\sum_{k=1}^{t} D^1{}_{nk} + \sum_{k=1}^{t} D^3{}_{nk}}{2} + \alpha \frac{\sum_{k=1}^{t} D^1{}_{nk} + \sum_{k=1}^{t} D^2{}_{nk}}{2}, \forall n \in N \quad (47)$$

$$X_{njt} \leq (1-\alpha)\frac{C^2{}_{nj} + C^3{}_{nj}}{2} + \alpha \frac{C^1{}_{nj} + C^2{}_{nj}}{2} \quad (48)$$
$\forall n \in N,, \forall j \in J, \forall t \in T$

$$X_{njt} \leq (1-\alpha)\frac{(\sum_{k=1}^{t} D^2{}_{nk}).Y_{jt} + (\sum_{k=1}^{t} D^3{}_{nk}).Y_{jt}}{2} + \alpha \frac{(\sum_{k=1}^{t} D^1{}_{nk}).Y_{jt} + (\sum_{k=1}^{t} D^2{}_{nk}).Y_{jt}}{2} \quad (49)$$
$\forall n \in N,, \forall j \in J, \forall t \in T$

$$\sum_j \sum_{t=1}^{T} X_{njt} \geq (\frac{\alpha}{2})\frac{\sum_{t=1}^{T} D^2{}_{nt} + \sum_{t=1}^{T} D^3{}_{nt}}{2} + (1-\frac{\alpha}{2})\frac{\sum_{t=1}^{T} D^1{}_{nt} + \sum_{t=1}^{T} D^2{}_{nt}}{2}$$
$$\sum_j \sum_{t=1}^{T} X_{njt} \leq (1-\frac{\alpha}{2})\frac{\sum_{t=1}^{T} D^2{}_{nt} + \sum_{t=1}^{T} D^3{}_{nt}}{2} + (\frac{\alpha}{2})\frac{\sum_{t=1}^{T} D^1{}_{nt} + \sum_{t=1}^{T} D^2{}_{nt}}{2} \quad (50)$$

$\forall n \in N$

$$X_{njt} \geq 0 \quad (51)$$

$$Y_{jt} = 0 or 1 \quad (52)$$

$$\sum_t \sum_n \sum_r [(1-\alpha)\frac{a^2{}_{rnt} + a^3{}_{rnt}}{2} + \alpha \frac{a^1{}_{rnt} + a^2{}_{rnt}}{2}] R_{rnt} \leq$$
$$\sum_t \sum_n \sum_r PR_{rnt} [(1-\alpha)\frac{a^2{}_{rnt} + a^3{}_{rnt}}{2} + \alpha \frac{a^1{}_{rnt} + a^2{}_{rnt}}{2}] \quad (53)$$

$$R_{rnt} = 0, or, 1 \tag{54}$$

$$\sum_n \sum_j \sum_t X_{njt}.(1-\alpha)[\frac{(P_{njt}^2 I^2 + P_{njt}^2) + (P_{njt}^3 I^3 + P_{njt}^3)}{2}] + \sum_n \sum_j \sum_t X_{njt}.\alpha[\frac{(P_{njt}^1 I^1 + P_{njt}^1) + (P_{njt}^2 I^2 + P_{njt}^2)}{2}] \geq$$
$$\sum_{k=1}^{t} \alpha(\frac{D_{nk}^2 + D_{nk}^3}{2}) + \sum_{k=1}^{t} \alpha(\frac{D_{nk}^1 + D_{nk}^2}{2}) \tag{55}$$

$$\sum_j \sum_t (1-\alpha)[\frac{(O_j^2 I^2 + O_j^2) + (O_j^3 I^3 + O_j^3)}{2}]Y_{jt} + \sum_j \sum_t \alpha[\frac{(O_j^1 I^1 + O_j^1) + (O_j^2 I^2 + O_j^2)}{2}]Y_{jt} \geq 0 \tag{56}$$

$$\sum_n \sum_j \sum_t X_{njt}[(1-\alpha)\frac{E^2{}_{nj} + E^3{}_{nj}}{2} + \alpha \frac{E^1{}_{nj} + E^2{}_{nj}}{2}] \leq 50$$

(57)

$$LV_{nqt} = LV_{nq(t-1)} + \sum_j Z_{jqnt} - L_{qnt} + L_{qn(t-1)}$$
$$\forall n \in N, \forall q \in Q, \forall t \in T \tag{58}$$

$$\sum_{j \in B} F_{bj\alpha t} - \sum_{j \in B} F_{jb\alpha t} = 0$$
$$\forall \alpha \in \mathrm{A}, \forall b \in B, \forall t \in T \tag{59}$$

$$L_{qnt}, LV_{nqt}, Z_{jqnt} \geq 0$$
$$\forall n \in N, \forall t \in T, \forall q \in Q, \forall j \in J \tag{60}$$

$$LV_{nq0} = 0, L_{qn0} = 0 \tag{61}$$

$$F_{bj\alpha t} = 0 \tag{62}$$

$$L_{qnt}, LV_{nqt}, Z_{jqnt} \geq 0$$
$$\forall n \in N, \forall t \in T, \forall q \in Q, \forall j \in J \tag{63}$$

Where, α is the feasibility degree of the constraints. This value is assigned by the decision maker considering the acceptable risk of violating the constraints in the solution (Wang and Fang, 2001; Lotfi and Torabi, 2011). This study considers 0.8 for the parameter α. γ represents the decision maker's optimism. This value can vary between zero and one (Yaghin, Torabi, and Ghomi 2012). This research assigns a value of 0.3 to parameter γ.

4.0 Initial Results

The proposed mathematical model is formulated in a fuzzy environment. A fuzzy goal programming approach is applied to solve the crisp model. According to this approach, the multi-objective model should be converted to an equivalent single-objective one. The max-min operator of Bellman and Zadeh (1970) is applied to convert the model to a single objective formulation. The multi-objective crisp model is shown as follows. Equation 64 gives the objective function of the crisp model. Equations 65-82, 11, 12, 14, 18 and 19-23 are the constraints of the final multi-objective crisp model.

$$Max \;\; \varphi \tag{64}$$

Subject to $\varphi \le \mu_{EV_{0.3}(Z1)}$ (65)

$$\varphi \le \mu_{EV_{0.3}(Z2)} \tag{66}$$

$$\varphi \le \mu_{EV_{0.3}(Z3)} \tag{67}$$

$$\varphi \le \mu_{EV_{0.3}(Z4)} \tag{68}$$

$$\varphi \le \mu_{EV_{0.3}(Z5)} \tag{69}$$

$$\varphi \le \mu_{EV_{0.3}(Z6)} \tag{70}$$

$$\mu_{EV_{0.3}(Z1)} = \frac{0.75\mathrm{E}+11 - \mathrm{EV}(\mathrm{Z1})}{0.75\mathrm{E}+11 - 0.5499079\mathrm{E}+11} \tag{71}$$

$$\mu_{EV_{0.3}(Z2)} = \frac{\mathrm{EV}(\mathrm{Z2}) - 1412650}{6588708 - 1412650} \tag{72}$$

$$\mu_{EV_{0.3}(Z3)} = \frac{\mathrm{EV}(\mathrm{Z3}) - 2174141}{4727752 - 2174141} \tag{73}$$

$$\mu_{EV_{0.3}(Z4)} = \frac{\mathrm{EV}(\mathrm{Z4}) - 1465661}{5347366 - 1465661} \tag{74}$$

$$\mu_{EV_{0.3}(Z5)} = \frac{0.1890264\mathrm{E}+08 - \mathrm{Z5}}{0.1890264\mathrm{E}+08 - 0} \tag{75}$$

$$\mu_{EV_{0.3}(Z6)} = \frac{0.6320292\mathrm{E}+12 - \mathrm{EV}(\mathrm{Z6})}{0.6320292\mathrm{E}+12 - 0.6048987\mathrm{E}+12} \tag{76}$$

$$EV_{0.3}(Z1)=(1-0.3).\frac{Z_1^1+Z_1^2}{2}+(0.3).\frac{Z_1^2+Z_1^3}{2} \quad (77)$$

$$EV_{0.3}(Z2)=(1-0.3).\frac{Z_2^1+Z_2^2}{2}+(0.3).\frac{Z_2^2+Z_2^3}{2} \quad (78)$$

$$EV_{0.3}(Z3)=(1-0.3).\frac{Z_3^1+Z_3^2}{2}+(0.3).\frac{Z_3^2+Z_3^3}{2} \quad (79)$$

$$EV_{0.3}(Z4)=(1-0.3).\frac{Z_4^1+Z_4^2}{2}+(0.3).\frac{Z_4^2+Z_4^3}{2} \quad (80)$$

$$EV_{0.3}(Z5)=(1-0.3).\frac{Z_5^1+Z_5^2}{2}+(0.3).\frac{Z_5^2+Z_5^3}{2} \quad (81)$$

$$EV_{0.3}(Z6)=(1-0.3).\frac{Z_6^1+Z_6^2}{2}+(0.3).\frac{Z_6^2+Z_6^3}{2} \quad (82)$$

Equations 11, 12, 14 ,18

Equations 19-23

Positive and negative ideal solutions of the objective function Zi are required to solve the model. Therefore, to obtain the negative ideal solution of an objective function, one of the following equations should be applied:

$$Z_i^{NIS}=\max\left\{z_i(v_j*;\mathrm{i}\neq \mathrm{j}\right\} \quad (83)$$

In case of having a minimization objective

$$Z_i^{NIS}=\min\left\{z_i(v_j*;\mathrm{i}\neq \mathrm{j}\right\} \quad (84)$$

In case of having a maximization approach objective

Z_i^{PIS} and Z_i^{NIS} indices are applied to show the positive and negative ideal solutions of objective function Zi. The proposed methodology of Abd. El-Wahed and Lee (2006) is applied in this research. According to this study, the Z_i^{PIS} is achieved by solving the model based on a single objective of Zi and ignoring other objective functions. In addition, v_j* is the positive ideal solution of objective function Z_i. The proposed model is coded and solved in LINGO 11 software. Table 4 shows the payoff table applied to obtain the positive and negative ideal solutions of the case study.

Table 4 Positive and negative ideal solutions (Payoff Table)

Objective Function	PIS	NIS
Ev(Z1)	0.5499079E+11	0.75E+11
Ev(Z2)	6588708	1412650
Ev(Z3)	4727752	2174141
Ev(Z4)	5347366	1465661
Z5	0	0.1890264E+08
Ev(Z6)	0.6048987E+12	0.6320292E+12

By applying the PISs and NISs shown in Table 4, the membership functions are formulated in Equations 85-90.

$$\mu_{Ev(Z1)} = \begin{cases} 1 & EV(Z1) \leq 0.5499079E+11 \\ \dfrac{0.75E+11 - EV(Z1)}{0.75E+11 - 0.5499079E+11} & 0.5499079E+11 \leq EV(Z1) \leq 0.75E+11 \\ 0 & EV(Z1) \geq 0.75E+11 \end{cases} \tag{85}$$

$$\mu_{Ev(Z2)} = \begin{cases} 1 & EV(Z2) \leq 1412650 \\ \dfrac{EV(Z2) - 1412650}{6588708 - 1412650} & 1412650 \leq EV(Z2) \leq 6588708 \\ 0 & EV(Z2) \geq 6588708 \end{cases} \tag{86}$$

$$\mu_{Ev(Z3)} = \begin{cases} 1 & EV(Z3) \leq 2174141 \\ \dfrac{EV(Z3) - 2174141}{4727752 - 2174141} & 2174141 \leq EV(Z3) \leq 4727752 \\ 0 & EV(Z3) \geq 4727752 \end{cases} \tag{87}$$

$$\mu_{Ev(Z4)} = \begin{cases} 1 & EV(Z4) \leq 1465661 \\ \dfrac{EV(Z4) - 1465661}{5347366 - 1465661} & 1465661 \leq EV(Z4) \leq 5347366 \\ 0 & EV(Z4) \geq 5347366 \end{cases}$$

(88)

$$\mu_{Ev\,(Z5)} = \begin{cases} 1 & EV\,(Z5) \le 0 \\ \dfrac{0.1890264E+8 - Z5}{0.1890264E+8 - 0} & 0 \le EV\,(Z5) \le 0.1890264E+8 \\ 0 & EV\,(Z5) \ge 0.1890264E+8 \end{cases}$$

(89)

$$\mu_{Ev\,(Z6)} = \begin{cases} 1 & EV\,(Z6) \le 0.6048987E+12 \\ \dfrac{0.6320292E+12 - EV\,(Z5)}{0.6320292E+12 - 0.6048987E+12} & 0.6048987E+12 \le EV\,(Z6) \le 0.6320292E+12 \\ 0 & EV\,(Z6) \ge 0.6320292E+12 \end{cases}$$

(90)

In addition to Equations 85-90, the membership functions of all objective functions are shown in Figure 3 to 8.

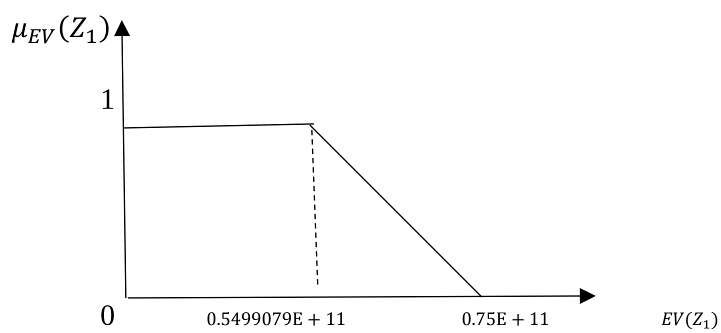

Figure 3 Membership function for Objective Function 1

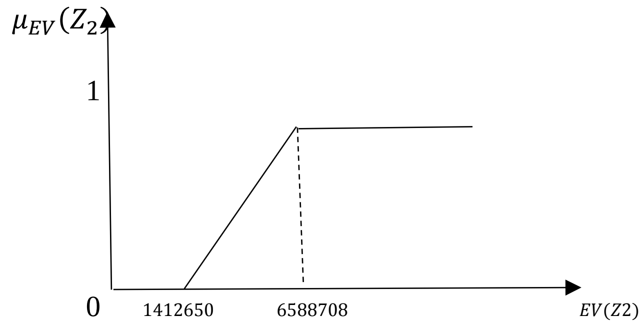

Figure 4 Membership function for Objective Function 2

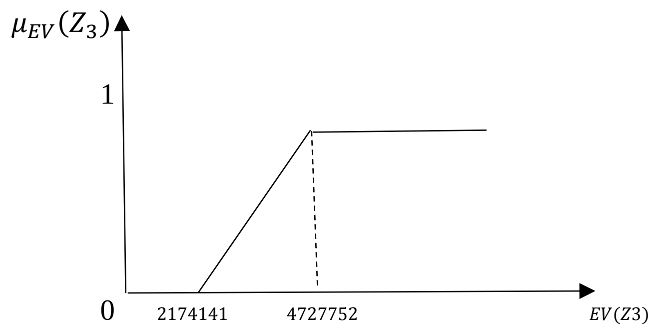

Figure 5 Membership function for Objective Function 3

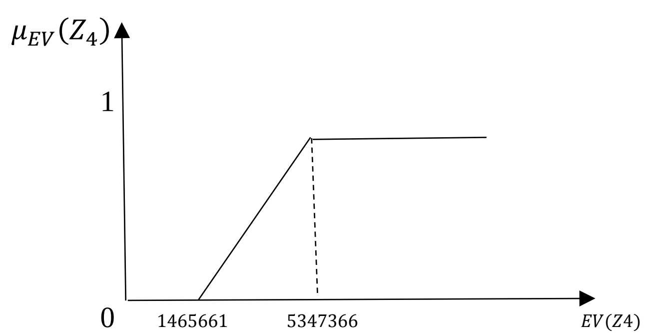

Figure 6 Membership function for Objective Function 4

Figure 7 Membership function for Objective Function 5

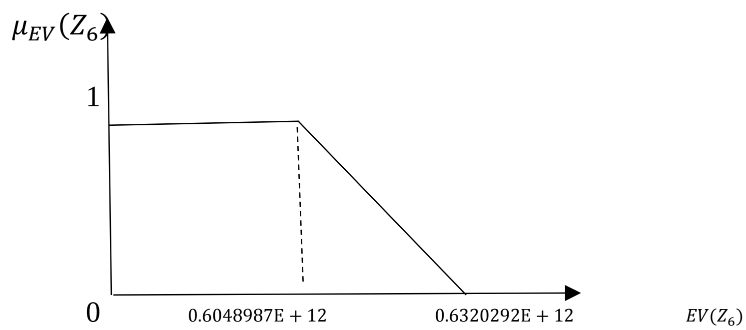

Figure 8 Membership function for Objective Function 6

5.0 Sensitivity Analysis

This section discusses the sensitivity analysis of the proposed mathematical model. It investigates how the output uncertainty of a proposed mathematical model can be allocated to different uncertainties in model inputs. Figure 9 displays the model variation with regard to pairs of objective functions in the absence of other objective functions. The top left figure displays the sensitivity analysis of the first objective function with regard to the second objective function. According to this figure, the value of the first objective function is increased when the second objective function is omitted from the model. Figure 10 displays a different analysis. This figure shows the sensitivity analysis of the first objective function ignoring other objective functions, and shows that the value of the first objective function is decreased when other objective functions are omitted from the model.

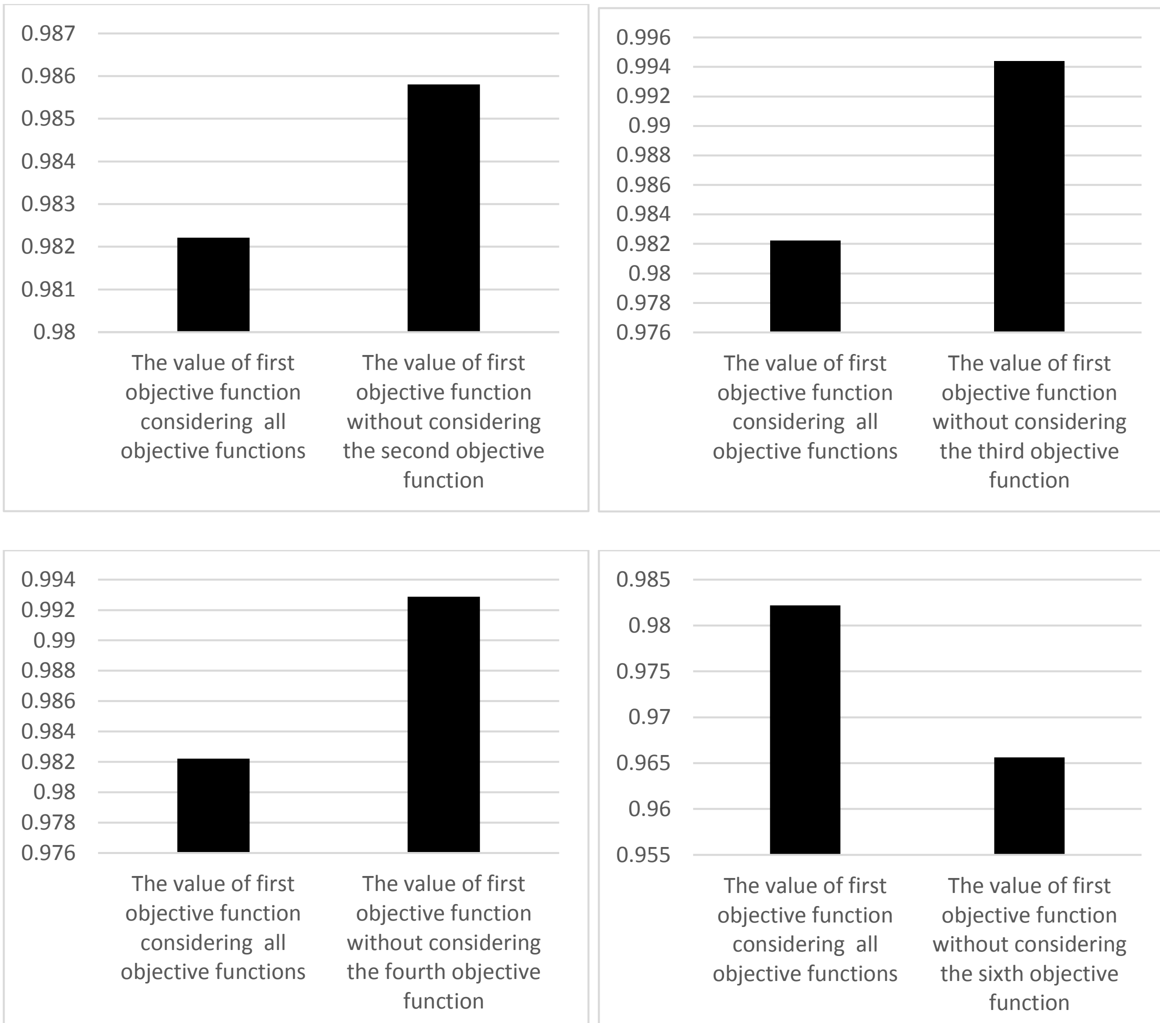

Figure 9 Sensitivity analysis of objective functions with regard to the second objective function

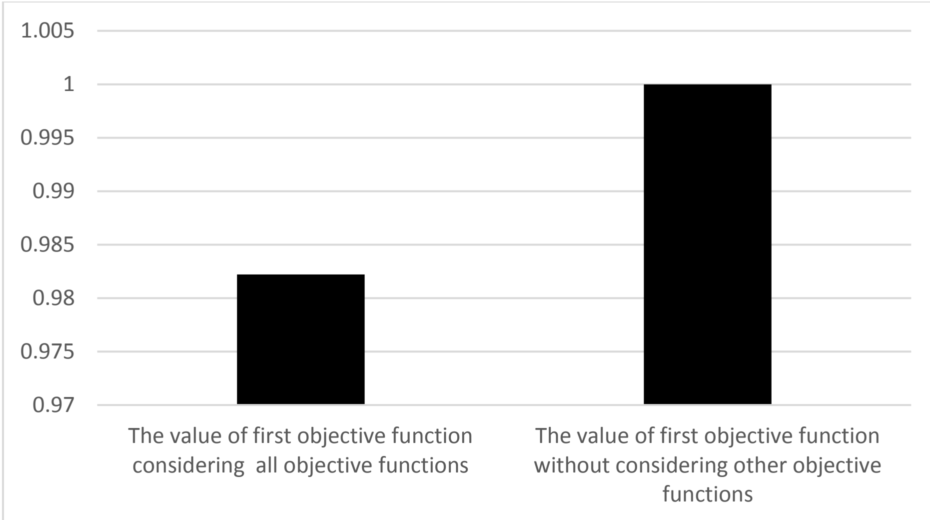

Figure 10 Sensitivity analysis of the first objective ignoring other objective functions

In addition to sensitivity analysis conducted on objective functions, Figures 11 and 12 display the sensitivity analysis for parameters α and γ. α is the feasibility degree of the constraints. This value is assigned by the decision maker considering the acceptable risk of violating the constraints imposed. This study considers the value of 0.8 for parameter α. γ is the decision maker's optimism. This value can be varied between zero and one. This research assigns a value of 0.3 to γ. The result change when the decision maker assigns different values of these two parameters. Different values of first objective functions are investigated with different values of α and γ parameters. This sensitivity analysis is shown in Figure 11. The sensitivity analysis of ϕ with different gamma values is shown in Figure 12.

According to Figure 9, the value of the first objective function (cost minimization) is increased in the absence of other objective functions. This is mainly due to the fact that some constraints of the model are removed by omitting the objective function. Therefore, the solution process takes place in a different feasible area. Managers, practitioners and researchers who are interested in seeing the effect of different objective functions on cost, can apply this method and investigate the significance of each objective function. An important implication of the sensitivity analysis is that removing some objective functions can affect the results, and managers may be interested in investigating different scenarios when dealing with mathematical models. The sensitivity analysis in this research helps them to check the variation of the results in the presence or absence of different objective functions. Finally, there are some parameters which are set by the decision makers. Different values of these parameters can affect the results and should be carefully investigated.

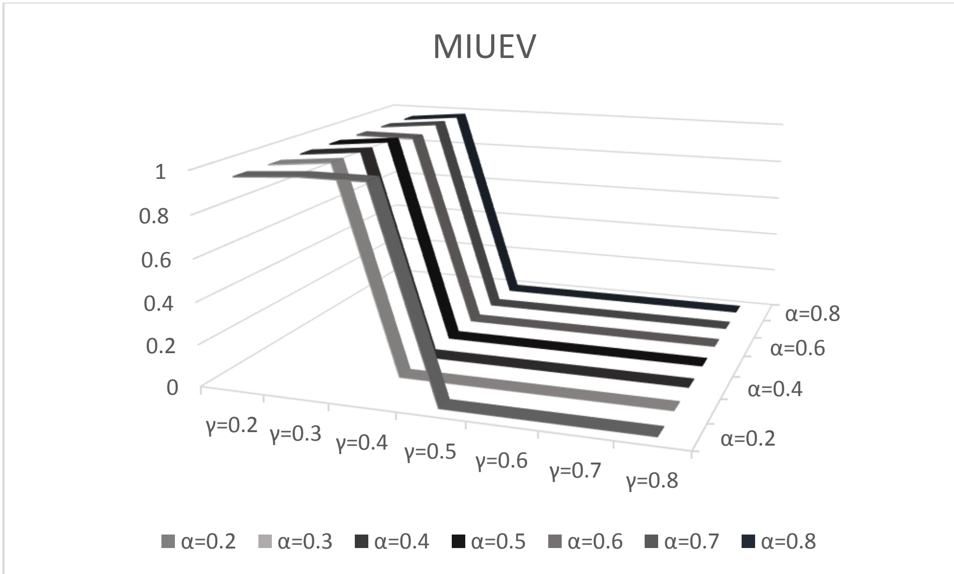

Figure 11 Sensitivity analysis of first objective function with different alpha and gamma values

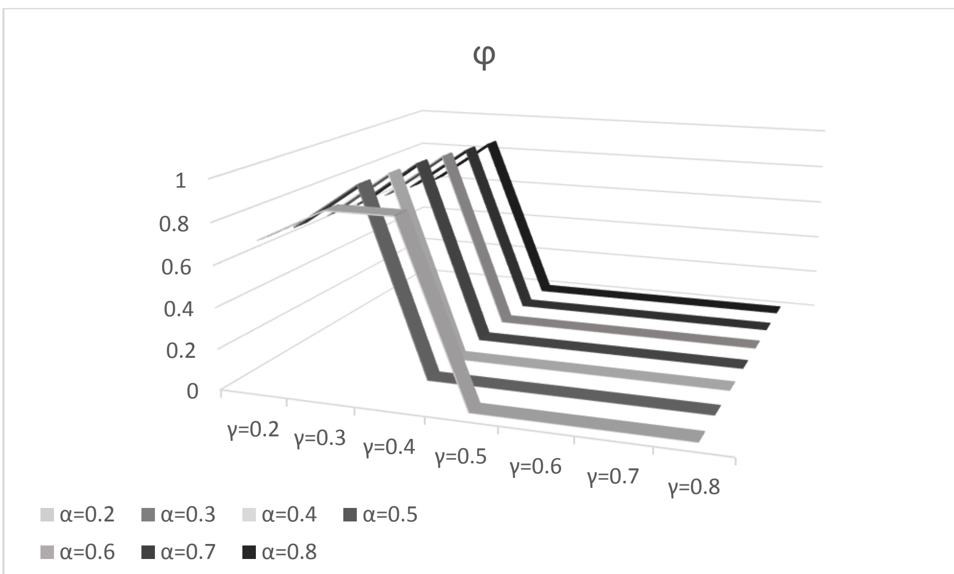

Figure 12 Sensitivity analysis of ϕ with different gamma values

6.0 Generalizing the model to a large scale problem

As is it important for any mathematical model to be generalized, the model developed in this research was extended a large scale. The model found a feasible solution. The following shows the results of the large scale application. As with the main application, the large scale application found a feasible solution. This satisfies the generalization concerns. In other words, the model can be generalized to apply to any sustainable supplier selection and order allocation problem. As the main output of the model is order allocation quantities, the other results are omitted from this section to save space.

Table 5 Order allocation results from the large scale problem

X(1, 1, 1)	93200.00	X(3, 1, 1)	1490400	X(5, 1, 1)	0	X(7, 1, 1)	975200
X(1, 1, 2)	0	X(3, 1, 2)	0	X(5, 1, 2)	0	X(7, 1, 2)	0
X(1, 1, 3)	0	X(3, 1, 3)	0	X(5, 1, 3)	0	X(7, 1, 3)	1033800
X(1, 1, 4)	0	X(3, 1, 4)	0	X(5, 1, 4)	0	X(7, 1, 4)	0
X(1, 2, 1)	0	X(3, 2, 1)	0	X(5, 2, 1)	809600	X(7, 2, 1)	0
X(1, 2, 2)	0	X(3, 2, 2)	0	X(5, 2, 2)	2101000	X(7, 2, 2)	0
X(1, 2, 3)	0	X(3, 2, 3)	0	X(5, 2, 3)	0	X(7, 2, 3)	0
X(1, 2, 4)	0	X(3, 2, 4)	0	X(5, 2, 4)	0	X(7, 2, 4)	0
X(1, 3, 1)	0	X(3, 3, 1)	0	X(5, 3, 1)	0	X(7, 3, 1)	0
X(1, 3, 2)	0	X(3, 3, 2)	0	X(5, 3, 2)	0	X(7, 3, 2)	0
X(1, 3, 3)	0	X(3, 3, 3)	0	X(5, 3, 3)	0	X(7, 3, 3)	0
X(1, 3, 4)	0	X(3, 3, 4)	0	X(5, 3, 4)	0	X(7, 3, 4)	0
X(1, 4, 1)	2190200	X(3, 4, 1)	0	X(5, 4, 1)	0	X(7, 4, 1)	0
X(1, 4, 2)	0	X(3, 4, 2)	0	X(5, 4, 2)	0	X(7, 4, 2)	0
X(1, 4, 3)	0	X(3, 4, 3)	322600	X(5, 4, 3)	0	X(7, 4, 3)	0
X(1, 4, 4)	0	X(3, 4, 4)	0	X(5, 4, 4)	0	X(7, 4, 4)	0
X(2, 1, 1)	74400.00	X(4, 1, 1)	0	X(6, 1, 1)	0	X(8, 1, 1)	0
X(2, 1, 2)	0	X(4, 1, 2)	0	X(6, 1, 2)	351400	X(8, 1, 2)	0
X(2, 1, 3)	0	X(4, 1, 3)	0	X(6, 1, 3)	0	X(8, 1, 3)	0
X(2, 1, 4)	0	X(4, 1, 4)	0	X(6, 1, 4)	0	X(8, 1, 4)	0
X(2, 2, 1)	0	X(4, 2, 1)	478400	X(6, 2, 1)	0	X(8, 2, 1)	0
X(2, 2, 2)	0	X(4, 2, 2)	0	X(6, 2, 2)	0	X(8, 2, 2)	0
X(2, 2, 3)	0	X(4, 2, 3)	147913.7	X(6, 2, 3)	0	X(8, 2, 3)	92000
X(2, 2, 4)	0	X(4, 2, 4)	0	X(6, 2, 4)	0	X(8, 2, 4)	174400
X(2, 3, 1)	0	X(4, 3, 1)	0	X(6, 3, 1)	128800	X(8, 3, 1)	0
X(2, 3, 2)	0	X(4, 3, 2)	0	X(6, 3, 2)	0	X(8, 3, 2)	0
X(2, 3, 3)	0	X(4, 3, 3)	0	X(6, 3, 3)	0	X(8, 3, 3)	0
X(2, 3, 4)	0	X(4, 3, 4)	0	X(6, 3, 4)	0	X(8, 3, 4)	0
X(2, 4, 1)	1748400	X(4, 4, 1)	0	X(6, 4, 1)	0	X(8, 4, 1)	929200
X(2, 4, 2)	0	X(4, 4, 2)	0	X(6, 4, 2)	0	X(8, 4, 2)	0
X(2, 4, 3)	0	X(4, 4, 3)	853486.3	X(6, 4, 3)	0	X(8, 4, 3)	0
X(2, 4, 4)	0	X(4, 4, 4)	0	X(6, 4, 4)	0	X(8, 4, 4)	0

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